QCD in Beauty Decays : Successes and Puzzles

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- We have the QCD-based theory of B decays

- It works at the nonperturbative level

  impressive agreement with experiment
  gives nontrivial predictions
  allows precision extraction of $|V_{cb}|$ and $|V_{ub}|$
  makes suggestions for next generation experiments

- There are puzzles[1] which are to be clarified

  Theoretical insights plus experimental data are needed

[1] of more than simply technical nature

Expansion in $\frac{\Lambda_{\rm QCD}}{m_b}$ requires dynamics

Physics of a heavy quark is simple. Dynamics of light degrees of freedom in the presence of the heavy quark

Bound-state $\longleftrightarrow$ nonperturbative effects

Can they be controlled?

QCD allows to establish a number of facts

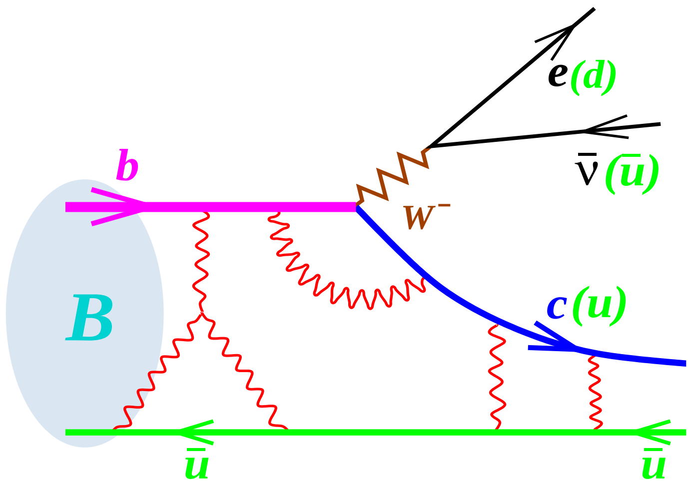
Lifetime hierarchy of beauty (and even charm) particles

Most informative are inclusive decays admit local OPE

Certain dynamical predictions are quite nontrivial

More precise statements are those of most general nature, hence independent of a possible mechanism of confinement, resulting hadron spectrum, ...

Lifetimes and inclusive decay widths



Quark level:

analogy with muon decay

$$\Gamma = \frac{G_F^2 M_b^5}{192\pi^3} \cdot z(m_c, m_b) \cdot N_c \cdot \left(1 - \frac{\alpha_s}{\pi} \ldots\right) \cdot \ldots \cdot \left\{ \begin{matrix} |V_{cb}|^2 \\ |V_{ub}|^2 \end{matrix} \right\}$$

How accurate is this?

Nonperturbative effects? $\sim \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^k$

Can they be computed?

M_B or m_b ?

$$M_B - m_b \approx 500\,\text{MeV} \sim \Lambda_{\rm QCD}$$
$$M_{\Lambda_b} - M_B \approx 370\,\text{MeV} \sim \Lambda_{\rm QCD}$$

$$\frac{M_B^5}{m_b^5} \approx 1.5 \text{ to } 2 \propto 1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

Can we go beyond this 'accuracy'?

QCD theorem (1992)

Bigi, Uraltsev, Vainshtein

No $\Lambda_{\rm QCD}/m_b$ corrections to inclusive widths of heavy flavor hadrons

Applies to all types: semileptonic, nonleptonic, $b \to s+\gamma$, $b \to s\,\ell^+\ell^-$, ...

Part II : $1/m_b^2$ effects are computed

$1/m_b^3$ effects are known as well

$$B,\; B_s,\; \Lambda_b, \ldots \qquad \frac{\Delta M}{M} \sim \frac{\Lambda_{\rm QCD}}{m_b} \qquad \text{yet} \qquad \frac{\Delta\Gamma}{\Gamma} \sim \frac{\Lambda^2_{\rm QCD}}{m_b^2}$$

$$M_B = m_b + \overline{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_b} + \frac{\mu_3^2}{m_b^2} + \ldots$$

$\overline{\Lambda}$ does not affect the width!

Exclusive property of QCD. Follows from the gauge nature of QCD interaction

Exact cancellation of the bound state effects with the final state interaction

Bound state & hadronization effects are given by `local` HQ operators $\bar{b}Ob$

Order $1/m_b^2$: $\quad \mu_\pi^2 = \langle B|\,\bar{b}\,(i\vec{D})^2 b\,|B\rangle\,, \quad \mu_G^2 = \langle B|\bar{b}\frac{i}{2}\sigma G b|B\rangle$

Order $1/m_b^3$: $\quad \langle B|\bar{b}\,\Gamma b\,\bar{q}\,\Gamma' q|B\rangle$

etc.

Practical applications

Extracting $|V_{cb}|$, $|V_{ub}|$ from $\Gamma_{\rm sl}(B)$

Need accurate values of QCD parameters

m_b, m_c $(m_b - m_c)$, μ_π^2, μ_G^2, ρ_D^3, ...

Replace models and their attributes used early on

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s + \gamma)$ decay distributions themselves

BSUV, 1993-1994

Nowadays is being implemented in experiment

With `comprehensive` approach we can do robust analysis without relying on $1/m_c$ expansion, or invoking unknown *nonlocal correlators*

Expansion in $1/m_c$ is questionable

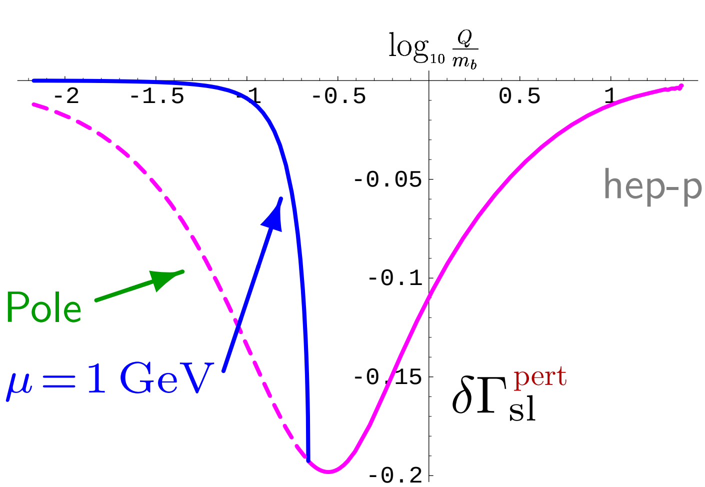
Theoretical status

Can aim at $1\,\%$ level in $|V_{cb}|$ assumes technical progress in theory

$|V_{ub}|$? – underway, 5% accuracy is realistic

An often question: How can this be true?

Perturbative corrections ? ...



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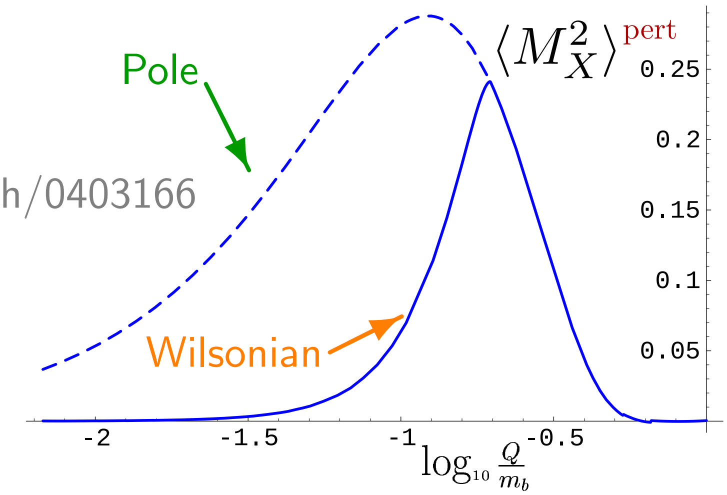
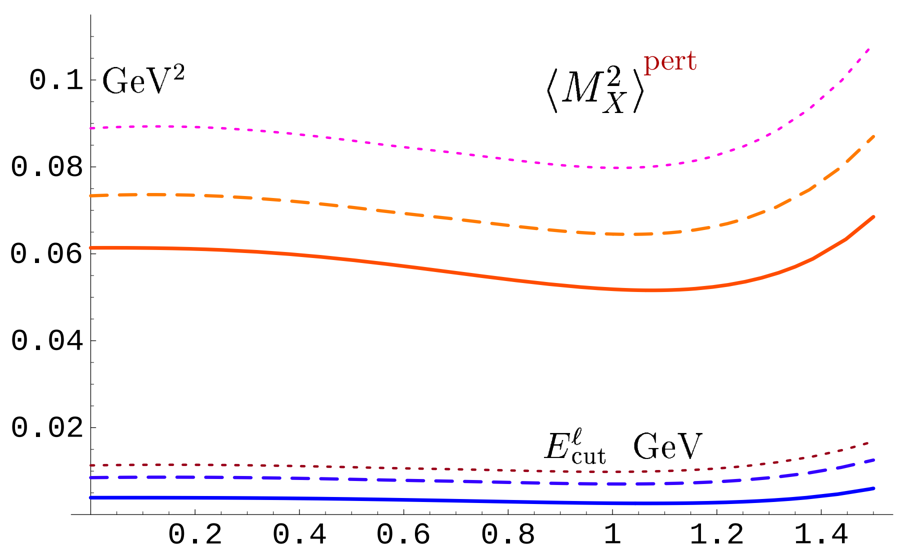


With the IR piece cut off according to Wilson

we can `work for precision!`

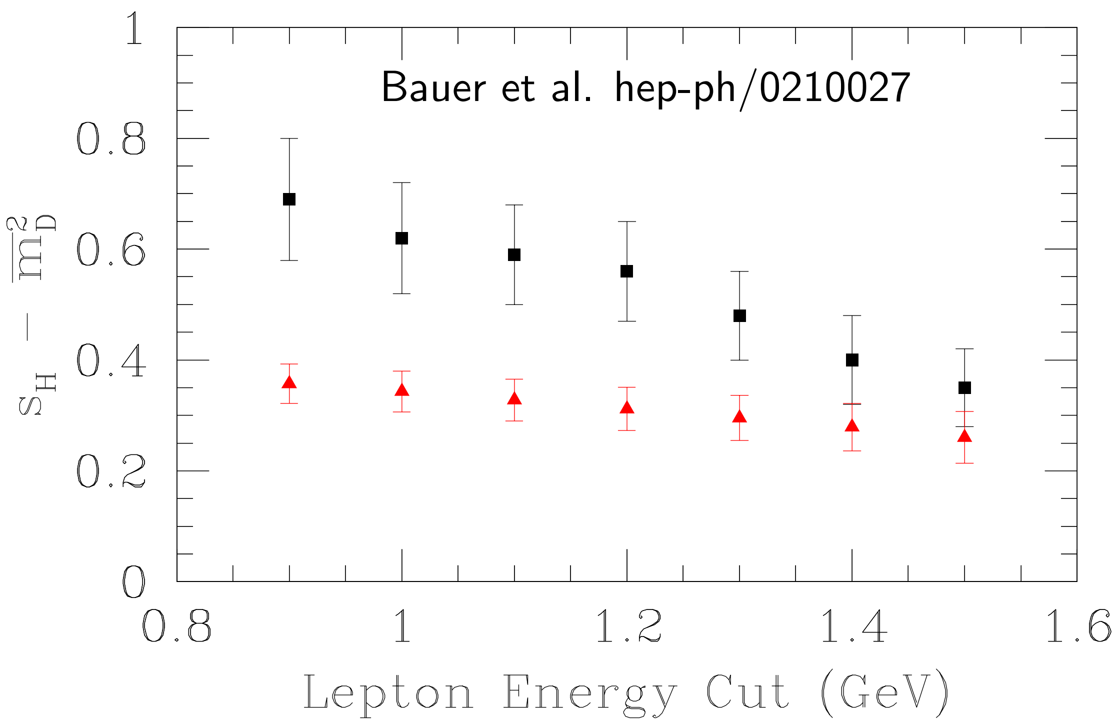
Corrections in the scheme with the hard cutoff, $\mu = 1\text{GeV}$. Within pole-type approaches the correction is 4-6 times larger and strongly decreases at larger $E_{\rm cut}^{\ell}$



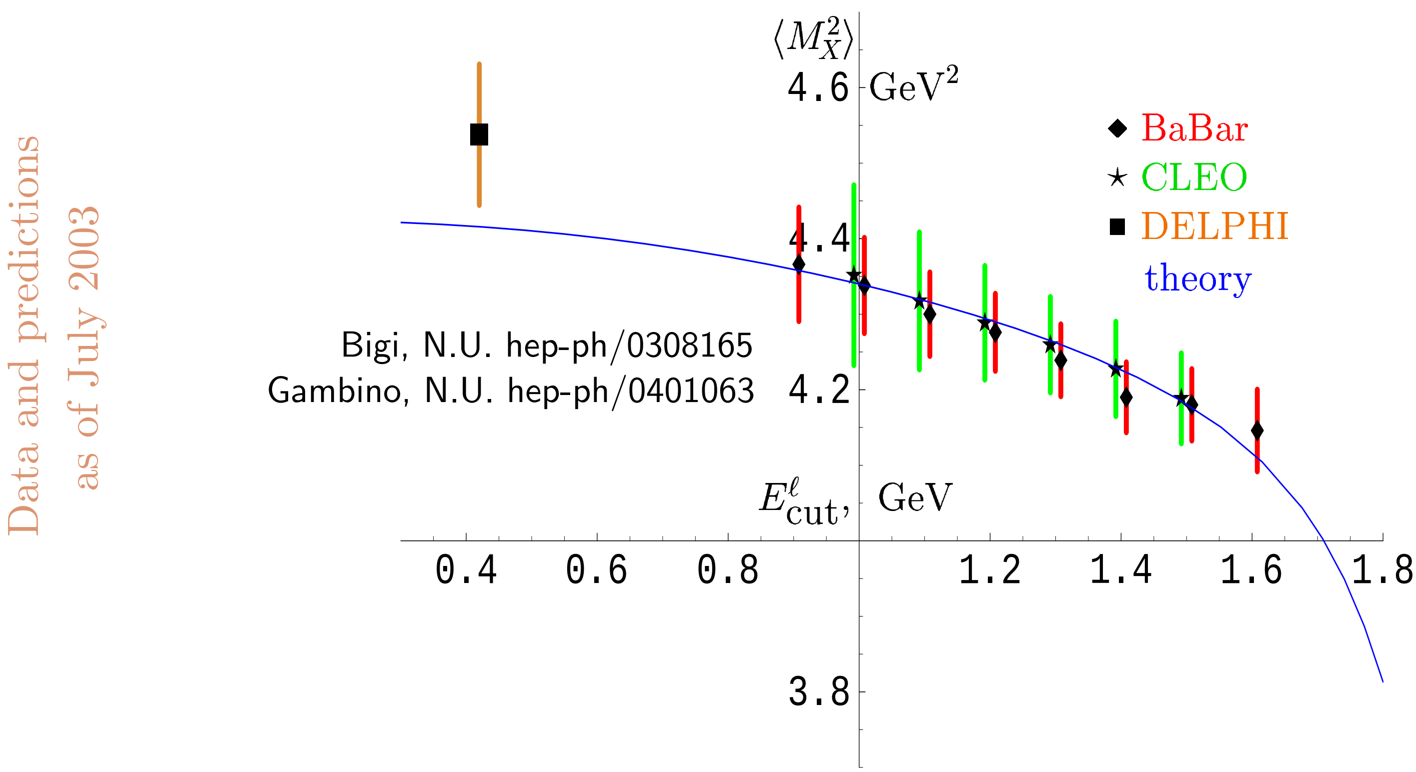
Now all pure perturbative corrections have been calculated

N.U.; M. Trott

Problem for theory with $\langle M_X^2 \rangle$ vs. $E_{\rm cut}^\ell$?



Robust OPE approach a lá Wilson, $\mu = 1$GeV:



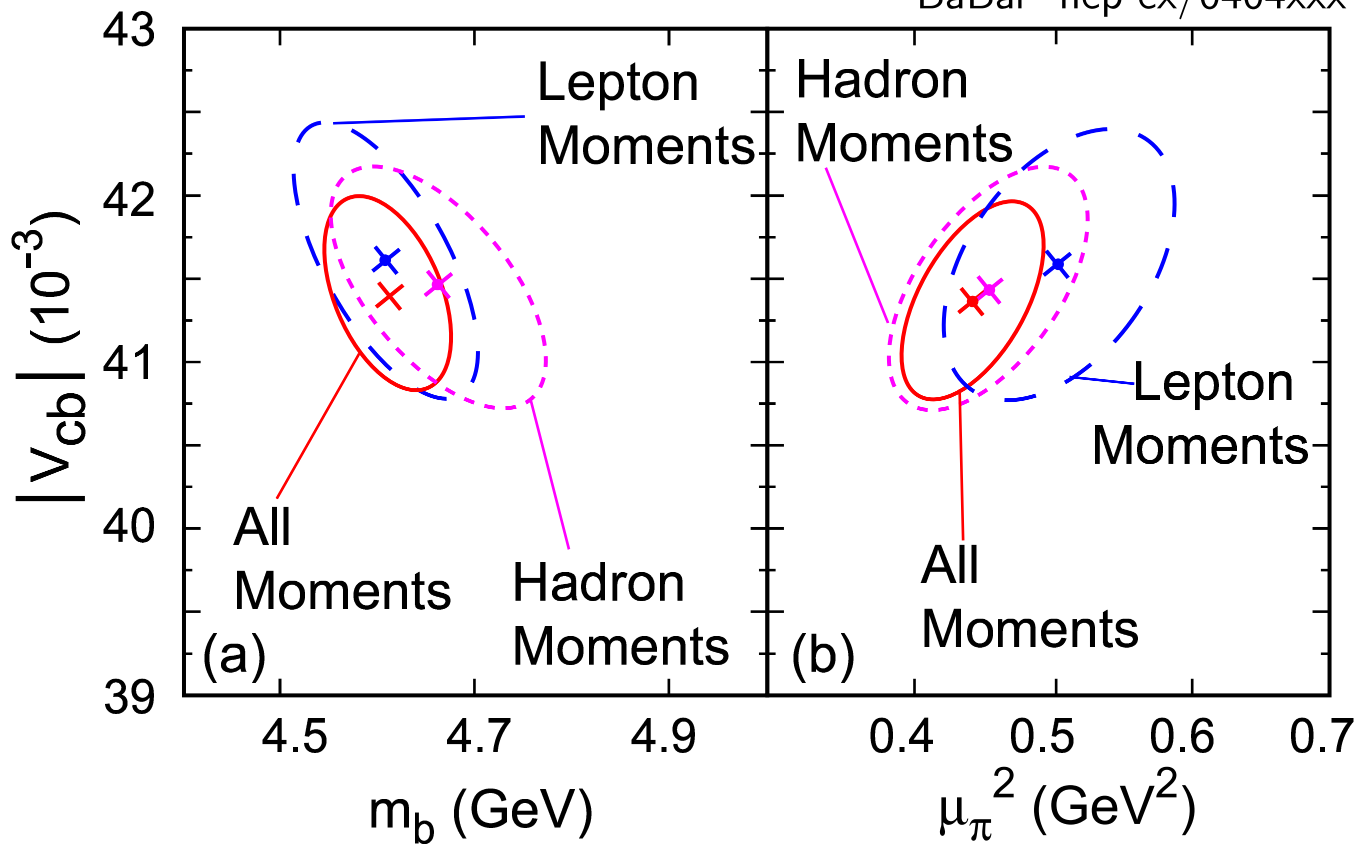
OPE works even where may be expected to break down

Present stage:

- ♠ Have an accurate and reliable determination of many HQ parameters from experiment
- ♣ Extracting $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ has good accuracy and solid grounds
- ♥ Have precision checks of the OPE at the nonperturbative level

I think the most impressive is good consistency between $\langle M_X^2\rangle$ and $\langle E_\ell\rangle$: A sensitive check of the nonperturbative sum rule for $M_B - m_b$

BaBar hep-ex/0404xxx



Surprise: SL decays at BaBar yielded accurate m_b itself

The combination $m_b - 0.74\, m_c$ is determined with only $17\,\mathrm{MeV}$ error bar!

Running mass is an observable and has `no` intrinsic limitation on precision

Theoretical expectation:

$$m_b(1\,\mathrm{GeV}) = (4.57 \pm 0.05)\,\mathrm{GeV}$$

Voloshin 1995–1996

Melnikov, Yelkhovsky
Beneke, Signer 1998–1999
Hoang

$\mu_\pi^2, \;\; \mu_G^2$ — primary nonperturbative values in the HQE

$$\mu_G^2 = \frac{1}{2M_B}\langle B|\,\bar{b}\,\frac{i}{2}\,g_s\,\sigma_{\mu\nu}G^{\mu\nu}\,b\,|B\rangle \;\longleftrightarrow\; \langle B|-g_s\,\vec{\sigma}_b\vec{B}_{\rm chr}(0)\,|B\rangle_{\rm QM}$$

$$\mu_\pi^2 = \frac{1}{2M_B}\langle B|\,\bar{b}\,(i\vec{D}\,)^2\,b\,|B\rangle \;\longleftrightarrow\; \langle B|\,\vec{p}_b^{\,2}\,|B\rangle_{\rm QM}$$

$$\vec{p}_b \to \vec{\pi}_b = -i\vec{D} = -i\vec{\partial} - g_s\vec{A}$$

μ_G^2 determines `hyperfine` splitting: $\quad M_{B^*} - M_B \simeq \frac{2}{3}\frac{\mu_G^2}{m_b}$

$$\mu_G^2(1\,{\rm GeV}) = 0.35^{+.03}_{-.02}\,{\rm GeV}^2$$

N.U. 2001

$$\mu_\pi^2(\mu) > \mu_G^2(\mu) \quad \text{at any } \mu$$

rigorous inequality

BSUV, Voloshin 1993–1994

Theory: $\quad \mu_\pi^2 \approx (0.45 \pm 0.1)\,{\rm GeV}^2$

Right at the central experimental value

Next order in $1/m_b$: Darwin and spin-orbital values

familiar from atomic physic

$$\rho_D^3 \;\propto\; \frac{1}{2M_B}\langle B|\,\bar{b}\,\Gamma b\,\bar{q}\,\Gamma' q\,|B\rangle \;\longleftrightarrow\; |\Psi(0)|^2_{\rm QM}$$

$$\rho_{LS}^3 \;\propto\; \langle \vec{\sigma}\cdot\vec{\pi}\times\vec{E}\rangle_b$$

- Inconsistency with $b \to s+\gamma$ moments ?

Relying on relations *imprecise* with a high cut on E_γ

$$\langle E_\gamma \rangle = \frac{m_b}{2} + \ldots \qquad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = \frac{\mu_\pi^2}{12} + \ldots$$

A good way to accurately measure HQ parameters?

Bottle neck: 'Hardness' $\mathcal{Q}$ often gets too low with the cuts

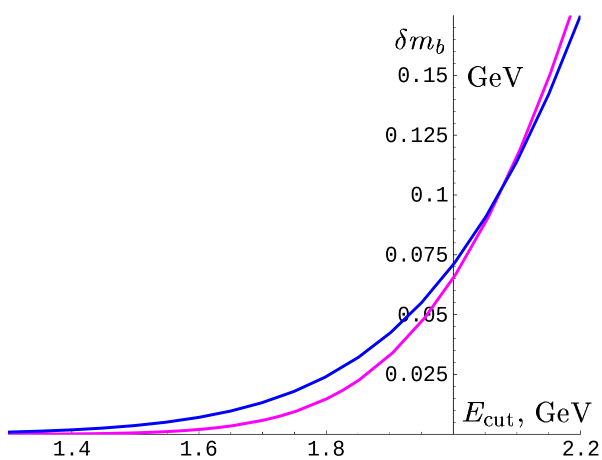
$\mathcal{Q} \simeq m_b - m_c$ for total widths, but

$\mathcal{Q}$ is `below` $1\,\mathrm{GeV}$ for $E_\ell > 1.7\,\mathrm{GeV}$

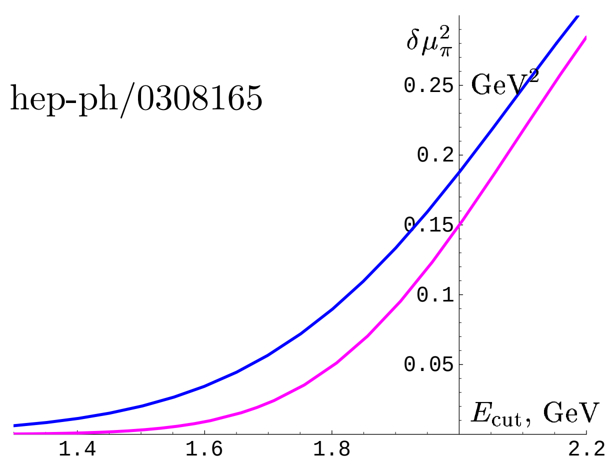
A complementary consideration suggests the expansion for M_X^2 loses sense for $E_{\rm cut} \gtrsim 1.7\,\mathrm{GeV}$

In $b \to s+\gamma$ $\quad \mathcal{Q} \simeq M_B - 2E_{\rm min} \simeq 1.2\,\mathrm{GeV}$

if the cut is at $E_\gamma = 2\,\mathrm{GeV}$



Bigi, N.U. hep-ph/0308165



Accounting for these biases yielded good agreement between `all` measurements

Experiment must strive to lower the cuts ! Bigi, N.U. 2002

BELLE 2004: With $E_\gamma > 1.8\,\mathrm{GeV}$ cut *biases* are not that much an issue

$$\langle E_\gamma \rangle = 2.289 \pm 0.026_{\rm stat} \pm 0.0034_{\rm sys}\ \mathrm{GeV}$$
$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0311 \pm 0.0073_{\rm stat} \pm 0.0063_{\rm sys}\ \mathrm{GeV}^2$$

For BaBar's HQ values we would obtain

$$\langle E_\gamma \rangle = 2.317\,\mathrm{GeV} \qquad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0329\,\mathrm{GeV}^2$$

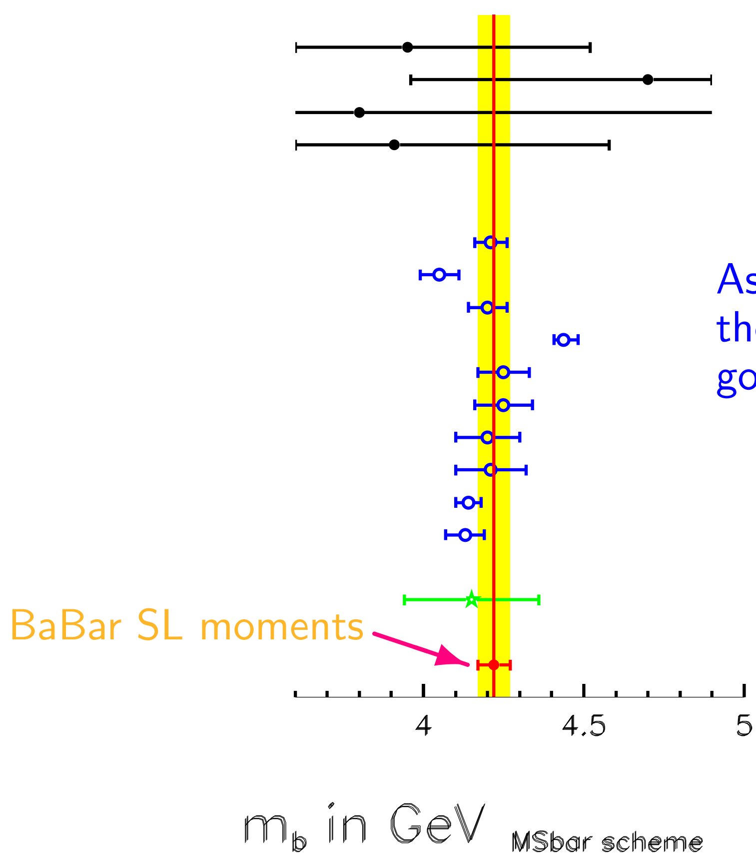
Quite consistent!

Adding this to the BaBar data yields only minor shifts in the fit:

$$m_b(1\mathrm{GeV}) \simeq 4.58\mathrm{GeV}, \qquad \mu_\pi^2(1\mathrm{GeV}) \simeq 0.45\mathrm{GeV}^2$$

no visible change in $|V_{cb}|$

More conventional in HEP is $m_b^{\overline{\rm MS}}(m_b)$:



As expected from the comparison for the low-scale running mass, here also a good agreement

What all this means?

OPE works well, the heavy quark parameters derived from experiment are consistent with the theoretical expectation based on independent theoretical analyses

Perturbative corrections have been calculated and are expectedly well behaved in the proper Wilsonian approach. No obstacles for precision calculations of truly inclusive short-distance observables

Theory needs calculation of the perturbative corrections to the Wilson coefficients of power-suppressed operators (μ_π^2, μ_G^2, ρ_D^3)

This becomes a limiting factor

Kinetic value μ_π^2 emerges as theoretically expected

Does the precise value matter? It appears that

$$\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$$

Very interesting regime!

Need to recall Heavy Quark Sum Rules

Recent development: D'Orsay sum rules Le Yaouanc et al.

Good example: $\varrho^2 > \frac{3}{4}$

N.U. 2000

Assuming the spin sum rule is saturated at $\mu = 1\,\mathrm{GeV}$ we have

$$\mu_\pi^2 - \mu_G^2 = 3\,\tilde{\varepsilon}^2 \cdot (\varrho^2 - \tfrac{3}{4})$$

Quite a constraint: $\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_\pi^2 - \mu_G^2}{3\tilde{\varepsilon}^2} \lesssim 0.2 \;\; (0.3)$

at $\mu_\pi^2 = 0.43\;(0.5)\;\mathrm{GeV}^2$ since $\tilde{\varepsilon} > 0.4\,\mathrm{GeV}$

ϱ^2 is probed in experiment

important for V_{cb}

radically affects $B \to D^*$

extrapolation to zero recoil

Recent UKQCD lattice is quite compatible with the prediction:

$$\varrho^2 = 0.83\,^{+.15\,+\,.24}_{-.11\,-\,.01}$$

hep-lat/0202029

Another application, for $B \to D\,\ell\nu$: expanding in $\mu_\pi^2 - \mu_G^2$

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) \;=\; 1.04 \pm 0.01 \;\pm\; 0.01$$

$\mu_\pi^2 \simeq \mu_G^2$ is a remarkable point for B and D mesons!

'BPS' expansion: Expand around $\mu_\pi^2 = \mu_G^2$

N.U. 2001-3

At $\mu_\pi^2 = \mu_G^2$ there is a `functional` relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$

Often extends Heavy Flavor (but not Spin) symmetry to all orders in $1/m_Q$

No formal power corrections to $m_b - m_c = M_B - M_D$

only exponential in $2m_c/\mu_{\mathrm{hadr}}$

Miracles of the BPS limit

N.U. 2003

- $\varrho^2 = \frac{3}{4}$

  inclusive hadronic moments can tell us about the slope of the $B \to D^{(*)}$ formfactor!

- No power corrections to $M = m_Q + \overline{\Lambda}$ for the ground state

  $M_B - M_D = m_b - m_c$ to all orders in $1/m_Q$

- For $B \to D$ amplitude

  $f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2)$ to any order in $1/m_Q$

- Zero recoil $B \to D$ amplitude: $\delta_{1/m^k} = 0$

  regardless of mass ratio

- In $B \to D$ at zero recoil

  $f_+ = \frac{M_B + M_D}{2\sqrt{M_B M_D}}$ to all orders in $1/m_Q$

- At arbitrary velocity power corrections in $B \to D$ vanish

  $$f_+(q)^2 = \frac{M_B + M_D}{2\sqrt{M_B M_D}}\, \xi\left(\frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}\right)$$

  Decay rate directly gives the IW function

Experiment: $B \to D$ slope much closer to $\varrho^2 \simeq 0.9$

Corrections to the shape of the $B \to D^*$ formfactor are way too significant

Quantifying Corrections to BPS

How significant are corrections to 'BPS' relations in actual QCD ? It depends

The deviation parameter: Dimensionful parameter is

$\alpha = \|(\vec{\sigma}\vec{\pi})\,|B\rangle\| \equiv \sqrt{\mu_\pi^2 - \mu_G^2}$ The dimensionless one is

$$\beta = \|\pi_0^{-1}(\vec{\sigma}\vec{\pi})\,|B\rangle\| \equiv \sqrt{3\left(\varrho^2 - \tfrac{3}{4}\right)} = 3\left(\sum_n\, |\tau_{1/2}^{(n)}|^2\right)^{\frac{1}{2}}$$

Numerically β is not a too small number, similar in size to generic $1/m_c$ expansion parameter β^2 should be good

We can count together powers of $1/m_c$ and β to judge the real quality of the HQ relations

At which order in β the BPS relations can be violated to `all orders` in $1/m_Q$? N.U. 2003

- Absence of corrections to $M_D = m_c + \overline{\Lambda}$,
  $M_B - M_D = m_b - m_c$ holds up to β^2
- Zero recoil $B \to D$ amplitude is unity up to β^2
- At arbitrary velocity relation between f_+ and f_- in $B \to D$ holds only to the leading order

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2) + \mathcal{O}(\beta)$$

- At arbitrary velocity the relations between $f_\pm$ in $B \to D$ and the IW function may receive corrections $\propto \beta^1$

- f_+ near zero recoil receives only second order corrections in β to any order in $1/m_Q$:

$$f_+\left((M_B - M_D)^2\right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2)$$

Analogue of the Ademollo-Gatto theorem for the BPS expansion

the same applies to f_-

Must be quite accurate, f_-/f_+ can be checked in $B \to D\,\tau\nu_\tau$

We know that all power corrections are small

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.04 \pm 0.01 \pm 0.01$$

All orders in $1/m$ in BPS, to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

This formfactor is known better than for 'gold-plated' $B \to D^*$

If this can be measured, nothing else exclusive may be required for $|V_{cb}|$

Are all skies crystal blue in SL decays? Not quite...

A "$\frac{1}{2} > \frac{3}{2}$" puzzle

Much of a priori theory knowledge comes from the Heavy Quark Sum Rules + the known size of μ_G^2
Sum rules explain why B^* is heavier than B; they set the scale of $\overline{\Lambda}$, μ_π^2, ...
The HQ sum rules are `exact` relations in the heavy quark limit

Two classes: first for ϱ^2, $\overline{\Lambda}$, μ_π^2, ρ_D^3, ... These are saturated by both $\frac{3}{2}$ and $\frac{1}{2}$ P-wave heavy quark states

Second are '`spin`' sum rules for $\varrho^2 - \frac{3}{4}$, $\overline{\Lambda} - 2\overline{\Sigma}$, $\mu_\pi^2 - \mu_G^2$, ... These include only $\frac{1}{2}$ states

Spin sum rules strongly suggest that $\frac{3}{2}$ P-wave states must dominate over $\frac{1}{2}$ states. This automatically happens in all quark models respecting QCD and Lorentz covariance
Orsay quark models

Experiment: $\frac{3}{2}$ charm P-wave states are narrow and well identified. They seem to contribute too little. Wide $\frac{1}{2}$ states might saturate the spin-singlet sum rules, but in aggregate they should be subdominant to $\frac{3}{2}$ states!

The most natural solution of HQSRs:

$\frac{3}{2}$ states at $\mathcal{E}_{\frac{3}{2}} \approx 450\,\mathrm{MeV}$ and $\tau_{\frac{3}{2}}^2 \approx 0.3$ while

$\tau_{\frac{1}{2}}^2 \approx 0.07 \div 0.12$ with $\mathcal{E}_{\frac{1}{2}} \approx 300 \div 500\,\mathrm{MeV}$

Possible resolutions:

Contribution of the excited P-wave states ...

Charm is too light to apply this classification itself, valid only for heavy quarks; extraction of τ's is not justified

Need a good physical reason to invert the hierarchy

Too light c quark... Lattices can help

Resolution of this controversy is an important task, needs joint efforts from both theory and experiment

Conclusions:

Heavy quark theory has become a mature branch of QCD
Experiment has entered a new era of detailed studies of both "electroweak" and "strong" aspects

The dynamic OPE has finally undergone and passed critical precisions checks at the nonperturbative level

Experiments find consistent heavy quark parameters from quite different measurements

$|V_{cb}|$ extraction has high accuracy and is based on reliable theory

Similar robust results are anticipated soon for $|V_{ub}|$

Inclusive studies yield crucial info for HQ physics, even for exclusive amplitudes   Formerly viewed as antipodes

Power corrections to HQ symmetry are very significant in charm.   There is a subset of relations which are stable, they are limited to the ground-state pseudoscalar B and D mesons, but exclude `spin` symmetry for charm

Experiment must verify the kinetic expectation value with even higher accuracy and fidelity,   extract more reliably ρ_D^3 in inclusive decays

Theory must provide perturbative corrections to coefficients of power-suppressed operators

$B \to D$ decays can be reliable theory-wise

If $\mu_\pi^2 \lesssim 0.45\,\mathrm{GeV}^2$ is firmly established then

$\mathcal{F}_+(0) \simeq 1.04$ is an accurate prediction for $B \to D$

Many nontrivial consequences of the BPS regime

Slope ϱ^2 is close to 1-, and

fits of $B \to D^*$ should incorporate constraints on $\hat{\varrho}^2$

$B \to D^{(*)}\tau\nu$ and $B \to X_c\tau\nu$ offer a number of interesting possibilities

Recent success of the QCD-based dynamic theory of nonperturbative physics in heavy quarks also raises new problems

A "$\frac{1}{2} > \frac{3}{2}$" puzzle

needs both theoretical and experimental scrutiny