Bumpy Inflation: subleading effects in axion inflation

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Rencontres de moriond 2016

Based on
1509.07049 w/kooner & parameswaran
1602.02812 w/parameswaran & Tasinato
COSMOLOGICAL INFLATION

• A period of accelerated quasi dS expansion in the very early universe (solving flatness, horizon and relic problems), driven by a scalar field, whose quantum fluctuations seeded the LSS

[Mukhanov-Chibisov ’81]
[Guth ’81, Linde ’82, Albrecht-Steinhardt ’82]

• Cosmological observables are encoded in the generalised slow-roll parameters \( (\dot{H} = \frac{\dot{a}}{a}) \)

\[
\begin{align*}
\epsilon & \equiv 2 \frac{H''^2}{H^2}, \\
\delta & \equiv \frac{H''}{H}, \\
\xi & \equiv \frac{H'''}{H^2}
\end{align*}
\]

\[
\mathcal{P}_\zeta \sim A_s k^{n_s-1}, \quad A_s = \frac{1}{8\pi^2 M_{Pl}^2} \frac{H^2}{\epsilon} = 2.1 \times 10^{-9}
\]

\[
n_s = 1 - 4\epsilon + 4\delta = 0.9667 \pm 0.0066
\]

\[
\alpha_s = -8 \xi + 20 \epsilon \delta - 8 \epsilon^2 \approx -0.013^{+0.010}_{-0.009}
\]

\[
r = 16\epsilon < 0.07
\]

[Planck ’15]
[BICEP2/Keck ’15]
COSMOLOGICAL INFLATION

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- Cosmological observables are encoded in the generalised slow-roll parameters \( \left( ' = \frac{d}{d\phi} \right) \)

\[
\begin{align*}
H &= \frac{\dot{a}}{a} \\
\epsilon &= 2 \frac{H'^2}{H^2}, \quad \delta &= \frac{H''}{H}, \quad \xi &= \frac{H^{'''} H'}{H^2}
\end{align*}
\]

\[
\mathcal{P}_\zeta \sim A_s k^{n_s - 1}, \quad A_s = \frac{1}{8\pi^2 M_{Pl}^2} \frac{H^2}{\epsilon} = 2.1 \times 10^{-9}
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[Mukhanov-Chibisov ’81] [Guth ’81, Linde ’82, Albrecht-Steinhardt ’82]
Tensor to scalar ratio $r$ is related to

- **The scale of inflation**
  
  $$V^{1/4} \approx 1.8 \times 10^{16} \text{GeV} \left( \frac{r}{0.1} \right)^{1/4}$$

- **The inflaton field excursion**
  
  $$\frac{\Delta \phi}{M_{Pl}} \geq \mathcal{O}(1) \left( \frac{r}{0.002} \right)^{1/2}$$

[Lyth, ’96; Boubekeur-Lyth, ’05]
[Garcia-Bellido, Roest, Scalisi, IZ ’14]
INFLATION NEEDS FUNDAMENTAL THEORY

• Inflation is sensitive to Planck scale physics. Higher order corrections to $V(\phi)$

$$\mathcal{O}_{p \geq 6} \rightarrow V(\phi) \left( \frac{\phi}{M_P} \right)^{p-4}$$

generically spoil slow-roll: $\eta$-problem

$$\Delta\eta \rightarrow \left( \frac{\phi}{M_P} \right)^{p-6} \gtrsim 1 \quad \left( \eta \equiv M_{Pl}^2 \left| \frac{V''}{V} \right| \ll 1 \right)$$

• (Large field) inflationary models ($\Delta\phi \gtrsim M_{Pl}$) are sensitive to all Planck suppressed interactions unless a symmetry protects the potential
Axion inflation in string theory (at leading order)

Inflaton is an axion (PNGB) with continuous global shift symmetry \( \phi \rightarrow \phi + \alpha \) [Freese-Frieman-Linto, '90] [Croon-Sanz, '14-15]

- **Natural Inflation**, \( V = V_0(1 - \cos(\phi/f)) \). Axions abound in string theory, but to realise \( f \gg M_{Pl} \) is hard \((f \gtrsim 7M_{Pl})\) [Banks, Dine, Fox & Gorbatov, '03] [Kim-Nilles-Peloso, '04; Dimopoulos et al '05; Avgoustidis, IZ, '08, Kenton-Thomas, '14, Kooner, Parameswaran, IZ, '15]

- **Chaotic Inflation (monomial) inflation**
  
  Axion monodromy

  \( V \sim V_0\phi^n \) \((\Delta \phi \sim 15M_{Pl})\) [Westphal-Silverstein, '08, '14] [Kaloper-Sorbo '08]
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Disfavoured by data
String theory models of inflation rely on 4D LEEFT, weakly coupled, perturbative string expansion \((g_s < 1, \quad L/\ell_s > 1)\)

- The string scale in regimes of perturbative control is

\[
M_s = \frac{g_s}{\sqrt{4\pi V_6^w}} \ M_{Pl} \lesssim 10^{17}\text{GeV}
\]

- For a 4D effective field theory description to be valid during inflation, we require the hierarchy:

\[
M_{inf} \lesssim M_{kk} \lesssim M_s \lesssim M_{Pl}
\]

- Otherwise we cannot neglect massive string excitations, Kaluza-Klein modes and extra dimensions
Subleading instanton effects give contributions to effective potential:
\[ \sum_n \Lambda_n \cos \left( \frac{n \phi}{f} \right) \]

Effect on inflaton dynamics depends on the size of corrections, their frequency and amplitude.

If corrections dominate \( \Rightarrow \) new minima are introduced. Inflaton trapped in local minimum and slow-roll inflation stops.

[Banks-Dine-Fox-Gorbatov, '03]
Subleading Corrections to Axion Potentials

- Subleading tiny modulations ⇒ inflaton’s background trajectory is hardly affected, but imprints seen in CMB – large, possibly oscillating, running of scalar spectral index.

Corrections are subleading, but significant - this talk

[Westphal-Silverstein-McAllister, ’08; Kobayashi-Takahashi, ’10; Kappl-Nilles-Winkler, ’15; Choi-Kim, ’15]
BUMPY CHAOTIC INFLATION

Axionic chaotic inflation with non-perturbative correction, e.g.

\[ V(\phi) = A + \frac{m^2 \phi^2}{2} + \lambda \phi \cos \left( \frac{\phi}{f} \right) \]

smooth monomial chaotic inflation for \( \lambda = A = 0 \),

giving for \( N \sim 60 \), \( \Delta \phi \sim 15 M_{Pl} \), \( r \sim 0.12 \), \( M_{inf} \sim 10^{16} \text{ GeV} \)

[Westphal, Silverstein, '08; Kobayashi, Oikawa, Otsuka, '15; Cabo-Bizet, Loaiza-Brito, IZ, to appear]
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We consider the chaotic inflationary setup with non-perturbative correction:

\[ V(\phi) = A + \frac{m^2 \phi^2}{2} + \lambda \phi \cos \left( \frac{\phi}{f} \right) \]

- Consider \( \lambda/f < m^2 \): corrections lead to sharp cliffs and gentle plateaus in potential

\[ H^2 = \frac{1}{3M_{Pl}} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \]

\[ \ddot{\phi} + 3H + V'(\phi) = 0 \]

- Friedmann/KG equations can be solved numerically

\( m \sim 3 \times 10^{-7} M_{Pl}, \quad f = 1/3 M_{Pl}, \quad \lambda \sim (3 \times 10^{-5} M_{Pl})^3 \)
BUMPY CHAOTIC INFLATION

- Slow roll last longer (wrt smooth case for same i.c.)

\[
\dot{\phi}(0) = 5M_{Pl}, \quad \dot{\phi}(0) = 0, \quad a(0) = 1
\]
BUMPY CHAOTIC INFLATION

- Slow roll last longer (wrt smooth case for same i.c.)
- Almost all inflation occurs on the plateaus

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**FIG. 4**: Solution to the Friedmann equations with the bumpy potential (2) with $A/d = 0.35M_{Pl}^4$, $f = 1/3M_{Pl}$ and initial conditions $\dot{a}(0) = 5M_{Pl}$, $\dot{a}(0) = 0$ and $a(0) = 1$.

**FIG. 5**: Hubble slow-roll parameters with respect to the number of e-folds before the end of inflation, for the bumpy model solution, Fig. 4. Observable today up to the end of inflation), the Hubble slow-roll parameters undergo strong oscillations, when the inflaton rolls down the steep slopes of the bumps. However, during the shorter range of e-folds which can be probed observationally by the CMB, ($\sim 10$ e-folds around $N = 50$), all the slow-roll parameters are small and smoothly varying. This implies that we do not expect consequent features in the power spectrum or non-Gaussian observables (like the ones explored for example in [21, 38]), since they can occur only at scales not probed by current CMB observations.
BUMPY CHAOTIC INFLATION

- Slow roll last longer (wrt smooth case for same i.c.)
- Almost all inflation occurs on the plateaus
- CMB observables at horizon crossing $N \sim 50$
  \[
  n_s = 0.9667, \quad r = 3.1 \times 10^{-5}, \quad \alpha_s = -0.015
  \]
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- Reduced field range (wrt smooth case $\Delta\phi \sim 15 M_{Pl}$)
  
  $$\Delta\phi \sim 3.2 M_{Pl}$$
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- Reduced inflationary scale (wrt smooth case \( M_{inf} \sim 1.8 \times 10^{16} \text{GeV} \))
  \[ M_{inf} \sim 2.4 \times 10^{15} \text{GeV} \]
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Possible to embed in string theory; back to favoured region
Natural inflation with subleading non-perturbative corrections

\[ V(\phi) = \Lambda^4 \left( 1 - \cos \left( \frac{\phi}{f} \right) \right) + \tilde{\Lambda}^4 \left( 1 - \cos \left( \frac{\phi}{\tilde{f}} \right) \right) \]

smooth natural inflation for \( \tilde{\Lambda} = 0 \), gives for \( N \sim 60 \)
\[ f \gtrsim 6.8 \, M_{Pl}, \quad r \sim 0.1, \quad M_{inf} \sim 10^{16} \, GeV \]

[ Banks-Dine-Fox-Gorbatov, '03; Kappl-Nilles-Winkler, '15 ]
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- We consider \( \Lambda = 3 \times 10^{-4} M_{Pl} \), \( f = 1 M_{Pl} \), \( \tilde{\Lambda} = 2 \times 10^{-4} M_{Pl} \), \( \tilde{f} = 1/3 M_{Pl} \)

- Platteau gives sufficient e-folds for single field with \( f = 1 M_{Pl} \)!

\( N_{tot} \sim 136(18) \) for bumpy (smooth)
Distinctive signatures in CMB

- CMB observables at horizon crossing $N \sim 55$

$$n_s = 0.9677, \quad r = 3.5 \times 10^{-7}, \quad \alpha_s = -0.0025$$

- Enough inflation for (sub-Planckian) decay constant $f = 1 M_{Pl}$. Planckian field range

$$\Delta \phi \sim 1 M_{Pl}$$

- Reduced inflationary scale (wrt smooth case)

$$M_{inf} \sim 7.8 \times 10^{14} \text{GeV}$$
Distinctive signatures in CMB

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Possible to embed in string theory; back to favoured region
CONCLUSIONS

- Non-perturbative corrections can introduce smooth step-like structures to leading order axion potential, whose plateaus give enhanced efolds of inflation.

- Sufficient inflation can be obtained for (sub)-Plackian field ranges & lower inflationary scales: possible to embed in string theory \((M_{inf} < M_s, f \lesssim M_{Pl})\).

- Features in potential give distinctive signatures in CMB (large negative running \(\alpha_s \simeq -10^{-2}, -10^{-3}\)).

- Tensor2Scalar ratio two to four orders of magnitude below future bounds \((r \lesssim 10^{-3})\).

- Can bumpy potentials and parameters emerge from concrete string theory constructions?
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- Tensor2Scalar \(r\) two to four orders of magnitude below future bounds \((r \lesssim 10^{-3})\).

- Can bumpy potentials and parameters emerge from concrete string theory constructions? maybe: from complex structure inflation in type IIB.

\[\begin{align*}
\text{Figure 4: IZ: could we show a much higher order case? \text{The 1 and 4th order look almost the same!}}
\end{align*}\]

\[\begin{align*}
\text{For a flux configuration with a Minkowski vacuum with fluxes} \quad F_1 = 20, \\
H_3 = 8, \\
H_4 = 1, \\
\text{we show the scalar potential as a function of } \theta \text{ with the rest of the moduli fixed at their vev's.}
\end{align*}\]