THE CLASSICAL CHROMODYNAMICS OF RELATIVISTIC HEAVY ION COLLISIONS

D. Kharzeev

BNL
1. High parton density regime in QCD is semi-classical (parton saturation, Color Glass Condensate) talks by E. Iancu, K. Itakura

2. Simple "classical" predictions for hadron production at RHIC seem to work

\[ n \sim \frac{2}{N_{\text{part}}} \frac{dN}{dy} \sim \frac{1}{Q_s^2} \]

\# of participants \( N_{\text{part}} \)

3. Strong classical fields modify the properties of the vacuum

\[ n(p) \sim \frac{1}{Q_s^2} \]

4. Much more work is needed, in theory and experiment (e.g., azimuthal correlations, fluctuations)
\[ t \sim \frac{2xP}{k_1^2} \]

large $X$ partons are "frozen"

\[ p_p \sim A^{\frac{1}{3}} \]
Gluon saturation

\[ n \sim \frac{1}{Q^2} \]

when \( \delta \cdot N_g \sim \pi R_A^2 \)
interactions become coherent

\[ 6 \sim \frac{\delta_s}{Q^2} \Rightarrow N_g \sim \frac{R_A^2 Q_s^2}{\delta_s} \]

or \( Q_s^2 \sim \frac{\delta_s N_g}{R_A^2} \)

- In cold nuclear matter,
  \[ N_g \sim A \cdot [xG] \]
  \[ Q_s^2 \sim \delta_s [xG] \cdot A^{1/3} \]

  "multiple scattering"

- Dimensionful scale \( Q_s \) depends on \( s, A, \) centrality
Quasi-classical limit in Quantum Mechanics

\[ \Delta p \Delta q \approx 2\pi \hbar \]

Bohr–Sommerfeld quantization:

\[ \frac{1}{2\pi \hbar} \oint pdq = n + \frac{1}{2} \]

\[ S = \frac{1}{2\pi} \oint pdq \quad \text{in classical mechanics, "adiabatic invariant," does not change under "slow" perturbations} \]

\[ n_{\text{initial}} \approx n_{\text{final}} \]
What is saturation?

\[ L_{\text{glue}} = -\frac{1}{4} \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \]

introduce \( \tilde{A}_\mu = g A_\mu \), \( \tilde{\mathcal{G}}_{\mu\nu} = g \mathcal{G}^{\mu\nu} \)

then \( L = -\frac{1}{4g^2} \tilde{\mathcal{G}}_{\mu\nu} \tilde{\mathcal{G}}^{\mu\nu} \),

and the action \( S \sim \int \frac{1}{g^2} \tilde{\mathcal{G}}_{\mu\nu} \tilde{\mathcal{G}}^{\mu\nu} \, d^4x \)

the number of gluons in a given field configuration is

\[ N_g \sim \frac{S}{\hbar} \sim \frac{1}{\hbar d_s} \]

\( \hbar \to 0 \) classical limit, equivalent to \( d_s \to 0 \)

non-linear interactions when \( \partial \tilde{A} \sim \tilde{A}^2 \)

\[ Q_s^2 \sim \tilde{A}^2 \sim \sqrt{\mathcal{G}^2} \sim \sqrt{p_4} \]
Consider a nucleus

Volume

Density of gluons

\[ q_s \sim \frac{d_s \cdot N_g}{V} \sim \frac{d_s N_g}{R_a^2} \frac{Q_s^2}{R_a^2} \sim Q_s^4 \]

(non-linear interaction)

\[ Q_s^2 \sim \frac{d_s N_g}{R_a^2} ; \quad N_g \sim \frac{R_a^2 Q_s^2}{d_s} \]

General properties of classical fields

\[ \Rightarrow MV \text{ model}, \ldots \]
What are the consequences for observables?

\[ \frac{1}{d_s} \ldots \]

\[ d_s \text{ runs with } Q_s^2 \ldots \]

\[ \Rightarrow S, A, \text{ centrality dependence} \]

determine centrality from the data:

![Graph showing data analysis]

D.K., M. Nardi
PLB '01
nucl-th/0012025
Figure 10: Summary of $\alpha_s(Q)$.
$\frac{dN}{d\eta}$ vs Centrality at $\eta=0$

Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV

- PHOBOS
- PHOBOS PRL
- $pp$
- Kharzeev/Nardi
- HIJING
- EKRT

Peter Steinberg

BROOKHAVEN NATIONAL LABORATORY
Energy and rapidity dependence

HERA: \( a + Q_s^2 \sim 1 \div 2 \text{ GeV}^2 \)

\[ xG \sim \frac{1}{x^\lambda}, \quad \lambda = 0.25 \div 0.3 \]

Even more interesting:
scaling in the data when plotted as a function of

\[ \tau = \frac{Q^2}{Q_0^2} \left( \frac{x}{x_0} \right)^\lambda \]

In high density QCD, this is a consequence of the existence of dimensionful scale

\[ Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right) \]
ZEUS

$Q^2 = 1 \text{ GeV}^2$

- ZEUS NLO-QCD Fit (Prel.) 2001

$Q^2 = 2.5 \text{ GeV}^2$

- tot. error ($\alpha_s$-free Fit)
- tot. error ($\alpha_s$-fixed Fit)
- stat. error ($\alpha_s$-fixed Fit)

$Q^2 = 7 \text{ GeV}^2$

$Q^2 = 20 \text{ GeV}^2$

$Q^2 = 200 \text{ GeV}^2$

$Q^2 = 2000 \text{ GeV}^2$

$x$
Figure 1: Experimental data on $\sigma_{\gamma p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$. 
Centrality, rapidity dependences

The gluon structure function

\[ xG_A(x, P_t^2) = \int d{k_t^2} \varphi_A(x, k_t^2) \]

when \( k_t^2 > Q_s^2 \),

\[ \varphi_A(x, k_t^2) \sim \frac{d \sigma}{d^2 k_t^2} \frac{1}{k_t^2} \]

in the saturation region,

\[ \varphi_A(x, k_t^2) \sim \frac{S_A}{d \sigma} \]
Nuclear collisions:

\[ \frac{E}{N_c} \frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2 - 1} \frac{1}{p_t^2} \int dk_z^2 ds \, \varphi_A(x, k_z^2). \]

\[ \varphi_A(x, (p_t - k_z)^2) \]

differential cross section

\[ \rho_{dN/dy} = \frac{1}{6_{AA}} \int d^2P_t \left( E \frac{d\sigma}{d^3p} \right) \]

GLR; Gyulassy - McLerran

rapidity density
Integrate over transverse momentum:

\[
\frac{dN}{dy} = \text{const} \quad S_A \frac{Q_s^2}{Q_{s,\text{min}}^2} \ln \left( \frac{Q_{s,\text{min}}^2}{\Lambda_{\text{QCD}}^2} \right) 
\]

\[
\cdot \left[ 1 + \frac{1}{2} \ln \left( \frac{Q_{s,\text{max}}^2}{Q_{s,\text{min}}^2} \right) \left( 1 - \frac{Q_{s,\text{max}}}{\sqrt{s}} \right)^4 \right]
\]

Since \( Q_{s,\text{min}}^2 = Q_s^2(s, y=0) e^{\pm \lambda y} \)

\[ x G \sim (1-x)^4 \quad x \rightarrow 1 \]

\[
\frac{dN}{dy} = c \quad N_{\text{part}} \frac{1}{s} e^{-\lambda y} \left[ \ln \left( \frac{Q_s^2}{\Lambda_{\text{QCD}}^2} \right) - \lambda y \right]
\]

\[
\cdot \left[ 1 + \lambda y \left( 1 - \frac{Q_s}{\sqrt{s}} e^{\frac{\lambda y}{2}} \right)^4 \right]
\]

\[ y = 0: \quad \text{coincides with previous} \]
First, transform gluon rapidity distribution into gluon pseudo-rapidity distribution.

Assume $\eta$ distribution (distribution in angle) is $\approx$ conserved during the hadronization.

$$h(\eta; p^\perp; m) = \frac{\cosh \eta}{\sqrt{m^2 + p^\perp^2 + \sinh^2 \eta}}$$

$p^\perp = Q_3$; $m^2 = 2m_Q Q_3$ mini-jets decay into $p$'s
Parton Saturation

- Saturated initial state gives predictions about final state.

\[ N(\text{hadrons}) = c \times N(\text{gluons}) \]
(parton-hadron duality)

\[ \Rightarrow \text{Describes energy, rapidity, centrality dependence of charged particle distributions} \]
Figure 2:

\[ \frac{dN}{d\eta^+} \bigg|_{\eta=0} \bigg|_{\sqrt{s}=200\text{ GeV}} = 620 \]

\[ \lambda = 0.25 \Rightarrow 10\% \text{ increase} \]
Saturation at 200 GeV

Data: 200 GeV shape from nucl-ex/0108009
Prediction: Kharzeev & Levin, nucl-th/0108006

Shape agrees well in central region...
Au+Au Collisions

$dN_{ch}/d\eta/(0.5N_{part})$

$\sqrt{s}=56$ GeV

HIST: HIJING

ISR, UA5, PHOBOS

Saturation model

M. Gyulassy, X.-N. Wang,
Centrality Dependence

\[ |\eta| \leq 1 \text{ AuAu} \]

\[ \frac{dN_{ch}}{d\eta/(N_{\text{part}}/2)} \]

- **PHOBOS** \( \sqrt{s_{NN}} = 200 \text{ GeV} \)
- **PHOBOS** \( \sqrt{s_{NN}} = 130 \text{ GeV} \)
- **UA5** \( \sqrt{s_{NN}} = 200 \text{ GeV} \)
- **pp interpolation** 130 GeV
- **Saturation Model** (\( \lambda = 0.25 \))
- **Two-component Fit**

130 GeV Data nucl-ex/0105011
Accepted in PRC
200 GeV Data submitted to PRC

Does the vacuum "melt"?

Instantons are suppressed at high $T, \mu$

$\Rightarrow$ Systematic calculations possible

- What happens at high gluon density?

Also suppressed in nuclear collisions

\[ n(g) = n_0(g) \exp \left( -c \frac{g^4 Q_s^4}{32 \pi^2 \kappa s N_c (Q_s T)^2} \right) \]

- Melting of the vacuum
- Topological defects?

\approx NO EFFECT FOR A SINGLE NUCLEUS

DK, Lovin, Kovchegov, NPA'01 hep-ph/010624

Lattice: DK, Krasnitz, Venugopalan

hep-ph/0109253
"Phase Diagram" of High Energy QCD

- Classical
  - "Saturation"
  - $q_s \ll 1$
  - $n_g \sim \frac{1}{q_s}$

- Perturbative
  - $q_s \ll 1$
  - $n_g \sim 1$

Axes:
- $E/A$
- $1/x$
- $k_t$, transverse momentum
- $Q^2$

Energy, atomic number, centrality.