Practical Aspects of the EFT of LSS
- The Eastcoast Story -

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[1406.4135],[1504.04366],[1505.07098],[1507.02255] and [1507.02256]

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Rencontres de Moriond, La Thuile
21.03.2016
1. Introduction
2. Lagrangian Space
3. Eulerian Space
4. Summary

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Practical Aspects of the EFT of LSS
LSS Clustering Statistics

Structure of the Power Spectrum

- Linear Theory valid on large scales
- Perturbative regime
- Non-perturbative virialized regime
- bias, redshift space . . .

Why bother with difficulties in LSS?

- Number of modes in CMB is saturated
- Large number of 3D modes in LSS (scale as $k_{\text{max}}^3$)
- Provides distinct signatures not present in CMB
LSS Clustering Statistics

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- Provides distinct signatures not present in CMB
Don’t get lost – Take Home Messages

- Detection of stochastic term
- One parameter one-loop bispectrum
- Detection of EFT counterterms at the field level
- One parameter two-loop power spectrum
- Parameter forecasts using the EFT
Credits - Most of EFT is SPT++

Lagrangian Perturbation Theory and resummations
Bouchet, Colombi, Hivon, Matsubara, Juskiewicz et al.

Standard Perturbation Theory and resummations
Bernardeau, Crocce, Grinstein, Pietroni, Scoccimarro, Wise et al.

EFT
Basic Idea [Baumann, Nicolis, Senatore, Zaldarriaga 2010]
One-Loop Power Spectrum [Carrasco, Hertzberg, Senatore 2012]
Two-Loop Power Spectrum [Carrasco, Foreman, Green, Senatore 2013]
Bispectrum [Angulo, Foreman, Schmittfull, Senatore 2014]
Lagrangian Space [Porto, Senatore, Zaldarriaga 2014]
IR-resummation [Senatore and Zaldarriaga 2014]
Bias [Senatore 2014] [Mirbabay, Schmidt, Zaldarriaga 2014]
Redshift Space Distortions [Senatore and Zaldarriaga 2014]

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EoM from the Vlaslov Equation

Vlaslov - Collisionless Boltzmann Equation

\[
\frac{\partial f}{\partial \tau} + \frac{p}{ma} \frac{\partial f}{\partial x} - ma \nabla \phi \frac{\partial f}{\partial p} = 0
\]

Definition of Density and Momentum

\[
\rho = ma^{-3} \int d^3 p \ f(x, p, \tau) \quad \rho v_i = \int d^3 p \ \frac{p_i}{ma} f(x, p, \tau) / \int d^3 p \ f(x, p, \tau)
\]

Fluid Equations

\[
\delta' + \partial_j [(1 + \delta)v_j] = 0
\]

\[
v_i' + \mathcal{H}v_i + \partial_i \phi + v_j \partial_j v_i = -\frac{1}{1 + \delta} \partial_j [(1 + \delta)\sigma_{ij}]
\]
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\]

Pressureless Perfect Fluid Equations

\[
\delta' + \partial_j [(1 + \delta)v_j] = 0 \\
v'_i + \mathcal{H}v_i + \partial_i \phi + v_j \partial_j v_i = 0
\]

Tobias Baldauf
Practical Aspects of the EFT of LSS
Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \nabla \cdot \mathbf{v}$)

\begin{align*}
\delta' + \theta &= -\alpha [\theta \ast \delta] \\
\theta' + \mathcal{H} \theta + \frac{3}{2} \mathcal{H}^2 \delta &= -\beta [\theta \ast \theta]
\end{align*}
Perturbative Solution of EoM

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\]

Perturbative Ansatz

\[
\delta(k) = \sum_{n} F_n(q_1, \ldots, q_n) \delta^{(1)}(q_1) \ldots \delta^{(1)}(q_n)
\]
Perturbative Solution of EoM

Equations of Motion for Pressureless Perfect Fluid ($\theta = \nabla \cdot \mathbf{v}$)

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\delta = 0$$

Matter Power Spectrum

$$P_{mm}(k, t) = D^2(t)P_{\text{lin}}(k)$$
Perturbative Solution of EoM

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Matter Power Spectrum

\[
P_{mm}(k, t) = D^2(t)P_{\text{lin}}(k) + D^4(t)[P_{22}(k) + 2P_{13}(k)]
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\]

UV-behaviour of loop terms

\[
P_{13} \xrightarrow{k \to 0} k^2 \int_\sigma \frac{P_{\text{lin}}(q)}{q^2} \propto \sigma_d^2
\]

UV-divergent for \(n > -1\)

\[
P_{22} \xrightarrow{k \to 0} k^4 \int_\sigma \frac{P_{\text{lin}}(q)}{q^4} \propto \sigma_d^4
\]

UV-divergent for \(n > 1/2\)
Problems of SPT

Large Densities

on small scales densities and velocities are large ⇒ no clear expansion parameter and no control over the size of subleading corrections

Shell Crossing

shells cross and haloes virialize ⇒ deviation from pressureless perfect fluid

Divergencies

loops are sensitive to the UV and can diverge for particular sets of initial conditions

Performance

agreement with simulations is rather meagre and there are deviations even on very large scales
Performance of SPT for the Power Spectrum
Equations of Motion for the Long Modes

**Coarse grained variables**

\[ f_\Lambda(x) = \int d^3x \ W_\Lambda(x - x') f(x') \]

Integrating out the small scales

\[ \delta'_\Lambda + \partial_j [(1 + \delta_\Lambda) v_{\Lambda,j}] = 0 \]

\[ v'_{\Lambda,i} + H v_{\Lambda,i} + \partial_i \phi_{\Lambda} + v_{\Lambda,j} \partial_j v_{\Lambda,i} = - \frac{1}{1 + \delta} \partial_j \tau_{\Lambda,ij} \]

Description of Short Scale Dynamics

\[ (f_s g_s)_\Lambda = \langle f_s g_s \rangle_\Lambda + \frac{\partial \langle f_s g_s \rangle}{\partial \delta_\Lambda} \delta_\Lambda + f_s g_s - \langle f_s g_s \rangle_\Lambda + \ldots \]

Effective Stress Tensor - Parametrizing the Ignorance about small scales

\[ \tau_{\Lambda,ij} = p \delta_{ij}^{(K)} + c_s^2 \delta_{ij}^{(K)} \delta_\Lambda + c_{v,b}^2 \delta_{ij}^{(K)} \partial_m v_{\Lambda,m} + c_{v,s}^2 \left[ \partial_i v_{\Lambda,j} + \partial_j v_{\Lambda,i} - \frac{2}{3} \delta_{ij}^{(K)} \partial_m v_{\Lambda,m} \right] + \Delta \tau_{ij} \]

[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]
Equations of Motion for the Long Modes

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⇒ all terms allowed by symmetries (second derivatives of the potential)

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[Baumann, Nicolis, Senatore, Zaldarriaga 2010][Carrasco, Hertzberg, Senatore 2012]
Effective Field Theory of LSS for Power Spectrum

Equations of Motion for Pressureless Perfect Fluid

\[ \delta_\Lambda' + \theta_\Lambda = -\alpha[\theta_\Lambda \ast \delta_\Lambda] \]
\[ \theta_\Lambda' + \mathcal{H}\theta_\Lambda + \frac{3}{2}\mathcal{H}^2\delta_\Lambda = -\beta[\theta_\Lambda \ast \theta_\Lambda] \]

Matter Power Spectrum

\[ P_{mm}(k, t) = P_{\text{lin}}(k) + P_{22}(k, \Lambda) + 2P_{13}(k, \Lambda) \]
Effective Field Theory of LSS for Power Spectrum

Equations of Motion including Effective Stress

\[
\delta'_\Lambda + \theta_\Lambda = - \alpha [\theta_\Lambda \ast \delta_\Lambda]
\]

\[
\theta'_\Lambda + \mathcal{H} \theta_\Lambda + \frac{3}{2} \mathcal{H}^2 \delta_\Lambda = - \beta [\theta_\Lambda \ast \theta_\Lambda] + \tilde{c}_s^2 k^2 \delta^{(1)}_\Lambda
\]

Matter Power Spectrum

\[
P_{mm}(k, t) = P_{\text{lin}}(k) + P_{22}(k, \Lambda) + 2P_{13}(k, \Lambda) - 2c_s^2(\Lambda)D^2(t)k^2P_{\text{lin}}(k)
\]
Sending cutoff to $\infty$ in $\Lambda$CDM

**Calculation of the Power Spectrum**

$$P(k) = P_{11}(k) + P_{22}(k, \Lambda) + 2P_{13}(k, \Lambda) - 2c_s^2(\Lambda)k^2P_{11}(k)$$

⇒ Result is $\Lambda$-independent

**Running of the speed of sound**

$$c_{s,\infty}^2 = c_s^2(\Lambda) - \frac{61}{210} \frac{1}{6\pi^2} \int_{\Lambda}^{\infty} dq \ P(q)$$

$c_{s,\infty}^2$ contains errors of PT and microscopic stress

**Scaling of Corrections for EdS power law**

$$P_{\text{loop}} \propto P_{11} \Delta^l \propto P_{11} \left(\frac{k}{k_{NL}}\right)^{(n+3)l} \quad P_{\text{stoch}} \propto \left(\frac{k}{k_{NL}}\right)^4$$

These scalings are important ingredients for forecasting – they form the theoretical error
Sending cutoff to \( \infty \) in \( \Lambda \text{CDM} \)

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$\Rightarrow$ Result is $\Lambda$-independent

Running of the speed of sound

$$c_{s,\infty}^2 = c_{s}^2(\Lambda) - \frac{64}{210} \frac{1}{6\pi^2} \int_{\Lambda}^{\infty} dq \ P(q)$$

$c_{s,\infty}^2$ contains errors of PT and microscopic stress

Scaling of Corrections for EdS power law

$$P_{l-loop} \propto P_{11} \Delta^l \propto P_{11} \left(\frac{k}{k_{NL}}\right)^{(n+3)l}$$

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**Sending cutoff to $\infty$ in $\Lambda$CDM**

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These scalings are important ingredients for forecasting – they form the theoretical error
Problems of SPT

Large Densities
on small scales densities and velocities are large \(\Rightarrow\) no clear expansion parameter and no control over the size of subleading corrections
smoothed fields are good expansion parameters

Shell Crossing
shells cross and haloes virialize \(\Rightarrow\) deviation from pressureless perfect fluid effective stress tensor can capture shell crossing

Divergencies
loops are sensitive to the UV and can diverge for particular sets of initial conditions effective corrections provide the required counterterms

Performance
agreement with simulations is rather meagre and there are deviations even on very large scales
Introduction

Lagrangian Space

Eulerian Space

Summary

Tobias Baldauf

Practical Aspects of the EFT of LSS
One-loop Lagrangian EFT

Displacement Field & Displacement Potentials

$$\Psi_i = x_i - q_i$$

$$\Psi = \nabla \phi + \nabla \times \omega$$

Displacement Potential with Counterterm

$$\phi_{1\text{-loop EFT}} = (1 + \alpha k^2) \phi^{(1)} + \phi^{(2)} + \phi^{(3)}$$

Measurement from Field Difference

$$\alpha_{\text{error}} = \frac{\phi_{\text{nl}} - \phi_{3\text{LPT}}}{k^2 \phi^{(1)}}$$

Measurement from Power Spectrum

$$\alpha_{\text{nl}} = \frac{1}{2} \frac{P_{\text{nl}} - P_{3\text{LPT}}}{k^2 P_{11}}$$

[Baldauf, Schaan, Zaldarriaga 2015]
One-loop Lagrangian EFT

Displacement Field & Displacement Potentials

$$\Psi_i = x_i - q_i$$

$$\Psi = \nabla \phi + \nabla \times \omega$$

Displacement Potential with Counterterm

$$\phi_{1-\text{loop EFT}} = (1 + \alpha k^2) \phi^{(1)} + \phi^{(2)} + \phi^{(3)}$$

Measurement from Field Difference

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Measurement from Power Spectrum

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[Baldauf, Schaan, Zaldarriaga 2015]
Transfer Functions - How far can we possibly push in $k$?

LPT provides a basis in field space

$$\phi_{ntLPT} = a_1^\perp \phi^{(1)} + \ldots + a_n^\perp \phi^{(n)}$$

$$\phi_{NL} = \phi_{ntLPT} + \phi_{error}$$

minimizing $P_{error} = \langle |\phi_{error}|^2 \rangle = \langle (\phi_{NL} - \phi_{ntLPT})^2 \rangle$

$$a_1^\perp = 1 + \frac{P_{13}}{P_{11}} + \alpha k^2 + \ldots$$

$$a_1^\perp = \frac{\langle \phi^{(1)} | \phi_{NL} \rangle}{\langle \phi^{(1)} | \phi^{(1)} \rangle}$$

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[Baldauf, Schaan, Zaldarriaga 2015]
Transfer Functions - How far can we possibly push in $k$?

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$$a_1^\perp = 1 + \frac{P_{13}}{P_{11}} + \alpha k^2 + ...$$

$$a_1^\perp = \frac{\langle \phi^{(1)} \perp |\phi_{NL} \rangle}{\langle \phi^{(1)} \perp |\phi^{(1)} \perp \rangle}$$

[Baldauf, Schaan, Zaldarriaga 2015]
Stochastic Term

\[ P_{\text{error}} = \langle |\phi_{\text{error}}|^2 \rangle = \langle (\phi_{\text{NL}} - \phi_{\text{ntLPT}})^2 \rangle \propto \left( \frac{k}{k_{\text{NL}}} \right)^4 \text{ with } k_{\text{NL}} \approx 0.32 \, h\text{Mpc}^{-1} \]
Stochastic Term in the Density Power Spectrum

\[ P_{\text{stoch}} = \langle \delta_{\text{NL}} - \delta_{\text{PT}} | \delta_{\text{NL}} - \delta_{\text{PT}} \rangle \Rightarrow P_{\text{NL}} = P_{\text{PT}} + P_{\text{stoch}} \]

\[ k_{1\%} = 0.25 \ h\text{Mpc}^{-1} \ \text{at} \ z = 0 \]

[Baldauf, Schaan, Zaldarriaga 2015]
Promised percent accuracy to $k = 0.6 \, h\text{Mpc}^{-1}$ . . .

Figure 4: Top: The prediction of the IR-resummed EFT at one-loop (in thick red) and two-loops (in thick blue). In thin dashed are the predictions from the Eulerian EFT, that is without IR-resummation, with the same colors respectively. The green band represents the estimated theoretical error from three-loops. The two-loops results have been renormalized at $k_{\text{ren}} = 0.2 h\text{Mpc}^{-1}$, and $c_2(s(1))$ has been approximately fit up to $k \approx 0.34 h\text{Mpc}^{-1}$. Since the equal-time matter power spectrum is IR-safe, we see that the effect of the IR-resummation is just to affect the oscillations, which are indeed now correctly taken into account. We see that the one-loop result matches to percent level the data up to $k \approx 0.34 h\text{Mpc}^{-1}$, while at two-loop matches all the way up to $k \approx 0.6 h\text{Mpc}^{-1}$. The spike at $k \approx 0.05 h\text{Mpc}^{-1}$ is due to the numerical interpolator, against which we compare, not to the EFT. It is also important to notice that the match stops exactly where the three-loop term is estimated to become relevant.

Bottom: We compare the predictions of the IR-resummed EFT with the ones of SPT. In thick magenta, red and blue we plot respectively the IR-resummed linear, one-loop and two-loops predictions of the EFT. With the same colors, but dashed, the same quantities in SPT. As we go to higher orders, SPT does not increase the agreement with the data. Furthermore, we notice that SPT has the same residual oscillatory features as the Eulerian EFT. In contrast, the IR-resummed EFT of LSS correctly predicts the size of the oscillations, and, at each order in perturbation theory, it improves the UV match to the data. Importantly, in the EFT of LSS, order by order in perturbation theory, it is possible to estimate up to where the theory should match the data.

[Senatore, Zaldarriaga 2014]
1 Introduction

2 Lagrangian Space

3 Eulerian Space

4 Summary
Two-Loop Power Spectrum in SPT

\[ P = P_{11} + P_{22} + 2P_{13} + 2P_{15} + 2P_{24} + P_{33-I} + P_{33-II} \]

1-loop

2-loop

[Baldauf, Mercolli, Zaldarriaga 2015]
Principled approach to higher loops and \( n \)-point functions

Parametrize stress tensor to third order in the long modes

\[
\begin{align*}
\tau^{(1)}_{ij} &= \epsilon^{(1)}_1 \delta^{(K)}_{ij} \partial^2 \phi + \epsilon^{(2)}_2 \partial_i \partial_j \phi \\
\tau^{(2)}_{ij} &= \epsilon^{(2)}_1 \delta^{(K)}_{ij} (\partial^2 \phi)^2 + \epsilon^{(2)}_2 \partial_i \partial_j \phi \partial^2 \phi + \epsilon^{(2)}_3 \partial_i \partial_l \phi \partial_l \partial_j \phi + \epsilon^{(2)}_4 \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_l \partial_m \phi \\
\tau^{(3)}_{ij} &= \epsilon^{(3)}_1 \delta^{(K)}_{ij} (\partial^2 \phi)^3 + \epsilon^{(3)}_2 \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_l \partial_m \phi \partial^2 \phi + \epsilon^{(3)}_3 \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_m \partial_n \phi \partial_n \partial_l \phi \\
&+ \epsilon^{(3)}_4 (\partial^2 \phi)^2 \partial_i \partial_j \phi + \epsilon^{(3)}_5 \partial_l \partial_m \phi \partial_l \partial_m \phi \partial_i \partial_j \phi \\
&+ \epsilon^{(3)}_6 \partial_i \partial_l \phi \partial_l \partial_m \phi \partial_m \partial_j \phi + \epsilon^{(3)}_7 \partial_i \partial_l \phi \partial_l \partial_j \phi \partial^2 \phi
\end{align*}
\]
Principled approach to higher loops and $n$-point functions

Parametrize stress tensor to third order in the long modes

\[
\begin{align*}
\tau^{(1)}_{ij} &= \epsilon_1^{(1)} \delta^{(K)}_{ij} \partial^2 \phi + \epsilon_2^{(2)} \partial_i \partial_j \phi \\
\tau^{(2)}_{ij} &= \epsilon_1^{(2)} \delta^{(K)}_{ij} (\partial^2 \phi)^2 + \epsilon_2^{(2)} \partial_i \partial_j \phi \partial^2 \phi + \epsilon_3^{(2)} \partial_i \partial_l \phi \partial_l \partial_j \phi + \epsilon_4^{(2)} \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_l \partial_m \phi \\
\tau^{(3)}_{ij} &= \epsilon_1^{(3)} \delta^{(K)}_{ij} (\partial^2 \phi)^3 + \epsilon_2^{(3)} \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_l \partial_m \phi \partial^2 \phi + \epsilon_3^{(3)} \delta^{(K)}_{ij} \partial_l \partial_m \phi \partial_m \partial_n \phi \partial_n \partial_l \phi \\
&\quad + \epsilon_4^{(3)} (\partial^2 \phi)^2 \partial_i \partial_j \phi + \epsilon_5^{(3)} \partial_l \partial_m \phi \partial_l \partial_m \phi \partial_i \partial_j \phi \\
&\quad + \epsilon_6^{(3)} \partial_i \partial_l \phi \partial_l \partial_m \phi \partial_m \partial_j \phi + \epsilon_7^{(3)} \partial_i \partial_l \phi \partial_l \partial_j \phi \partial^2 \phi
\end{align*}
\]

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.

*John von Neumann*
Principled approach to higher loops and \( n \)-point functions

Parametrize stress tensor to third order in the long modes

\[
\begin{align*}
\tau_{ij}^{(1)} &= \epsilon_1^{(1)} \delta_{ij}^{(K)} \partial^2 \phi + \epsilon_2^{(2)} \partial_i \partial_j \phi \\
\tau_{ij}^{(2)} &= \epsilon_1^{(2)} \delta_{ij}^{(K)} (\partial^2 \phi)^2 + \epsilon_2^{(2)} \partial_i \partial_j \phi \partial^2 \phi + \epsilon_3^{(2)} \partial_i \partial_l \phi \partial_l \partial_j \phi + \epsilon_4^{(2)} \delta_{ij}^{(K)} \partial_l \partial_m \phi \partial_l \partial_m \phi \\
\tau_{ij}^{(3)} &= \epsilon_1^{(3)} \delta_{ij}^{(K)} (\partial^2 \phi)^3 + \epsilon_2^{(3)} \delta_{ij}^{(K)} \partial_l \partial_m \phi \partial_l \partial_m \phi \partial^2 \phi + \epsilon_3^{(3)} \delta_{ij}^{(K)} \partial_l \partial_m \phi \partial_m \partial_n \phi \partial_n \partial_l \phi \\
&\quad + \epsilon_4^{(3)} (\partial^2 \phi)^2 \partial_i \partial_j \phi + \epsilon_5^{(3)} \partial_l \partial_m \phi \partial_l \partial_m \phi \partial_i \partial_j \phi \\
&\quad + \epsilon_6^{(3)} \partial_i \partial_l \phi \partial_l \partial_m \phi \partial_m \partial_j \phi + \epsilon_7^{(3)} \partial_i \partial_l \phi \partial_l \partial_j \phi \partial^2 \phi
\end{align*}
\]

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk. 

\textit{John von Neumann}

Let’s be less general and more pragmatic → focus on the UV-sensitivity of SPT
Two Loop Regularization

most UV-sensitive diagrams: single-hard diagrams containing an isolated power spectrum in the loop - all proportional to $\sigma_d^2$

[Baldauf, Mercoli, Zaldarriaga 2015]
UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

\[ P_{13}^{q_1 \to \infty} = -k^2 P_{\text{lin}} \frac{61}{630} \int_q \frac{P_{\text{lin}}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\text{lin}} \]

Two Loop UV-Sensitivity

\[ P_{15}^{q_1 \to \infty} \propto \sigma_d^2 \quad P_{24}^{q_1 \to \infty} \propto \sigma_d^2 \quad P_{33-II}^{q_1 \to \infty} \propto \sigma_d^2 \]

One Parameter Ansatz

\[ P_{\text{ctr}} \propto c_s^2 \left[ 2P_{13}^{q_1 \to \infty} + 2\bar{P}_{15}^{q_1 \to \infty} + 2P_{24}^{q_1 \to \infty} + P_{33-II}^{q_1 \to \infty} \right] \]

[Baldauf, Mercolli, Zaldarriaga 2015]
UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

\[ P^{q_1 \to \infty}_{13} = -k^2 P_{\text{lin}} \frac{61}{630} \int_q \frac{P_{\text{lin}}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\text{lin}} \]

Two Loop UV-Sensitivity

\[ P^{q_1 \to \infty}_{15} \propto \sigma_d^2 \quad P^{q_1 \to \infty}_{24} \propto \sigma_d^2 \quad P^{q_1 \to \infty}_{33-\Pi} \propto \sigma_d^2 \]

One Parameter Ansatz

\[ P_{\text{ctr}} \propto c_s^2 \left[ 2P^{q_1 \to \infty}_{13} + 2P^{q_1 \to \infty}_{15} + 2P^{q_1 \to \infty}_{24} + P^{q_1 \to \infty}_{33-\Pi} \right] \]

[Baldauf, Mercolli, Zaldarriaga 2015]
UV Sensitivity and Two Loop Ansatz

One Loop UV-Sensitivity

\[ P_{13}^{q_1 \to \infty} = -k^2 P_{\text{lin}} \frac{61}{630} \int_q \frac{P_{\text{lin}}(q)}{q^2} = -\frac{61}{210} \sigma_d^2 k^2 P_{\text{lin}} \]

Two Loop UV-Sensitivity

\[ P_{15}^{q_1 \to \infty} \propto \sigma_d^2 \quad P_{24}^{q_1 \to \infty} \propto \sigma_d^2 \quad P_{33-ll}^{q_1 \to \infty} \propto \sigma_d^2 \]

One Parameter Ansatz

\[ P_{\text{ctr}} \propto c_s^2 \left[ 2P_{13}^{q_1 \to \infty} + 2\bar{P}_{15}^{q_1 \to \infty} + 2P_{24}^{q_1 \to \infty} + P_{33-ll}^{q_1 \to \infty} \right] \]

[Baldauf, Mercalli, Zaldarriaga 2015]
Where can one measure $c_s^2$ in isolation?

\[
\frac{P - P_{\text{lin}} - P_{1\text{loop}}}{2k^2 P_{\text{lin}}} = c_s^2 - \frac{\bar{P}_{2\text{loop}} + P_{\text{ctr,2loop}}}{2k^2 P_{\text{lin}}}
\]

[Baldauf, Mercoli, Zaldarriaga 2015]
Where can one measure $c_s^2$ in isolation?

\[
\frac{P - P_{\text{lin}} - P_{\text{1loop}}}{2k^2 P_{\text{lin}}} = c_s^2 = \frac{P_{\text{2loop}} + P_{\text{ctr,2loop}}}{2k^2 P_{\text{lin}}}
\]

![Graph showing the relationship between $k$ and the various terms involving $c_s^2$ and $P$.](image)

[Baldauf, Mercolli, Zaldarriaga 2015]
EFT can be evaluated for a given simulation initial condition & compared to the result ⇒ cancellation of cosmic variance & test at the field level

[Baldauf, Mercolli, Zaldarriaga 2015]
Performance for the Power Spectrum

[Baldauf, Mercolli, Zaldarriaga 2015], see also [Foreman, Perrier, Senatore 2015]
Performance for the Power Spectrum

[Baldauf, Mercolli, Zaldarriaga 2015], see also [Foreman, Perrier, Senatore 2015]
Problems of SPT

**Large Densities**
on small scales densities and velocities are large ⇒ no clear expansion parameter and no control over the size of subleading corrections smoothed fields are good expansion parameters ✔

**Shell Crossing**
shells cross and haloes virialize ⇒ deviation from pressureless perfect fluid effective stress tensor can capture shell crossing ✔

**Divergencies**
loops are sensitive to the UV and can diverge for particular sets of initial conditions effective corrections provide the required counterterms ✔

**Performance**
agreement with simulations is rather meagre and there are deviations even on very large scales EFT parameters measured in the IR improve range in the UV ✔
Effective Field Theory of LSS for the Bispectrum

Equations of Motion including Effective Stress

\[
\begin{align*}
\delta' + \theta &= -\alpha[\theta \ast \delta] \\
\theta' + \mathcal{H}\theta + \frac{3}{2} \mathcal{H}^2 \delta &= -\beta[\theta \ast \theta] + \tilde{c}_s^2 k^2 \delta^{(1)}
\end{align*}
\]

Matter Power Spectrum

\[
P_{\text{mm}}(k) = P_{\text{lin}}(k) + P_{\text{1loop}}(k) - 2c_s^2 k^2 P_{\text{lin}}(k)
\]

Matter Bispectrum

\[
B_{\text{mmm}}(k_j) = B_{\text{tree}}(k_j) + B_{\text{1loop}}(k_j)
\]

[Baldauf, Mercolli, Mirbabayi, Pajer 2014], see also [Angulo, Foreman, Schmittfull, Senatore 2015]
Effective Field Theory of LSS for the Bispectrum

Equations of Motion including Effective Stress

\[
\delta' + \theta = - \alpha [\theta \ast \delta]
\]
\[
\theta' + \mathcal{H} \theta + \frac{3}{2} \mathcal{H}^2 \delta = - \beta [\theta \ast \theta] + \tilde{c}_s^2 k^2 \delta^{(1)} + \tilde{c}_s^2 k^2 \delta^{(2)} + \tilde{e}_i E_i
\]

Matter Power Spectrum

\[
P_{mm}(k) = P_{\text{lin}}(k) + P_{1\text{loop}}(k) - 2 c_s^2 k^2 P_{\text{lin}}(k)
\]

Matter Bispectrum

\[
B_{mmm}(k_j) = B_{\text{tree}}(k_j) + B_{1\text{loop}}(k_j) + B_{c_s}(k_j) + B_{e_i}(k_j)
\]

[Baldauf, Mercoli, Mirababai, Pajer 2014], see also [Angulo, Foreman, Schmittfull, Senatore 2015]
Bispectrum Performance

[Baldauf, Mercolli, Mirbabayi, Pajer 2014]
Bispectrum Performance

[Baldauf, Mercolli, Mirbabayi, Pajer 2014]
Summary

Achievements

- detected non-zero counterterms for EFT at the field level in the low-$k$ limit for densities and displacements
- detected the stochastic term $\Rightarrow$ leads to percent level corrections at $k = 0.3 \ h\text{Mpc}^{-1}$
- One-loop Eulerian EFT $k_{1\%} = 0.1 \ h\text{Mpc}^{-1}$, two-loop Eulerian EFT $k_{1\%} = 0.3 \ h\text{Mpc}^{-1}$ at $z = 0$
- Improved description of the bispectrum using one-loop EFT

Outlook

- detailed measurements of the second order counterterms
- understanding of the origin of the ”speed of sound” - microscopic or loop errors?
Don’t get lost – Take Home Messages

1. solid constraints on the maximum range of validity of perturbative part of EFT approaches
2. solid detection of EFT corrections at the field level in the low-$k$
3. one-parameter two-loop EFT model for power spectrum
4. one-parameter one-loop EFT model for bispectrum
5. performance of EFT in constraining fundamental physics (Marko’s talk)