Massive neutrinos
and LSS beyond
linear approximation

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Neutrinos and neutrino masses

In the SM of particle physics neutrinos are massless

$$SU(3)_c \times SU(2)_l \times U(1)_Y$$

$$L_f = \sum_{\psi=e,\mu,\tau} \left[ i\bar{\psi}\gamma^\mu \psi - m_\psi \bar{\psi} \psi + i(\bar{\nu}_\psi)L \bar{\nu}_L (\nu_\psi)L - \frac{g}{\sqrt{2}} W^\mu (\bar{\nu}_\psi)L \gamma_\mu \psi_L + ... \right]$$

all terms are ‘flavour’ preserving

adding a mass term (Dirac or Majorana):

$$L_m = \sum_{\psi=e,\mu,\tau} \left[ m_{\psi\psi'} (\bar{\nu}_{\psi'})L (\nu_\psi)L + ... \right]$$

mixes ‘flavour’

electroweak interactions create mixed states that oscillate

$$\partial_t \psi = \Omega_{\psi\psi'} \psi'$$

$$\psi(t) = e^{\Omega_{\psi\psi'} t} \psi'(0)$$

depends on mixing and mass split
Particle physics constraints on $m_{\nu}$

\[ 0.06 \text{ eV} < \sum_{\psi=e,\mu,\tau} m_{\nu,\psi} \]

the sum can be constrained by decaying processes:

\[ ^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_k \]

\[ \sum_{\psi=e,\mu,\tau} m_{\nu,\psi} < 2 \text{ eV} \ (95\% \ CL) \]

This small scale may be the door to physics BSM!
(and cosmology has the key!)
(Massive) neutrinos in cosmology

Cosmology is very sensitive to neutrinos properties

- Baryogenesis scenarios
- Neutrino decoupling: CvB
- BBN p/n chemical equilibrium
  contributes to $N_{eff}$
- CMB
- EarlyISW
- LateISW
- LSS
- all neutrino species are relativistic

Mostly sensitive to **massless** properties $N_{eff}$
Planck’15 bounds

\[ N_{\text{eff}} = 2.99 \pm 0.20 \ (95\% \ Planck \ TT, \ TE, \ EE + \ lowP) \]

\[ \sum m_\nu < 0.49 \text{ eV} \ (95\% \ Planck \ TT, \ TE, \ EE + \ lowP) \]

(improve by BAO data)
Effects of $m_\nu$ in large scale structure

thermal background $f_{\nu 0}(\eta, p) \equiv \left( e^{p/T_\nu} + 1 \right)^{-1}$

distribution function $\Psi_\nu(\eta; \bar{x}, \bar{p}) = \frac{f_\nu(\eta; \bar{x}, \bar{p})}{f_{\nu 0}(\eta, p)} - 1$

(linear) Boltzmann ($E(p) = \sqrt{p^2 + m_\nu^2}$)

$\dot{\Psi}_\nu + \frac{p}{E(p)} \hat{p} \cdot \nabla \Psi_\nu + \frac{\text{d} \ln f_{\nu 0}}{\text{d} \ln (ap)} \left[ \dot{\phi} - \frac{E(p)}{p} \hat{p} \cdot \nabla \psi \right] = 0$

Massless neutrinos $E(p) = p$ free-stream and do not cluster

$\delta \rho_\nu = \int d^3 p E(p) f_\nu(\eta, \bar{x}, \bar{p}), \quad \delta \rho_\nu(k)'' = (c^2(\eta)k^2 - 3a^2 H^2/2)\delta \rho_\nu(k) + \ldots$

still affect (through $N_{e,f}$) r-m equality and BAO production

Massive: when $p \ll E(p) \sim m$, neutrinos become **cold (cluster)**

$k_{fs} \sim aH/c, \quad c(\eta) \sim T_\nu(\eta)/m_\nu$
Effects of $m_\nu$ in large scale structure II

$$\delta \equiv \frac{\sum_{i=b,c,\nu} \delta \rho_i}{\sum_{i=b,c,\nu} \bar{\rho}_i}$$

$$\langle \delta(k, t)\delta(k', t) \rangle = P(k, t)\delta^{(3)}(k + k')$$

Effect on linear power-spectrum

$$m_\nu = 0.3 \text{ eV} \quad k_{fs}(\eta_{nr}) \sim (m_\nu/T_\nu)^{1/2}$$

Transition to CDM at large scales!
Observed Matter power spectrum

DES, Euclid, LSST,...: 1% level at different redshifts! (also higher n-point correlation functions)

Figure 1: Observable spectrum (top) and relative error on this spectrum (bottom), for the first redshift bin (left) and last redshift bin (right) of a Euclid-like galaxy redshift survey. The quantity displayed in the top is the galaxy power spectrum \( P_g(k_{\text{ref}}, \mu, z) \) as a function of the fiducial wavenumber \( k_{\text{ref}} \), for fixed redshift and perpendicularly to the line of sight (\( \mu = 0 \)), rescaled by the inverse squared bias \( b(z)^{-2} \) and by a factor \( H(z)/D_A(z)^2 \): it is therefore a dimensionless quantity. The upper plots show a comparison between a model with massless neutrinos and our fiducial model (\( M_\nu = 0.05 \text{ eV} / \text{massless} \)). Solid lines are derived from the non-linear matter power spectrum using the updated halofit version of ref. [24], while dashed lines are derived from the linear power spectrum. The lower plots show the part of the relative error coming from observational or theoretical errors only (cosmic variance is included in the observational error). In these plots, the individual 1- error on each data point has been rescaled by the square root of the number of points, in such a way that the edges of the error bands correspond to a shift between theory and observation leading to \( \chi^2 = 1 \), when only the observational or theoretical error is incorporated in the likelihood expression. In these lower plots, we also show for comparison the ratio between a massless model and a model with the minimum total mass allowed by neutrino experiments, \( M_\nu = 0.05 \text{ eV} / \text{massless} \).

We fit the mock and Euclid-like spectra using the MCMC code MontePython [27]. MontePython uses the Metropolis-Hastings algorithm like CosmoMC [28], but is interfaced with class [29, 30] instead of camb [31], is written in python, and has extra functionality; it will soon be released publicly, including the Euclid-like likelihood codes.

DES, Euclid, LSST,...: 1% level at different redshifts! (also higher n-point correlation functions)
Beyond **linear** theory to close the mass interval!

N-body (with warm components)  
Demanding (hard for MC)  
Halo model (~10% precision)  

The effect is 5% at BAO scales  
Mildly non-linear regime!  
Non-linear perturbation theory

**SPT:** DM as a **non-linear** pressureless perfect fluid

\[ \dot{\delta}_{DM} + \partial_i ([1 + \delta_{DM}] v^i_{DM}) = 0 \quad (+ \text{UV}) \]

\[ \dot{\alpha}_{DM} + H \alpha_{DM} + \beta \delta_{DM} v^i_{DM} = -\Delta \phi \]

\[ \theta \equiv \partial_i v^i \]

**Neutrinos**  
**CDM-like**  
**Phase-space**

\[ \Delta \phi = \frac{3}{2} H^2 \Omega_m [f_\nu \delta_\nu + (1 - f_\nu) \delta_{cb}] \]

\[ f_\nu \equiv \frac{\Omega_\nu}{\Omega_m} = \frac{1}{\Omega_m h^2} \sum_i m_{\nu,i} \]

\[ k \text{ (h/Mpc)} \]

\[ 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \]
Massive neutrinos in SPT

CDM-like

Neutrinos

Phase space

QI: Since $\delta_\nu$ is small, can it be treated as linear?

$\dot{\theta}_{cb} + H\theta_{cb} + \frac{3}{2}H^2\Omega_m[f_\nu\delta^L_\nu + (1 - f_\nu)\delta_{cb}] = -\beta\delta_{cb}\theta_{cb}$ (+ UV)

$\dot{\delta}_{cb} + \theta_{cb} = -\alpha\theta_{cb}\delta_{cb}$

Non-linear DM ($cb$)

$\delta_{cb}(t, k) = \sum_{n} F_n(t, k_1, ..., k_n)\delta_{cb}(0, k_1)....\delta_{cb}(0, k_n)\delta^{(3)}(k - \sum k_i)$

$\langle \delta(k, t)\delta(k', t) \rangle = P(k, t)\delta^{(3)}(k + k')$
Momentum conservation, EP and $\sim k^2 P(k)_L$

$\sim k^2 P(k)_L$ : suppressed non-linearities at low $k$

Isolated mass at $t = 0$  

$\delta_k(t) = \int d^3 x \delta(x, t) e^{-ikx}$

$k \ll R^{-1}$ Power transfer from short scales  

$\delta_k(t) = \int d^3 x \delta(x, t) - i\bar{k} \int d^3 x \bar{x} \delta(x, t) + O(k^2)$

Total energy (conserved)  
Related to total momentum

$\frac{d^2}{dt^2} \int d^3 x \bar{x} \delta(x, t) = 0 + O(\mathcal{H})$

NL Euler and Conservation + Poisson

$\delta_k \sim k^2$ (this is crucial for EFT or BAO)

Not true if part is linear and part is non-linear!  

$\delta_k \sim O(1)$
**Linear vs Non-linear \( \nu \)'s II**

**Q1:** Since \( \delta_\nu \) is small, can it be treated as **linear**?

**A1:** ❗ it introduces a spurious large effect at NLO!

**Q2:** How to include \( \nu \) non-linearities?

(even linear order is **NOT** a fluid at **all** redshift)

**A2:** At low-redshift \((z < z_{nr} \sim 10^2)\) the fluid is very cold.

Non-cold corrections are \( O(T_\nu/m_\nu) \)

\[
\delta_\nu + \theta_\nu = -\alpha \theta_\nu \delta_\nu \\
\dot{\theta}_\nu + \mathcal{H}\theta_\nu + \frac{3}{2}\mathcal{H}^2\Omega_m[f_\nu \delta_\nu + (1 - f_\nu)\delta_{cb}] - k^2 c_s(t)^2 \delta_\nu = -\beta \delta_\nu \theta_\nu + O(T_\nu/m_\nu)
\]

i.e. from the Boltzmann equations at \( 10 > z > 10^2 \)

linear physics
Accuracy of approximations

DB, Garny, Konstandin, Lesgourgues 2014
CLASS CODE, Blas, Lesgourgues, Tram, 2011

Linear Fluid vs Boltzmann equation

\[ \sum m_\nu = 0.21 \text{ eV} \]

\[ z = 0 \]

Due to Newtonian limit
Results at NLO

NNLO (to compare with N-body and EFT) on the way

at least 5\% effect

Audren et al. 2013
EUCLID Forecast
Comparison of NLO Results

DB, Garny, Konstandin, Lesgourgues 2014

\[ \Delta P = P - P_L \]

**Schemes with linear** $\nu$

Wong 2008

no momentum conservation

**Ansatz** $\delta_\nu \rightarrow \delta_{cb} \frac{\delta^L_\nu}{\delta_{cb}}$

Lesgourgues et al 2009

No $\delta_\nu$ (modified IPS)

Saito et al 2008, BOSS 2014

$P_c(0, k)$

NNLO?
Does it matter?

NLO is not enough for 5% accuracy at BAO

NNLO and resummations/EFT much more sensitive to the short mode/long mode (de)coupling

Pure CDM

Predictive descriptions require a good $\sim k^2$ behaviour
Two point correlation function

\[
\xi(x, z) = \frac{4\pi}{x} \int_0^\infty dk \, k P(k, z) \sin(kx)
\]

Damping of BAO-peak sensitive to PS!

N-Body

RPT-inspired formula (modified IPS)

Peloso et al 2015

TSPT calculation result to appear soon!
Conclusions

- Neutrino masses are the only missing parameters of SM

- Cosmology is in an advantageous situation to fix them (Neutrinos are copiously produced in the early universe)

- After redshifting, they become cold and cluster

- The influence in LSS is at the level that may be reachable in upcoming surveys (for whole mass range)

- This requires to deal with mildly non-linear scales in the matter power spectrum: N-body or SPT with two fluids

- Conservation of momentum or spurious (big) effect!
Future

- Resummations for BAO
- Analytical understanding of other observables/systematics
- Comparison with N-body
- Effects of mass splitting
- EFT/UV effects
Neutrino mass and $N_{\text{eff}}$ affect many other LSS observables

Ly-α, Shear, Mass function, kSZ…

Costanzi et al 2014

Palanque-Delabrouille et al 2014
Pure DM NNLO results (two iterations)

Blas, Garny, Konstandin 2013
Taruya et al 2012

Convergence at low-k?

NNLO or better correction necessary for precision cosmology
$\sim k^2 P(k)_L$

Schemes with linear $\nu$

no momentum conservation