Joint Resummation in Electroweak Boson Production at Hadron Colliders

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1. NLO calculations

Drell-Yan process $AB \to \gamma^*(\to l^+l^-)$:

\[ \hat{s} \rightarrow Q^2, Q_T^2 \]

* Mass distribution ($\tau = Q^2/s$)

\[
\frac{d\sigma}{dQ^2} = \sigma_0 \sum_{a,b} \int dx_1 dx_2 dz f_a(x_1) f_b(x_2) \delta(zx_1 x_2 - \tau) \omega_{ab}(z)
\]

\[
\omega_{ab}(z) = \frac{\alpha_s}{\pi} \left[ A \left( \frac{\ln(1-z)}{1-z} \right)_+ + B \left( \frac{1}{1-z} \right)_+ + C(z) \right]
\]

* Transverse momentum distribution

\[
\frac{d\sigma}{dQ_T^2} = \sigma_0 \sum_{a,b} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \frac{d\tilde{\sigma}_{ab}}{dQ_T^2}
\]

\[
\frac{d\tilde{\sigma}_{ab}}{dQ_T^2} = \frac{\alpha_s}{\pi} \left[ A' \left( \frac{\ln(Q^2/Q_T^2)}{Q_T^2} \right)_+ + B' \left( \frac{1}{Q_T^2} \right)_+ + C'(Q_T^2) \right]
\]

⇒ large logarithmic corrections in the limit

$z \to 1$ and $Q_T \to 0$

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Logarithmic corrections

Higher orders

Multiple gluon emission:

Threshold:

\[ \alpha_s^N \frac{\ln^{2N-1}(1-z)}{1-z} \]

Recoil:

\[ \alpha_s^N \ln^{2N-1} \left( \frac{Q^2}{Q_T^2} \right) \]

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\( z \to 1, \, Q_T \to 0 \) close to the phase space edge

only soft radiation allowed

For \( \alpha_s \ln(...)^2 \gtrsim 1 \) traditional fixed-order perturbation theory breaks down
2. Resummation

Reorganization of the standard perturbative expansion in $\alpha_s$ by selecting and summing classes of logarithmic terms to all orders

$\Rightarrow$ In general, resummation follows from (re)factorization (factorization $\Rightarrow$ evolution eq. $\Rightarrow$ resummation)

* dynamical factorization (process independent)
* phase-space factorization (process dependent, often performed in a conjugate space)

Refactorized cross sections exist for both:

$\rightarrow$ recoil resummation (in Fourier conjugate to $Q_T$, b space) [Collins, Soper '83] [Collins, Soper, Sterman '85]
$\rightarrow$ threshold resummation (in Mellin $N$ space) [Sterman '87] [Catani, Trentadue '89]

and are of the form

$$\hat{\sigma} = \hat{\sigma}_0 \int_{\text{inv}} C \exp(S)$$

where

$$S = L f_1(\alpha_s L) + f_2(\alpha_s L) + \alpha_s f_3(\alpha_s L) + ...$$

$$L = \ln(b^2 Q^2), \ln(N)$$

$C$ contains finite contributions

All singular dependence on $L$ exponentiates

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4
Effects of soft gluon resummation:

- Recoil logarithms: Sudakov suppression
  \[
  \exp\left[-\alpha_s \ln^2 \left(\frac{Q^2}{Q_T^2}\right)\right]
  \]

- Threshold logarithms: Sudakov enhancement
  \[
  \exp\left[+\alpha_s \ln^2 (1 - z)\right]
  \]

Combined effect hard to predict:

- \( \uparrow \) extra \( k_T \) from soft radiation allows softer \( 2 \rightarrow 2 \) subprocess at hard scattering

- \( \downarrow \) extra energy for radiation leads to larger \( x \) in parton distribution functions

- \( \uparrow \) large \( x \) means being closer to threshold

\( \Rightarrow \) The two types of logs should be resummed together

\[\text{Joint resummation} = \text{resummation of threshold and recoil logarithms}\]
Joint resummation in electroweak annihilation

Refactorization of the cross section + eikonal approximation + 'webs' techniques
⇒ joint resummation [Laenen, Sterman, Vogelsang '00]

⇒ NLL accuracy
⇒ conserves energy and transverse momentum

\[
\frac{d\sigma^{\text{res}}}{dQ^2 \, dQ_T^2} = \sum_a \sigma_a^{(0)} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2 b}{(2\pi)^2} e^{iQ_T \cdot \vec{b}} \\
\times C'_a(Q, b, N, \mu, \mu_F) \text{ exp} \left[ E_{\alpha}^{\text{PT}}(N, b, Q, \mu, \mu_F) \right] \\
\times C'_a(Q, b, N, \mu, \mu_F)
\]

where

\[
\sigma_a^{(0)} = \frac{4\pi\alpha_s^2}{g^2 S^2} \varepsilon_a^2
\]

\[
C'_a(Q, b, N, \mu) = \sum_j C_{a/j}(N, \alpha_s(\mu)) f_j(N, \mu_F)
\]

\[
E_{\alpha}^{\text{PT}}(N, b, Q, \mu, \mu_F) = 2 \int_0^Q \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T)) \left[ J_0(b k_T) K_0 \left( \frac{2N k_T}{Q} \right) \\
+ \ln \left( \frac{N k_T}{Q} \frac{2N}{Q} \right) \right]
\]

\[
-2 \ln \left( \bar{N} \right) \int_{\mu_F^2}^{Q^2} \frac{dk_T^2}{k_T^2} A_a(\alpha_s(k_T))
\]

where \( \bar{N} = N e_{\gamma e} \) and (at NLL)

\[
A_a(\alpha_s) = \frac{\alpha_s}{\pi} A_a^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 A_a^{(2)} + \ldots
\]

\[
B_a(\alpha_s) = \frac{\alpha_s}{\pi} B_a^{(1)} + \ldots
\]

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\[
\frac{d\sigma_{\text{res}}}{dQ^2 dQ_T^2} = \sum_a \sigma_a^{(0)} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2b}{(2\pi)^2} e^{iQ_T b} \times \times C_a(Q, b, N, \mu, \mu_F) \exp \left[ E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) \right] \times C_{a\bar{a}}(Q, b, N, \mu, \mu_F)
\]

where now \( E_{a\bar{a}}^{\text{PT}} \) expanded at NLL

\[
E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) = \frac{2}{\alpha_s(\mu)} h_a^{(0)}(\lambda, \beta) + 2h_a^{(1)}(\lambda, \beta, Q, \mu)
\]

with

\[
h_a^{(0)}(\lambda, \beta) = \frac{A_a^{(1)}}{2\pi b_0^2} \left[ 2\beta + \ln(1 - 2\beta) \right],
\]

\[
h_a^{(1)}(\lambda, \beta, Q, \mu) = \frac{A_a^{(1)} b_1}{2\pi b_0^3} \left[ \frac{1}{2} \ln^2(1 - 2\beta) + \frac{2\beta + \ln(1 - 2\beta)}{1 - 2\beta} \right] + \frac{B_a^{(1)}}{2\pi b_0} \ln(1 - 2\beta) + \frac{1}{2\pi b_0} \left[ A_a^{(1)} \ln \left( \frac{Q^2}{\mu^2} \right) - A_a^{(2)} \right] \times \left[ \frac{2\beta}{1 - 2\beta} + \ln(1 - 2\beta) \right].
\]

In these equations,

\[
\lambda = b_0 \alpha_s(\mu) \ln(N e^{\gamma_E}),
\]

\[
\beta = b_0 \alpha_s(\mu) \ln(\chi)
\]

\[
C_a(Q, b, N, \mu, \mu_F) = \sum_{j,k} C_{a/j}(N, \alpha_s(\mu)) E_{j,k}(N, Q/\chi, \mu_F) f_k(N, \mu_F)
\]

\( \varepsilon \) - evolution matrix from \( \mu_F \) to \( Q \)

at LO \( \varepsilon = \exp \left( \frac{1}{2\pi} \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \alpha_s(k_T^2) \right) \)

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Joint resummation

* $\chi$ is a function of

\[ \tilde{N} \equiv N e^{\gamma_e} \text{ and } \tilde{b} \equiv b Q e^{\gamma_e}/2 \]

and is chosen to be

\[ \chi(\tilde{N}, \tilde{b}) = \tilde{b} + \frac{\tilde{N}}{1 + \eta \tilde{b} / \tilde{N}} \quad \eta = \text{const.} \]

⇒ Leading and next-to-leading logarithms of $\tilde{N}$ and $b$

correctly reproduced in the limits:

$\tilde{N} \to \infty$ (at fixed $b$)

$b \to \infty$ (at fixed $\tilde{N}$).

⇒ at large $b$ corrections to the leading term suppressed as $1/\tilde{b}^2$

→ when $\tilde{b} \gg \tilde{N}$ $\alpha_s$–expansion of $E_{a\bar{a}}^{PT}$ has

same logs of $(Q^2/Q_T^2)$ as fixed-order expression at NLO.
Joint resummation: transforms

* Inverse \( \bar{N} \) and \( b \) transforms – integrals along contours in \( (\bar{N}, b) \) complex space

**N** contour:

\[
N = C + ze^{\pm i\phi}
\]

Landau pole:

\[
\beta = \frac{1}{2} \quad \Rightarrow \quad \chi(\bar{N}, \bar{b}) = \exp\left[1/(2b_0\alpha_s(\mu))\right] = \rho_L
\]

rightmost pdf singularity \( C < \rho_L \)

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Joint resummation: transforms

* Singularities in $b$ space:

→ Landau pole

$$
\chi(\tilde{N}, \tilde{b}) = \tilde{b} + \frac{\tilde{N}}{1 + \frac{b}{4\tilde{N}}} = \exp \left[ \frac{1}{2b_0 \alpha_s(\mu)} \right] \equiv \rho_L
$$

→ $\chi = 0 \Rightarrow \tilde{b} = -2\tilde{N}$

→ $1 + \frac{b}{4\tilde{N}} = 0 \Rightarrow \tilde{b} = -4\tilde{N}$

**b contour:**

$$
\int d^2 b \ e^{iQ_T \tilde{b}} f(b) = 2\pi \int_0^\infty db \ J_0(bQ_T) f(b)
$$

$$
= \pi \int_0^\infty db \left[ h_1(bQ_T, v) + h_2(bQ_T, v) \right] f(b)
$$

where $h_{1,2}(z, v)$ are related to Hankel functions:

$$
h_1(z, v) \equiv -\frac{1}{\pi} \int_{-\pi + iv\pi}^{-\pi + iv\pi} d\theta e^{-iz \sin \theta}, \quad h_2(z, v) \equiv -\frac{1}{\pi} \int_{\pi + iv\pi}^{-i\pi} d\theta e^{-iz \sin \theta}.
$$

$$
0 < \left( \frac{C}{1 + b_0 Q/8C} + \frac{b_0 Q}{2} \right) e^{\gamma\epsilon} < \rho_L
$$

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Joint resummation

Matching prescription

\[
\frac{d\sigma}{dQ^2dQ_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2dQ_T^2} - \frac{d\sigma^{\text{exp}(k)}}{dQ^2dQ_T^2} + \frac{d\sigma^{\text{fixed}(k)}}{dQ^2dQ_T^2}
\]

* avoids double-counting of terms non-singular in \(Q_T\)
* returns positive cross section even at large \(Q_T\), no need to switch to pure NLO

Non-perturbative input

Next-to-leading log joint exponent in the limit of small \(k_T\) of soft radiation

\[
E \sim \left(-\frac{b^2}{2} + \frac{2N^2}{Q^2}\right) \int_0^\lambda^2 dk_T^2 A_a(\alpha_s(k_T)) \ln \left(\frac{Q}{Nk_T}\right)
\]

\(\Rightarrow\) Gaussian form in \(\tilde{N}\) and \(b\) of the non-perturbative function

* electroweak boson production - away from threshold
\(\Rightarrow\) small \(N\) \(\Rightarrow\) only \(b^2\) term kept

\[
E_{a\bar{a}}^{P^T}(N, b, Q, \mu) \rightarrow E_{a\bar{a}}^{P^T}(N, b, Q, \mu) - gb^2,
\]
Joint resummation

$Z$ production at the Tevatron, CDF data

[AK, Sterman, Vogelsang, hep-ph/0202251]
3. Conclusions

New joint resummation formalism

- successfully sums threshold and recoil corrections to next-to-leading log accuracy
- conserves energy and transverse momentum
- recovers threshold and recoil resummation formalisms in the appropriate limits
- does not require any extra dimensional scales
- suggests a form of non-perturbative input
- provides very good description of Z boson production at the Tevatron and much more...