The Movie:
O.P.E. and power corrections to the QCD coupling constant.

Casting:
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Produced by

The story:

Lattice data seems to show that power corrections should be convoked to describe appropriately the transition of the coupling constant running from U.V. to I.R. domains. Those power corrections for the Landau-gauge MOM coupling constant can be analysed in terms of Operator Product Expansion (O.P.E.) of two- and three-point Green functions, the gluon condensate $< A^2 >$ emerging from this study. The semi-classical picture given by instantons can be also used to look for into the nature of the power corrections and gluon condensate.
How can it be computed?

- **Other methods**

- **The "Ours": Green function Methods**
  - **A remake:** Ph. Boucaud et al. JHEP 10 (98) 017.
First Act: ... From the Lattice!

Then:

- 1000 confs. for:
  - $a = 0.033 \text{ fm} (\beta = 6.8)$,
    - $V = (0.53 \text{ fm})^4 (N = 16)$.
    - $V = (0.79 \text{ fm})^4 (N = 24)$.
  - $a = 0.072 \text{ fm} (\beta = 6.2)$,
    - $V = (1.7 \text{ fm})^4 (N = 24)$
  - $a = 0.100 \text{ fm} (\beta = 6.0)$,
    - $V = (1.2 \text{ fm})^4 (N = 12)$.
    - $V = (1.6 \text{ fm})^4 (N = 16)$.
    - $V = (2.4 \text{ fm})^4 (N = 24)$
    - $V = (3.2 \text{ fm})^4 (N = 32)$

Results:

Three-point

$\alpha_S^{\text{MOM}}(4.32 \text{ GeV}) = 0.269(3)$
$\Lambda_{\text{MS}}(4.32 \text{ GeV}) = 299(7) \text{ MeV}$

Two-point

$\alpha_S^{\text{MOM}}(9.6 \text{ GeV}) = 0.176(2)$
$\alpha_S^{\text{MOM}}(9.6 \text{ GeV}) = 0.193(3)$
$\Lambda_{\text{MS}} = 319(14) \text{ MeV}$

Moriond 2002
\[ \frac{d \ln Z_{\text{Mom}}(\mu^2)}{d \ln \mu^2} = - \Gamma^R(\alpha_s(\mu)) \]

\[ \frac{\partial \alpha_s(\mu)}{\partial \ln \mu} = \beta(\alpha_s) \]

**Network Diagram**

- Initial Conditions: \(Z(\mu_0), \alpha_s(\mu_0)\)
- Perturbative Region:
  - \(\Lambda_{\overline{MS}}\)
  - \(\alpha_s(\mu_0)\)
  \[ \Lambda_{\overline{MS}} = \Lambda e^{(\alpha_s(\mu_0) - \alpha_s(\text{inst.})) / 2 \ln \Lambda} \]

**Questions and Connections**

- \(Z_{\text{Mom}}(\Lambda)\) ?
- \(Z_{\text{Mom}}(\mu^2, \Lambda)\)
\( \beta = 6.2 ; \) Volume = 24

\( \alpha \)

\( \mu \)

\( Z_3 \)

\( \chi^2 = 0.52 \)

window = [2.78, 4.32] (GeV)
Lattice data at
\( \beta = 6.0 \) and \( \beta = 6.2 \)

High statistics data (1,000 config.s):
\( \beta = 6.0, 6.2, 6.8 \)
extrapolated to \( \nu \to \infty \)
A "cheapest" solution

Power corrections:

\[ G(p^2) = G(p^2)_{3\text{loops}} \left( 1 + \frac{c}{p^2} \right) \]

\[ \alpha_s(p^2) = \alpha_s(p^2)_{3\text{loops}} \left( 1 + \frac{c'}{p^2} \right) \]  

(1)

Notice: \( < A_\mu A^\mu > \neq 0 \) in the Landau gauge.

The power correction is sizeable at 10 GeV !!!

Indeed \( O(1/10^2) \sim 1 \% \), \( \alpha^3 \sim 0.5 \% \)

Result:

\[ \Lambda_{\overline{\text{MS}}} = 237 \pm 4 \pm 10\text{MeV} \]

To compare to Schrödinger functional (ALPHA collaboration):

\[ \Lambda_{\text{MS}} = 238 \pm 19 \text{ MeV} . \]
Second Act: In terms of OPE

⇒ O.P.E. and S.V.Z. QCD sum rules lead to:

\[ ( \ldots )_{N.P} \equiv ( \ldots )_{Pert.} \]

\[ + ( \ldots + \ldots + \ldots ) \times \langle A^2 \rangle \]

⇒ In M.O.M. scheme:

\[
\begin{align*}
c &= \frac{6\pi^2}{\beta_0(N_c^2-1)} \left( \ln \frac{p_\Lambda}{\Lambda} \right)^{\gamma_0+\hat{\gamma}_0-1} \langle A^2 \rangle_{\mu} \left( \ln \frac{\mu}{\Lambda} \right)^{-\gamma_0+\hat{\gamma}_0} \\
c' &= \frac{18\pi^2}{\beta_0(N_c^2-1)} \left( \ln \frac{p_\Lambda}{\Lambda} \right)^{\gamma_0+\hat{\gamma}_0-1} \langle A^2 \rangle_{\mu} \left( \ln \frac{\mu}{\Lambda} \right)^{-\gamma_0+\hat{\gamma}_0}
\end{align*}
\]

(2)

\[ \gamma_0 = 13/2, \beta_0 = 11, \hat{\gamma}_0 = 3N_c/4 \]
Remark about \( MOM \).

Operator Product Expansion for \( G^{(3)}(k, k, k) \) (MOM) leads directly to the \( \langle A^2 \rangle \) condensate, as in \( G^{(2)}(k^2) \), but in \( G^{(3)}(k, -k, 0) \) (MOM), due to the presence of the soft gluon,

\[
\begin{align*}
A^2 \ 	ext{condensate here emerges from vacuum insertion factorisation hypotheses:} \\
\langle : A(0)A(0)A(0) : \tilde{A}(0) \rangle & \rightarrow \langle A(0)A(0) | 0 \rangle \langle 0 | A(0) \tilde{A}(0) \rangle = \\
= \langle A^2 \rangle \tilde{G}^{(2)}(0)
\end{align*}
\]

The calculation of \( \langle A^2 \rangle \) anomalous dimension approves this approach, as terms breaking this hypotheses are negligible.
O.P.E. results. $\langle A^2 \rangle$.

$\Lambda_{\text{MS}}$ and of $\langle A^2 \rangle_{R,\mu}$ at $\mu = 10$ GeV condensate are fitted to be:

<table>
<thead>
<tr>
<th></th>
<th>3 loops</th>
<th>MOM</th>
<th>MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_{\text{MS}}$</td>
<td></td>
<td>233(28) MeV</td>
<td>260(18) MeV</td>
</tr>
<tr>
<td>$\left{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right}_{\text{prop}}$</td>
<td>1.55(17) GeV</td>
<td>1.39(14) GeV</td>
<td></td>
</tr>
<tr>
<td>$\left{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right}_{\text{alpha}}$</td>
<td>1.9(3) GeV</td>
<td>2.3(6) GeV</td>
<td></td>
</tr>
</tbody>
</table>

MOM:

\[
\left( \frac{\left\{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right\}_{\text{alpha}}}{\left\{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right\}_{\text{prop}}} \right)^{2\text{loops}} = 3.65(4) \rightarrow \left( \frac{\left\{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right\}_{\text{alpha}}}{\left\{ \sqrt{\langle A^2 \rangle_{R,\mu}} \right\}_{\text{prop}}} \right)^{3\text{loops}} = 1.7(3)
\]

Comments

- A clear convergence from two to three loops is observed.
- A value of $\Lambda_{\text{MS}}$ coherent with Schroedinger Functional's emerges from this study.

The OPE analysis seem to give a coherent picture of power corrections to QCD coupling constant and gluon propagator due to an $\langle A^2 \rangle$ condensate.
Epilogue: Conclusions.

- An $\langle A^2 \rangle$ condensate appears in Landau Gauge Green Functions whose induced power correction is still present at energies $\sim 10$ GeV.
- A coherent O.P.E. description of these power corrections has been presented.
- An estimation of $\Lambda_{\overline{MS}}$ coherent with previous results emerges from this study.
- A crude Free Instanton Approximation provides a rather good description of gluon propagator at high momentum for a cooled lattice configuration.
- Preliminary results based in an instanton background explain this $\langle A^2 \rangle$ condensate as a Instanton contribution.

This talk is based on:

- JHEP0004(2000)006
- PLB493(2000)315
- PRD63(2000)114003
- PRD64(2001)114003
- hep-ph/0203119