Listening for primordial gravitational waves

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based on arxiv:1603.01287
with Pierre Binétruy, Mauro Pieroni
Vanilla inflation - classical dynamics

end of inflation at slow-roll parameters:

3 single-field slow roll inflation:
changing the equation of state

\[ H = \frac{1}{\ln \left( \frac{H}{a} \right)} \]

\[ \dot{M}^2 = \frac{P}{\epsilon_0^2} \]

CMB pivot scale, \( \Theta = M^2 f \): 

\[ V_0 \]

reheating; \( \Theta \Rightarrow f \Rightarrow k \Rightarrow m \) 

Primordial vacuum fluctuations before the end of inflation today = comoving Hubble scale: inflation:

\[ f \rightarrow k \rightarrow N_k \rightarrow V(\phi_k), \dot{\phi}_k \]

vacuum fluctuations of inflaton field and metric during inflation

modes leave the horizon during inflation, \( k < a H \)

outside the horizon, modes are frozen

after inflation, modes re-enter and are red-shifted

1:1 correlation

GW spectrum sensitive to primordial spectrum (scalar potential) and post-inflationary expansion
Direct detection vs CMB polarisation

- Tensor anisotropies on last scattering surface
- Polarization of CMB photons through Thomson scattering
- GW travels freely until today

- Distortion of space as GW passes detector
- Lensing: T -> E
- Dust contaminates primordial signal
- B - modes most sensitive
- Ground-based interferometers
- Space-based interferometers
- Pulsar timing arrays

with $r \lesssim 0.1$, planned direkt GW detectors cannot see primordial vacuum fluctuations
Direct detection vs CMB polarisation

Listening for primordial gravitational waves
a generic coupling for a pseudoscalar inflaton:
\[
\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - V(\phi) - \frac{\alpha}{4 \Lambda} \phi F_{\mu \nu} \tilde{F}^{\mu \nu}.
\]
resulting background equations of motion:
\[
\ddot{\phi} + 3 H \dot{\phi} + \frac{\partial V}{\partial \phi} = \frac{\alpha}{\Lambda} \langle \tilde{E} \tilde{B} \rangle.
\]
\[
\frac{d^2 A^a_{\pm}(\tau, k)}{d \tau^2} + \left[ k^2 \pm 2 k \frac{\xi}{\tau} \right] A^a_{\pm}(\tau, k) = 0, \quad \xi = \frac{\dot{\phi}}{2 \Lambda H}
\]
\[\rightarrow\] tachyonic instability for the gauge field, controlled by \[\xi \propto \sqrt{\epsilon} = \dot{\phi}/(\sqrt{2}H)\]
\[\rightarrow\] exponential growth of gauge field modes towards end of inflation
\[\rightarrow\] backreaction on inflaton eom, new friction term: \[\langle \tilde{E} \tilde{B} \rangle \simeq N \cdot 2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2 \pi \xi}\]
+ additional source for scalar and tensor fluctuations
power spectrum of scalar and tensor perturbations affected
pseudoscalar inflation

\[
\ln \left( \frac{H}{a} \right) \quad \text{CMB pivot scale}
\]

\[
\ln (\cdot)
\]

Vanilla inflation - classical dynamics

\[
\ln a
\]

changing the equation of state

single-field slow roll inflation:

\[
3H \dot{\phi} = V_0
\]

slow-roll parameters:

\[
\epsilon = \frac{M_P^2}{V_0} \quad \sigma = \frac{M_P^2 V_0^2}{V_2} \quad \eta = \frac{M_P^2 V_0^2}{V_2^2}
\]

end of inflation at

\[
f \sim 1\quad 10^5
\]

scaling of the horizon

inflation:

\[
a = a_0 \exp(\frac{Ht}{N})
\]

comoving Hubble scale:

today \(N^*\) Hubble times before the end of inflation

\[
N^* = R_H d t \quad N^* \sim 50 \quad 60
\]
Consequently, this constrains the value of $\epsilon_v$. The precise value depends on the scale with an amplitude of $\beta/N^p + O(1/N^{p+1})$.

Evolution of the inflation field:

- additional friction, CMB observables evaluated at 'later' point on
  
  \[ n_s \approx 1 - \frac{O(1)}{N_*}, \quad r = \frac{16 \beta}{N_*}, \quad N_* < N_{CMB} \approx 60 \]

- rapid increase of $\xi$ for large values of $p$; enters eom exponentially

Large effects at the end of inflation, in particular for large $p$ (small $r$!)
perturbation power spectra

\[ \Delta_s^2(k) = \Delta_s^2(k)_{\text{vac}} + \Delta_s^2(k)_{\text{gauge}} = \left( \frac{H^2}{2\pi |\dot{\phi}|} \right)^2 + \left( \frac{\alpha \langle \bar{E} \bar{B} \rangle}{3bH\dot{\phi}} \right)^2 \]

\[ b = 1 - 2\pi \xi \frac{\alpha \langle \bar{E} \bar{B} \rangle}{3\Delta H \dot{\phi}} \]

\[ \Delta_s^2(k) \approx \frac{1}{N(2\pi \xi)^2} \cdot \]

\[ n_s = 1 - \frac{p}{N} - 6\epsilon. \]

\[ \Omega_{\text{gw}} = \frac{1}{12} \left( \frac{H}{\pi M_p} \right)^2 (1 + 4.3 \times 10^{-7} \frac{H^2}{M_p^2 \xi^6} e^{4\pi \xi}) \]

- amplitude @ N\text{CMB} fixes one parameter
- large power on small scales -> PBHs
- nearly universal amplitude on small scales
- increase at small scales to universal value
- low scale models feature stronger increase
- Onset of increase depends on coupling $\alpha$

we find both universal and inflation model specific features
The parameters of the GW spectrum can be described by (implicit) analytical equations.

- $f_1$: gauge contribution takes over in $\Omega_{GW}$
- $f_2$: gauge friction supersedes Hubble friction

- $\Omega_{GW}^{CMB}$: fixed by $r \times \Delta_s^2$
- $\Omega_{GW}^{\text{max}}$: determined by $\epsilon \leq 1$

→ described by (implicit) analytical equations

- $\epsilon_v \approx \beta / N^p$:
  - $p$: slope of increase and vacuum amplitude
  - $\beta$: vacuum amplitude

- $L \equiv \alpha / 4 \Lambda \phi F\tilde{F}$:
  - $\alpha / \Lambda$: shifts spectrum horizontally

$\Omega_{\text{max}} \rightarrow (f_2, \Omega_{GW,2})$

$p_1 < p_2$

$(f_2, \Omega_{GW,2})$

$(f_1, \Omega_{GW,1})$

$(f_1, \Omega_{GW,1})$

$f_{\text{CMB}}$

$f_{\text{max}}$

multi-frequency direct GW tests (eg SKA, eLISA, LIGO/VIRGO) plus CMB polarisation can constrain all 3 parameters

Listening for primordial gravitational waves
pseudoscalar Starobinsky inflation

\[ V(\phi) = V_0 \left( 1 - e^{-\gamma \phi} \right)^2 \]

parameters:
- \( p = 2 \)
- \( \beta = 1/(2\gamma^2) \)
- \( \alpha / \Lambda \)

observables:
- slow-roll observables \( n_s, r \)
- non-gaussianity parameter \( \xi_{\text{CMB}} \)
- direct GW sensitivity LIGO/VIRGO O1 (current), O2:15/16, O5:20-22

Remarkable complementarity between different measurements, multi-frequency and multi-messenger analysis of microphysics of inflation!
GW astronomy has begun - and we are only at the very beginning!

If the inflaton is a pseudoscalar, the GW signal of cosmic inflation can be enhanced by many orders of magnitude, in particular in the range of eLISA and LIGO/VIRGO.

The spectrum is then sensitive to the shape of the inflaton potential.

Universality classes of inflation describe the range of predictions based on only three parameters.

The complementarity of CMB and direct GW measurements provides a powerful probe of the physics of cosmic inflation.

For the future:

….. particle physics: identification of possible gauge groups
….. cosmology: reheating, baryogenesis

eg Kusenko, Schmitz, Yanagida ’14; Anber, Sabancilar ’15; Adshead, Sfakianakis ’15
backup slides
Some useful properties of GWs

perturbations of the background metric: \[ ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(x, \tau))dx^\mu dx^\nu \]
governed by linearized Einstein equation \((\ddot{h}_{ij} = a h_{ij}, \text{ TT - gauge})\)

\[ \ddot{h}_{ij}(k, \tau) + \left( k^2 - \frac{a''}{a} \right) \dot{h}_{ij}(k, \tau) = 16\pi G a \Pi_{ij}(k, \tau) \]
\[ \sim a^2 H^2 \]

\( k \gg aH : h_{ij} \sim \cos(\omega \tau)/a \), \( k \ll aH : h_{ij} \sim \text{const.} \)

a useful plane wave expansion: \[ h_{ij}(x, \tau) = \sum_{P=+,-} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2 \hat{k} \ h_P(k) \ T_k(\tau) \ e^{i P (\hat{k}\cdot\hat{x})} \]
\[ \sim a(\tau_i)/a(\tau) \]

transfer function \( \text{, expansion coefficients} \), polarization tensor \( P = +, \times \)

observational quantity in direct detection

\[ \Omega_{GW} = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(k, \tau)}{\partial \ln k} \], \[ \rho_{GW}(\tau) = \frac{1}{32\pi G} \left\langle \dot{h}_{ij}(x, \tau) \dot{h}^{ij}(x, \tau) \right\rangle \]
Primordial vacuum fluctuations  
(standard inflation)

\[ r = \Delta t^2 / \Delta s^2 \]

CMB

direct

\[ \Omega_{GW}(k) = \frac{\Delta t^2}{12} \frac{k^2}{a_0^2 H_0^2} T_k^2 \approx \frac{\Delta t^2}{12} \Omega_r \]

for \( k_{\text{eq}} \ll k \ll k_{\text{RH}} \)

Rubakov ‘82
Turner, White, Lidsey ’93
Seto, Yokoyama ’03
Smith, Kamionkowski ’05

BICEP2 ‘14
Rubakov ‘82
Turner, White, Lidsey ’93
Seto, Yokoyama ’03
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- hypothetical primordial contribution with \( r \sim 0.17 \)
- peak at horizon \( \text{at } t_{\text{cmb}} \)
- GW measured \( \text{at } t_{\text{cmb}} \)
- current bound \( r \sim 0.1 \)
- scale invariant spectrum
- sensitivity to entire cosmological history
- not detectable in near future

Listening for primordial gravitational waves
Other stochastic backgrounds

Preheating & cosmic strings at GUT scale

GUT-scale phase transition after hybrid inflation,
Buchmuller, Domcke, Kamada, Schmitz '12

new physics at TeV and beyond?

unresolved BH mergers,
first order phase transitions
LIGO/VIRGO collaboration '16,
Caprini et al '15

plot from Bhupal Dev, Mazumdar '16

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further observational signatures

A brief overview:

- CMB: scalar and tensor fluctuations, in particular non-gaussianities
- blue GW signal (enhanced on small scales), maximally chiral. suppressed at CMB scales but interesting for LISA, LIGO/VIRGO,…
- PBH formation due to enhanced scalar power on small scales
- indirekt GW bound from $N_{\text{eff}}$ in BBN and CMB
- primordial magnetic fields

→ very interesting setup for multi-messenger analysis

main focus here on CMB and direct GW observations, can the inflaton gauge field coupling enable us to probe the microphysics of inflation?

References:
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  Anber, Sorbo ‘12
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- Allen ‘96,
  Pagano, Salvati, Melchiorre ‘15
- Durrer, Hollenstein, Jain ‘11,
  Caprini, Sorbo ‘14,
  Fujita et al ‘15