The Colour Glass Condensate:
An Introduction

- A 2 year old concept (Moriond, 2000)
- A physical picture of the high density gluonic system at small x
  - derived from QCD
  - verifiable consequences (cf. next talk by Kazunori Itakura)
- Based on many years of work, involving many people:
  - McLerran, Venugopalan (94)
  - Jalilian-Marian, Kovner, Leonidov, Weigert (97)
  - Leonidov, McLerran, E. I. (twice 98)
  - Ferreiro, Itakura
- Results consistent with different approaches by
  - A. Mueller, Balitsky, Kovchegov, Weigert

A longer introduction: hep-ph/0202270, Cargèse lectures (74 pages !)
High Energy Scattering in QCD

- Relevant degrees of freedom = Small-x partons

\[ x = \frac{k^+}{P^+} = \text{longitudinal momentum fraction} \]

\[ x \sim \frac{Q^2}{s} \ll 1 \text{ as } s \to \infty \]

![Diagram of scattering process](image)

\[ k_z \ll P \]

maximal overlap

- Parton distributions rise strongly with \( \frac{1}{x} \)

- Linear evolution eqs. (BFKL, DGLAP) may explain this rise...

\[ dP(2) \sim \alpha_s \frac{d^2 \tau}{2^+} \to \alpha_s \ln \frac{1}{x} \]

\[ p^+ \gg q_1^+ \gg q_2^+ \gg \ldots \gg k^+ \]

\[ xG(x, Q^2) = \frac{dN_{\text{gluons}}}{d \ln \frac{1}{x}} \sim e^{\alpha_s \ln \frac{1}{x}} \]

...but suffer from inconsistencies at very small \( x \):

- Violation of unitarity bounds: \( G_{\text{BFKL}} \sim S^{\alpha_s} \)

- Infrared diffusion: \( k_1^2 \to 0 \) within the ladder

- At high density, non-linear effects should limit the growth.
Gluon Density Grows

Low Energy

High Energy
The Idea of Saturation

- When should non-linear effects become of order 1?

\[ \text{Radiation} = \text{Recombination} \Rightarrow \text{Saturation} \]

(Grivob, Levin, Ryshin, 83)

- Interaction probability = \( \Theta(1) \)

\[ \frac{\alpha_s N_c}{Q^2} \times \frac{G(x, Q^2)}{\pi R^2} \gg 1 \]

for \( Q^2 \leq Q_s^2(x) \equiv \text{saturation momentum} \)

\[ Q_s^2(x) \sim \alpha_s N_c \frac{xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} e^{C_s \log \frac{1}{x}} \]

- Very small \( x \) and/or very large \( A \)

\[ \Rightarrow Q_s^2(x) \gg \Lambda_{\text{QCD}}^2 \Rightarrow \alpha_s(Q_s^2(x)) \ll 1 \]

- Ordinary perturb. theory breaks down by non-linear effects

- Weak coupling & large occupation numbers (\( \sim \frac{1}{\alpha_s} \))

= Semi-Classical regime

High-density gluons \( \leftrightarrow \) Strong classical colour fields

(McLerran-Venugopalan, 94)
The Effective Theory for the Colour Glass

Small-\(x\) gluons \(\quad k^+ = x P^+ \quad\) Classical fields radiated by the fast moving partons \(\quad p^+ \gg k^+ \quad\)

\[
(D^\gamma F^\gamma M)_a(x) = S^{k^+} S(x^-) S_0(x_\perp)
\]

\(\quad p^+ \gg k^+ \quad\) \(\quad k^+ \quad\)

Fast d.o.f. are frozen in some \underline{random} configuration.

\[
W_z[p] : \text{probability density for } p (z = \ln \frac{1}{x})
\]

\[
\langle A^i_x A^j_y \ldots \rangle_z = \int dp \ W_z[p] \ A^i_x[p] A^j_y[p] \ldots
\]

\(\Rightarrow\) Colour Glass

- Spin Glass

\(S_i = \pm 1: \text{dynamical variables}\)

Random links \(f_{ij}\)

"Quenched" variables

- The opposite of a Plasma

At saturation: \(A^i[p] \sim \frac{1}{N} \rightarrow \) Exact solution required
The Classical Solution

A non-Abelian Weiszäcker-Williams field.

\[\begin{align*}
E_z &= B_z = 0 \\
E_x &= B_y; \quad E_y = -B_x \quad (\vec{E} \cdot \vec{B} = 0)
\end{align*}\]

\[E^i(x^1, x^2) = S(x^+ V(x^1) \Theta^i V^+(x^1) \quad (i=x,y)\]

\[V^+(x^1) = P \exp \left( ig \int dx^- \alpha^a_\alpha(x^+ x^1) T^a \right)\]

Wilson line

\[-\nabla_1^2 \alpha^a_\alpha = p^a : 2\text{-dimens Coulomb field}\]

[Light-cone gauge \( A^+ = 0 \)]
Non-Linear Quantum Evolution

$W_z[p]$: Integrate out fast gluons in layers of $\zeta \equiv \ln \frac{p^+}{\Lambda^+}$

\[ \frac{\partial W_z[p]}{\partial \zeta} = \int \frac{\alpha_s}{2} \frac{S}{S_p(x_1)} \chi_{x_1y_1}^{ab} [p] \frac{S}{S_p(y_1)} W_z[p] \]

A functional diffusion eq. on a manifold with metric $\chi[p]$

\[ \delta \langle p^a(x_1) p^b(y_1) \rangle = \chi^{ab}_{x_1y_1}[p] = \alpha_s \ln \frac{1}{b} \chi^{ab}_{x_1y_1} [p] \]

$\chi^{ab}_{x_1y_1}[p] = \int \frac{x^{i-2i} (x_1 - z_1)^2}{(x_1 - z_1)^2} \frac{y^{i-2i} (y_1 - z_1)^2}{(y_1 - z_1)^2} (1 - V_x^+ V_z)_{ac} (1 - V_z^+ V_y)_{cb}$

BFKL-like Non-linear in $p$

\[ \text{Weak fields} \Rightarrow V_x^+ \sim 1 + ig\alpha(x_1) \Rightarrow BFKL \]

\[ Q_s(z) \leftrightarrow \text{Inverse correlation length of } \langle V_x^+ V_y \rangle \]
Gluon Saturation

- \( Q_s^2(z) \approx A^2 e^{C \alpha_s z} \); \( C = \frac{4}{5} \frac{N_c}{\pi} \)
- Gluon distribution in the transverse phase-space

\[
N_z(k_t) = \frac{1}{4 \alpha_s^2} \frac{dN}{d^2k_t} \propto \left\langle |E_1(\vec{r})|^2 \right\rangle
\]

\[
\Lambda_{QCD} \quad Q_s(\tau_1) \quad Q_s(\tau_2) \quad k_t
\]

\[
N_z(k_t) \propto \begin{cases} 
\frac{\Lambda^2}{k_t^2} e^{\sqrt{4 \alpha_s \ln \left( k_t^2 / \Lambda^2 \right)}} & \text{for } k_t \gg Q_s(z) \\
\frac{1}{\alpha_s} \left( \frac{Q_s^2(z)}{k_t^2} \right)^{1-\delta} & \text{for } Q_s \leq k_t \leq \frac{Q_s^2}{\Lambda} \\
\frac{1}{\alpha_s} \ln \left( \frac{Q_s^2(z)}{k_t^2} \right) \sim Z & \text{for } k_t \ll Q_s(z)
\end{cases}
\]

- Geometric Scaling up to \( k_t \ll \frac{Q_s^2}{\Lambda} \)