NC and CC Cross-Sectons at High $Q^2$

Eram Rizvi

Birmingham University

Neutral Current Measurements
Charged Current Measurements
Proton PDFs
Neutral Current DIS Structure Functions

\[
\frac{d^2 \sigma_{NC}^\pm}{dx dQ^2} = \frac{2 \alpha \pi^2}{Q^4 x} \left[ Y_+ \tilde{F}_2 + Y_- x \tilde{F}_3 - y^2 F_L \right]
\]

\[
\tilde{F}_2 \equiv F_2 - \nu_e \frac{\kappa_w Q^2}{Q^2 + M_Z^2} F_2 \gamma^Z + (\nu_e^2 + a_e^2) \left[ \frac{\kappa_w Q^2}{Q^2 + M_Z^2} \right]^2 F_2^Z
\]

\[
x \tilde{F}_3 \equiv -a_e \frac{\kappa_w Q^2}{Q^2 + M_Z^2} x F_3 \gamma^Z + (2\nu_e a_e) \left[ \frac{\kappa_w Q^2}{Q^2 + M_Z^2} \right]^2 x F_3^Z
\]

In Leading Order:

\[
\tilde{F}_2 \propto \sum_{\text{quarks}} (xq + x \bar{q}) \quad x \tilde{F}_3 \propto \sum_{\text{quarks}} (xq - x \bar{q})
\]

\[
\kappa_w = \frac{1}{4 \sin^2 (\theta_w) \cos^2 (\theta_w)}
\]

Reduced cross section

\[
\tilde{\sigma}_{NC}^\pm = \tilde{F}_2 \quad \text{when} \quad F_L = x \tilde{F}_3 = 0
\]

\[
\tilde{\sigma}_{NC}^\pm = \frac{Q^4 x}{2 \alpha \pi^2} \frac{d^2 \sigma}{Y_+ dx dQ^2} = \left[ \tilde{F}_2 + \frac{Y_-}{Y_+} x \tilde{F}_3 - \frac{y^2}{Y_+} F_L \right]
\]
Neutral Current DIS – Reduced Cross Section

\[ \sigma_{NC}^\pm = \frac{x Q^4}{2 \pi \alpha^2} \frac{1}{Y_+} \frac{d^2 \sigma^{NC}}{dxdQ^2} \]

HERA Neutral Current

- **SM shows good agreement with data over all** \( Q^2 \) **range**
Neutral Current DIS – $x F_3$ Extraction

$$\frac{d^2 \sigma_{NC}^\pm}{dxdQ^2} = \frac{2 \alpha \pi^2}{Q^4 x} \left[ Y_+ \tilde{F}_2 \mp Y_- x \tilde{F}_3 - y^2 \tilde{F}_L \right]$$

- Sign of $x F_3$ term depends on lepton charge
- $F_L$ small at high $Q^2$

Extraction Method

$$\sigma^- = \frac{d^2 \sigma^-}{dxdQ^2} = \frac{2\pi \alpha^2}{xQ^4} \left[ Y_+^{920} \tilde{F}_2 - Y_-^{920} x \tilde{F}_3 \right]$$

$$\sigma^+ = \frac{d^2 \sigma^+}{dxdQ^2} = \frac{2\pi \alpha^2}{xQ^4} \left[ Y_+^{820} \tilde{F}_2 - Y_-^{820} x \tilde{F}_3 \right]$$

- Error limited by electron sample (16 pb$^{-1}$)
- Positron data sample (~100 pb$^{-1}$)
Neutral Current DIS – $xf_3$

Direct sensitivity to the valence quarks

Quantify level of agreement by testing sum-rule:

$$\int F_3^{xyz} \, dx = 2e_u a_u N_u + 2e_d a_d N_d = \frac{5}{3} \cdot O(1 - \alpha_s / \pi)$$

**H1:**

$$\int F_3^{xyz} \, dx = 1.88 \pm 0.35 \text{(stat.)} \pm 0.27 \text{(syst.)}$$

**QCD Fit:**

$$\int F_3^{xyz} \, dx = 1.11$$

Moriond QCD 2002 – Les Arcs  
NC and CC Cross-Sections at High $Q^2$  
Eram Rizvi – Birmingham University
Neutral Current DIS – $F_L$ Extraction

\[
\frac{d^2 \sigma_{NC}^+}{dx dQ^2} = \frac{2 \alpha \pi^2}{Q^4 x} \left[ Y_+ \tilde{F}_2(x, Q^2) + Y_+ x \tilde{F}_3(x, Q^2) - y^2 F_L(x, Q^2) \right]
\]

- $y^2 F_L$ small at high $Q^2$ → need high $y$
- H1 data extend to small electron energies

$F_L$ extracted from cross section by assuming $F_2$ from QCD fit:

\[
F_L = \frac{Y_+}{y^2} \left[ F_2 - A \sigma_{NC} \right]
\]

$A$ is normalisation factor between fit and measurement determined from data with $y<0.6$
\[ \tilde{\sigma}^-_{cc} = x \left[ u + c + (1 - y)^2 (\bar{d} + \bar{s}) \right] \]
\[ \tilde{\sigma}^+_{cc} = x \left[ \bar{u} + \bar{c} + (1 - y)^2 (d + s) \right] \]

**HERA Charged Current**

- Possibility to extract \( d/u \) using proton as target – Higher Statistics
High x PDFs

NC → xu\textsubscript{v}  \hspace{1cm} CC e^+ → xd\textsubscript{v}

H1 Preliminary

- QCD fit to H1 data alone yields precision on xu\textsubscript{v} ~6–10%; xd\textsubscript{v} ~20%
- H1 perform local extraction of PDFs with small theory uncertainty – independent of assumptions in a QCD fit.
- Both methods in agreement with global analyses.

Conclusion: 1fb\textsuperscript{-1} HERA data will have a large impact on SM predictions at LHC