The Hillipop likelihood and the $A_L$ parameter using Planck data

F. Couchot, S. Henrot-Versillé, O. Perdereau, S. Plaszczynski, B. Rouillé d'Orfeuil, M. Spinelli, M. Tristram

Appetizer: profile likelihoods
- Hillipop & $\Lambda$CDM
- The $A_L$ test

Based on: arXiv:1510.07600 (submitted to A&A)
**Appetizer: Profile Likelihoods**

Parameter estimation:

- in Cosmology: **Bayesian inference** using Monte Carlo Markov Chains to explore the likelihood function

- in Particle Physics(*): **Profile likelihoods**

\[ \chi^2 = -2 \ln(L) \]

1- Fix a value of \( \theta: \theta_1 \)
2- Find the minimum of the \( \chi^2 \), fitting over all the other dimensions (\( \chi^2(\theta_1) \))
3- Keep the corresponding value of the \( \chi^2 \)
4- Take another value of \( \theta=\theta_2 \), go on with Point 1-

Planck, A&A 566, A54 (2014)&arXiv:1507.02704 (E5); SHV et al., Class. Qu. Grav. 32 045003

(*) one can also use Neyman CL construction eg. Palanque-Delabrouille et al. arXiv. 1410.7244
Appetizer: Profile Likelihoods

- A potential difference between both methods (beyond philosophical matters) => The so-called Volume Effects: a 2D-example

1/ The Bayesian posterior distribution

Marginal posterior:

\[ P(\theta_1|D) = \int L(\theta_1, \theta_2)p(\theta_1, \theta_2)d\theta_2 \]

=> The mean value of the estimated parameter does not match the one of the best fit

Hopefully not the case in ΛCDM, beware with the extensions!
Appetizer: Profile Likelihoods

- A potential difference between both methods (beyond philosophical matters) => The so-called Volume Effects: a 2D-example

2/ The profile likelihood

Marginal posterior:
\[ P(\theta_1|D) = \int L(\theta_1, \theta_2) p(\theta_1, \theta_2) d\theta_2 \]

Profile likelihood:
\[ L(\theta_1) = \max_{\theta_2} L(\theta_1, \theta_2) \]

\[ \chi^2 = -2\ln(L) \]

=> the best fit determines the mean value of the parameter

\[ (\chi^2 - \chi^2_{\text{min}}) = 1 \] gives the \(1\sigma\) error
1/ The Bayesian posterior

Marginal posterior:

\[ P(\theta_1|D) = \int L(\theta_1, \theta_2)p(\theta_1, \theta_2)d\theta_2 \]

2/ The profile likelihood

Profile likelihood:

\[ L(\theta_1) = \max_{\theta_2} L(\theta_1, \theta_2) \]

=> Use a common software to compare both approaches in the same framework
=> Make sure the results do depend on the physics and not on the statistical approach and make sure we understand the discrepancies if any

CAMEL: camel.in2p3.fr

Cosmological analysis with MCMC + Profile Likelihood based on CLASS(*) for the Boltzmann code

(*) J. Lesgourgues, arXiv:1104.2932
A quick reminder on the released Planck likelihoods (*):

- **lowTEB**: a low-$\ell$ temperature and polarisation map based likelihood
- **Plik**: a gaussian high-$\ell$ temperature likelihood

To better constrain the **SZ sector** (thermal and kinetic), a constraint has been used, derived from ACT data:

$$A^{SZ} = A^{kSZ} + 1.6 A^{tSZ} = 9.5 \mu K^2 \quad \text{and a dispersion} \ 3\mu K^2$$

(*) Planck Collab. arXiv:1507.02704
The Hillipop likelihood

What is Hillipop?

A high-ℓ Planck likelihood (ℓ > 50) based on cross-spectra @ 100, 143 and 217 and 353GHz with 2015 temperature data, alternative likelihood to Plik(*)

The main differences with Plik?

- we use 15 X-Spectra from 6 maps (**)
- intercalibration coefficients are defined at the map level
- we use different masks to further reduce the contamination from foregrounds
- the parameterization of the foregrounds we use templates derived from the Planck data

\[ C_{\ell}^{XY} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^X a_{\ell m}^Y \]

(*) see « Planck 2015 results XI » (appendix D) arXiv:1507.02704
Results of Hillipop on $\Lambda$CDM

Comparison between Hillipop and Plik
- lowTEB data are used at low $\ell$

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<tr>
<th>$\Lambda$CDM par</th>
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<td>$\Omega_b h^2$</td>
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<td>$\Omega_c h^2$</td>
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<td>0.072 ± 0.020</td>
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<td>$\ln(10^{10}A_s)$</td>
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<td>3.068 ± 0.038</td>
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<td>$n_s$</td>
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- $\tau$ and $A_s$ are correlated through $C_\ell \propto A_s e^{-2\tau}$
  and mainly driven by the low $\ell$ data, why so different?
What if we remove lowTEB data?

\[ \tau = 0.067^{+0.023}_{-0.021} \text{ (lowTEB)} \quad \xrightleftharpoons{2.2\sigma} \quad \tau = 0.172^{+0.038}_{-0.042} \text{ (Plik)} \]
What if we remove lowTEB data?

Errors have ~ same order of magnitude, mean value is closer to the LowTEB estimation.

\[ \tau = 0.067^{+0.023}_{-0.021} \quad \text{(lowTEB)} \quad 1.3\sigma \quad \tau = 0.134^{+0.038}_{-0.048} \quad \text{(Hillipop)} \]
In Boltzman codes, the weak lensing enters the prediction of the CMB spectrum through a convolution of the unlensed spectrum with the lensing potential power spectrum $C_{\ell}^\Psi$. => the main impact is to smooth out the main peaks.

The $A_L$ parameter is a fudge factor defined as: $C_{\ell}^\Psi \rightarrow A_L C_{\ell}^\Psi$.

$A_L = 0$ : theory where the weak lensing is ignored

$A_L = 1$ standard $\Lambda$CDM

Measuring $A_L \neq 1$ indicates either a problem in the model or remaining systematics in the data.

Results from Planck(*): \( A_L = 1.22 \pm 0.10 \) (Plik+lowTEB, camb/MCMC)

Role of the Boltzmann solver
(CLASS)+ MarkovChain: \( A_L = 1.24 \pm 0.10 \) (Plik+lowTEB, class/MCMC)

Profile Likelihood Analysis: \( A_L = 1.26^{+0.11}_{-0.10} \) (Plik+lowTEB, class/profile)

2.6\( \sigma \) away from 1….

(*) Planck Collab, arXiv:1502.01589
$A_L$ from **Hillipop + lowTEB**

Profile Likelihood Analysis:  

\[ A_L = 1.22^{+0.11}_{-0.10} \text{ (Hillipop+lowTEB)} \]

$\Rightarrow$ Same order of magnitude...Still with a lower $\tau$ !

$\Rightarrow$ Correlated mainly with AkSZ

$\Rightarrow$ Use VHL data to further constrain the SZ sector
The VHL data

which VHL data do we use?

ACT [Das et al. 2014]
SPTHigh [Reichardt et al. 2012]
SPTLow [Story et al. 2012]

<table>
<thead>
<tr>
<th>Dataset</th>
<th>freq (GHz)</th>
<th>#spectra</th>
<th>#nuisances</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPT_low</td>
<td>150</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>SPT_high</td>
<td>95, 150, 220</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>ACT south/equat</td>
<td>148, 218</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

foreground subtracted combined spectrum @150GHz
**$A_L$ from Hillipop+lowTEB+VHL data**

Final result:  $A_L = 1.03 \pm 0.08$  (Hillipop+lowTEB+VHL).

$\Rightarrow$ fully compatible with 1, passing successfully the $A_L$ test

$\Rightarrow$ Allowing to get a coherent picture of $\Lambda$CDM:

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<td>$\Omega_b h^2$</td>
<td>$0.02200 \pm 0.00019$</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>$0.1200 \pm 0.0020$</td>
</tr>
<tr>
<td>$100\theta_s$</td>
<td>$1.04200 \pm 0.00040$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.059 \pm 0.017$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.9630 \pm 0.0054$</td>
</tr>
<tr>
<td>$\ln(10^{10} A_s)$</td>
<td>$3.045 \pm 0.032$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
<td>$0.315 \pm 0.012$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$67.19 \pm 0.88$</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>$0.811 \pm 0.013$</td>
</tr>
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</table>
Conclusion

• We have developed a MCMC and profile likelihood analysis of cosmological data, see: camel.in2p3.fr

• We have computed a high-\(\ell\) likelihood of Planck data Hillipop which gives a \(\tau\) value compatible with lowTEB data

• We have combined Hillipop with VHL data to further constrain the SZ foregrounds

• We end up with \(A_L\) fully compatible with 1, allowing to get a coherent picture of \(\Lambda CDM\)
Correlation Matrix of Hillipop+lowTEB
Correlation Matrix of Hillipop+lowTEB+VHL
low-$\ell$: pulls $\tau$ \downarrow
high-$\ell$: amplitude $C_\ell \propto A_s e^{-2\tau} \rightarrow A_s$ \downarrow
high-$\ell$: to preserve lensing information ($C_\ell^\Phi \propto A_s A_L$) : $A_L$ \uparrow

Plik+prior Tau=0.07

Plik+prior Tau=0.17
(a) ACT_equat 148 × 148 (b) ACT_equat 148 × 218 (c) ACT_equat 218 × 218

(d) ACT_south 148 × 148 (e) ACT_south 148 × 218 (f) ACT_south 218 × 218
(g) SPT_high 95 x 95
(h) SPT_high 95 x 150
(i) SPT_high 95 x 220
(j) SPT_high 150 x 150
(k) SPT_high 150 x 220
(l) SPT_high 220 x 220
\( (m) \ SPT\_\text{low} \ 150 \times 150 \ \text{GHz} \)
\[ A^{kSZ} + 3.5A^{tSZ} = 3.16 \pm 0.25 \]

\[ A_L = 1.26^{+0.12}_{-0.10} \quad (\text{Hillipop+lowTEB+SZ-cor}). \]