XXXIXth Rencontres de Moriond: QCD and High-Energy Hadronic Interactions
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Gluon radiation off massive quarks in a QCD medium

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1. Introduction.
2. Results.
3. Conclusions.

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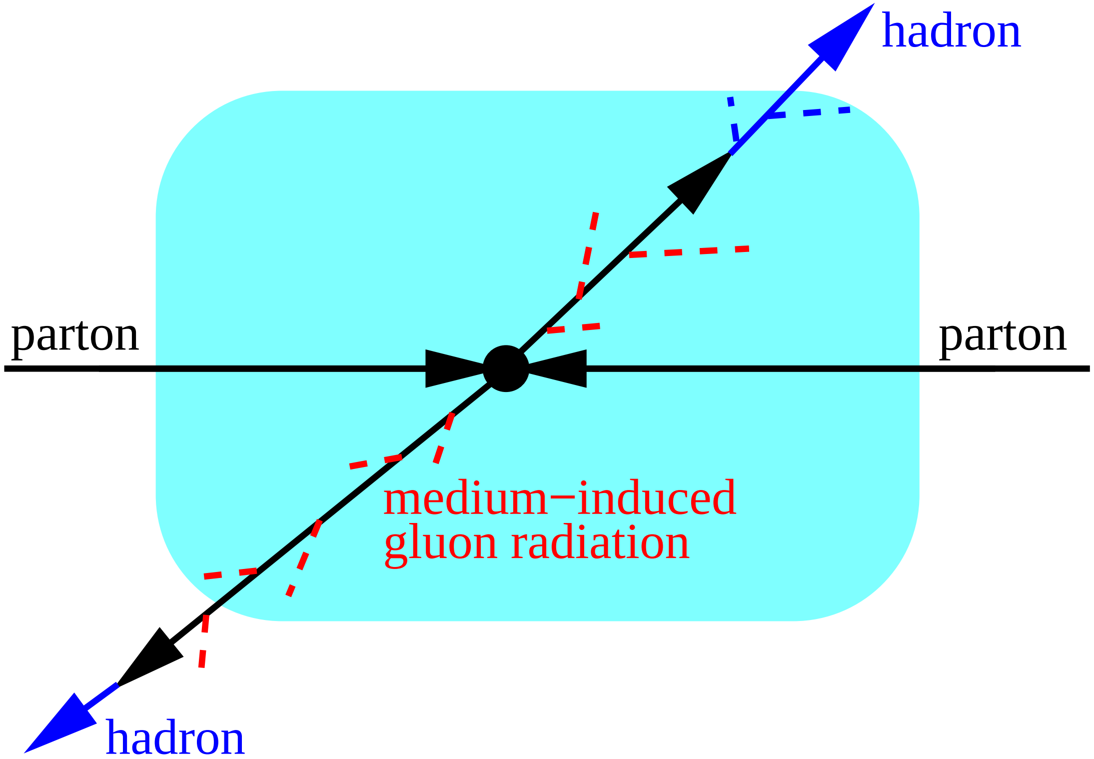
Related talks: Dainese, d'Enterria, Ferreiro, Kharzeev, Liu and Wang.

1. Introduction:



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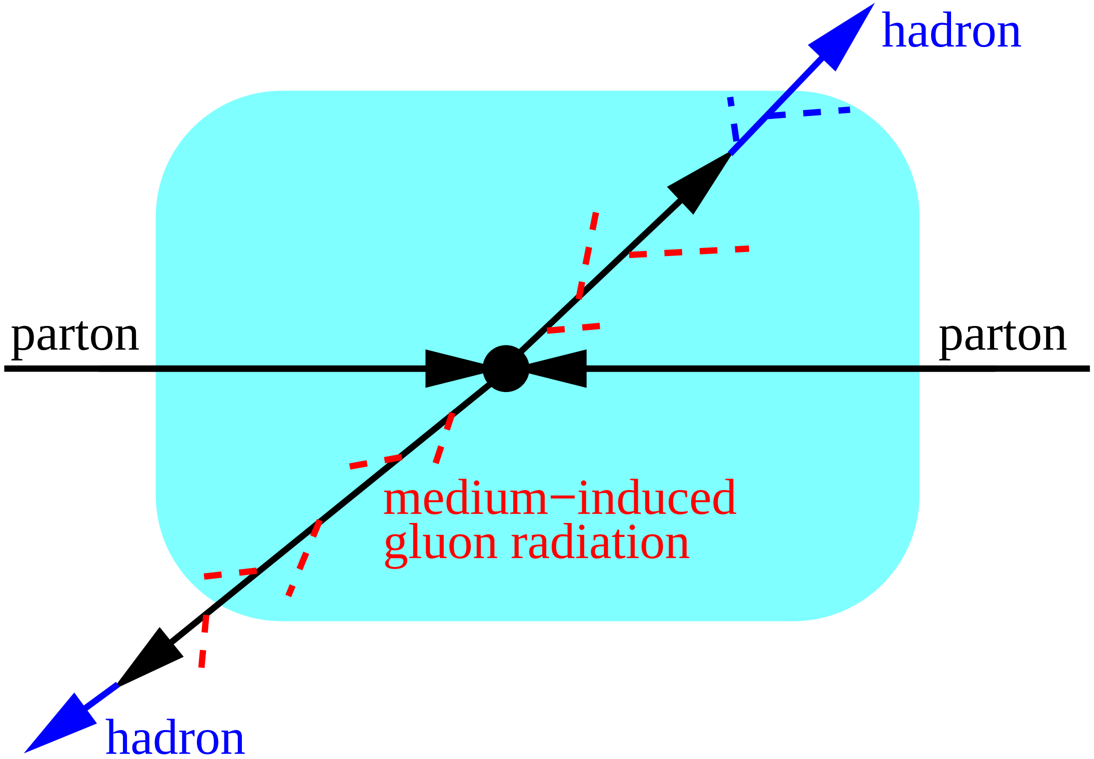
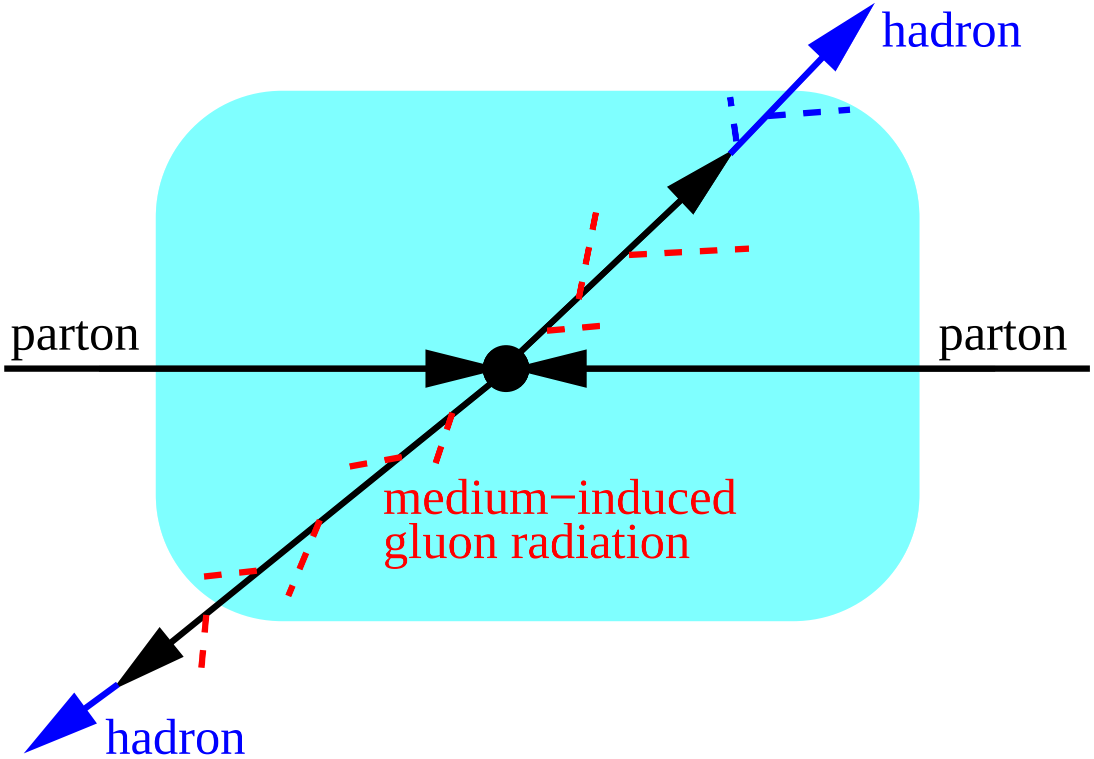


- Medium-induced gluon radiation implies:

⇒ Energy degradation of the leading parton.
⇒ Broadening of the parton shower.
⇒ Increase of the associated hadron multiplicity.



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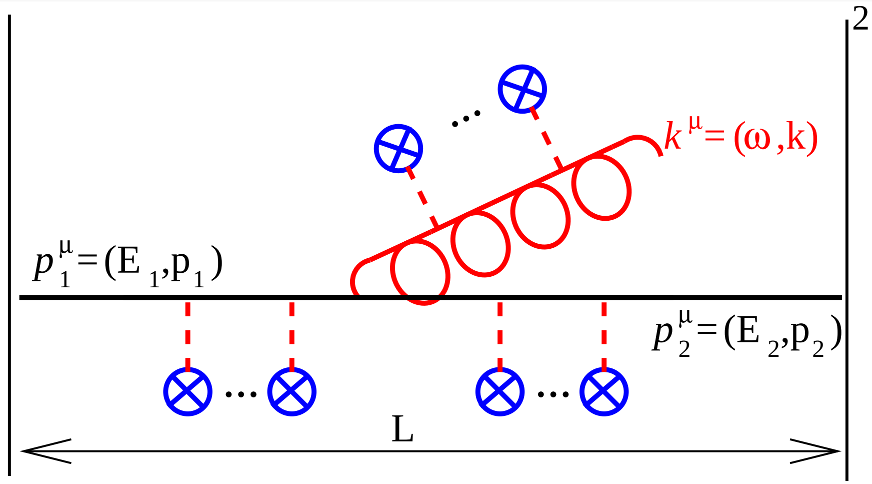
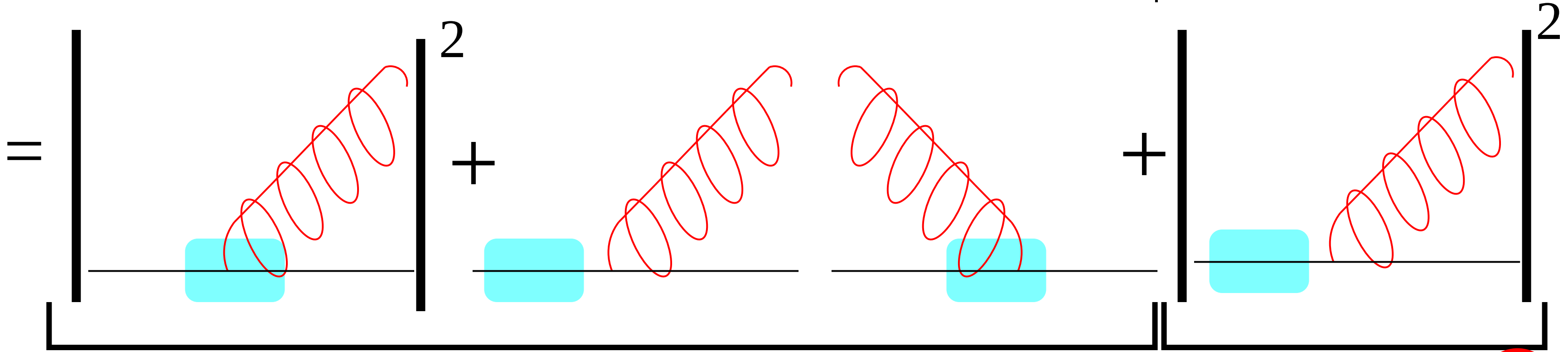
♣ How does the medium-induced gluon radiation off a massive quark differ from that off a massless parton?

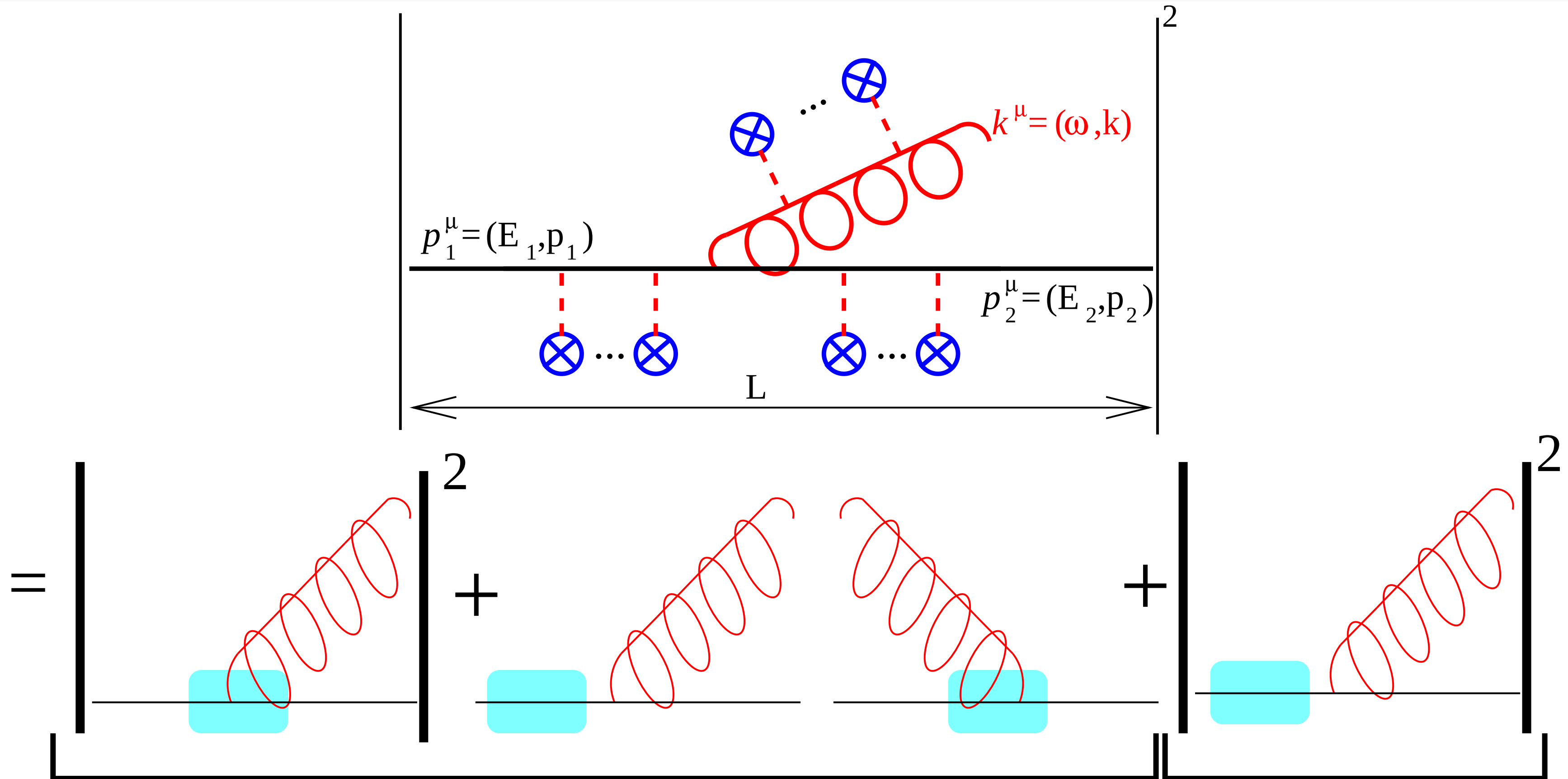
Medium-induced gluon radiation off massless quarks:





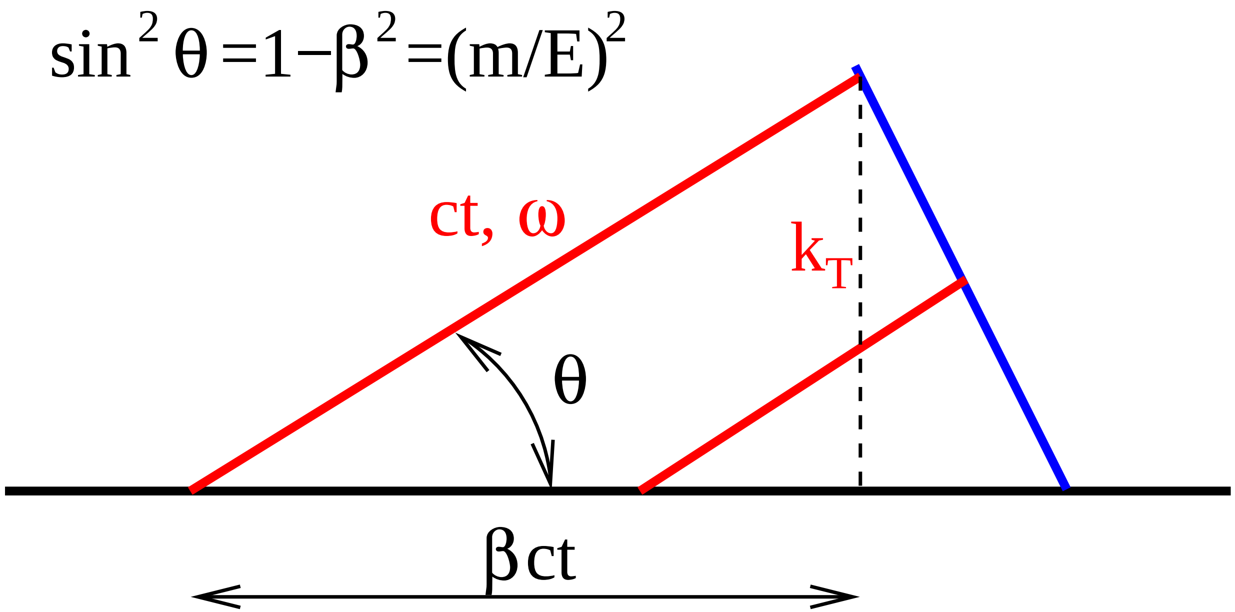
$$\omega \frac{dI_{\text{medium}}}{d\omega \, d\mathbf{k}_\perp}$$

$$\omega \frac{dI_{\text{vacuum}}}{d\omega \, d\mathbf{k}_\perp} \propto \frac{1}{\mathbf{k}_\perp^2}$$

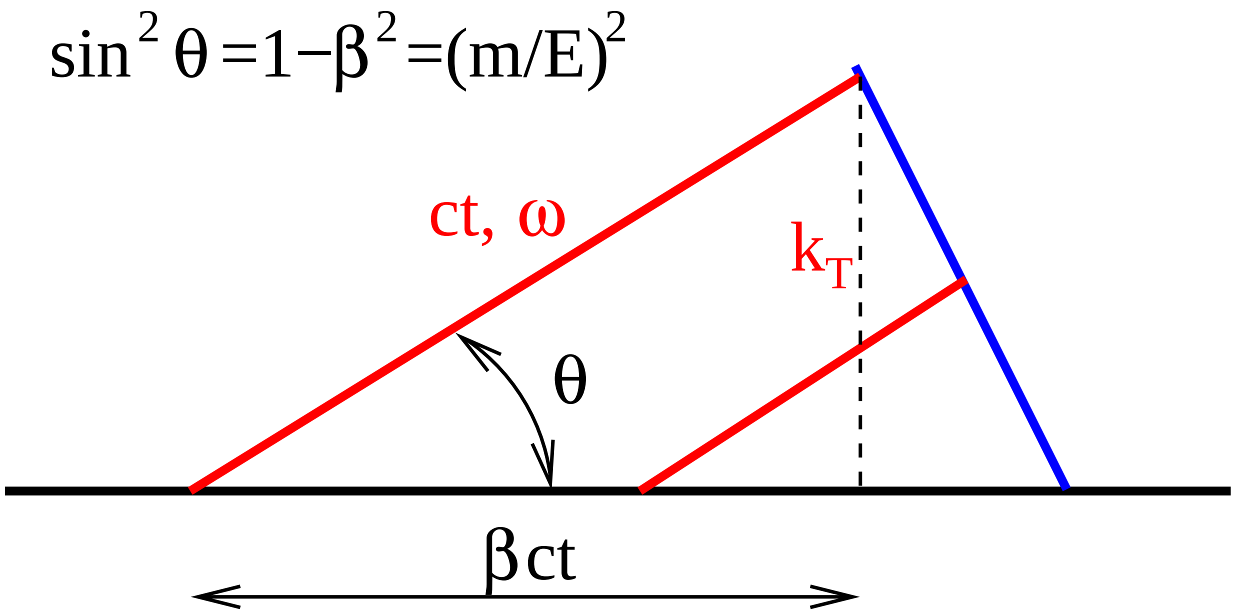


$$\omega \frac{dI_{\mathrm{medium}}}{d\omega\, d\mathbf{k}_\perp} \qquad\qquad \omega \frac{dI_{\mathrm{vacuum}}}{d\omega\, d\mathbf{k}_\perp} \propto \frac{1}{\mathbf{k}_\perp^2}$$

- The BDMPS formalism describes the medium-modification of the vacuum radiation pattern (Baier-Dokshitzer-Mueller-Peigné-Schiff; Zakharov; Wiedemann; Gyulassy-Levai-Vitev; Wang-Guo; see reviews in: Baier, Schiff, Zakharov, ARNPS50(00)37; Kovner, Wiedemann in Quark Gluon Plasma 3; Gyulassy, Vitev, Wang, Zhang, ibid).



- Gluon radiation in the vacuum is modified by a mass of the parent quark: radiation for angles $\theta < m/E$ is suppressed, the dead cone effect (Dokshitzer, Khoze, Troyan, JPG17(91)1481):



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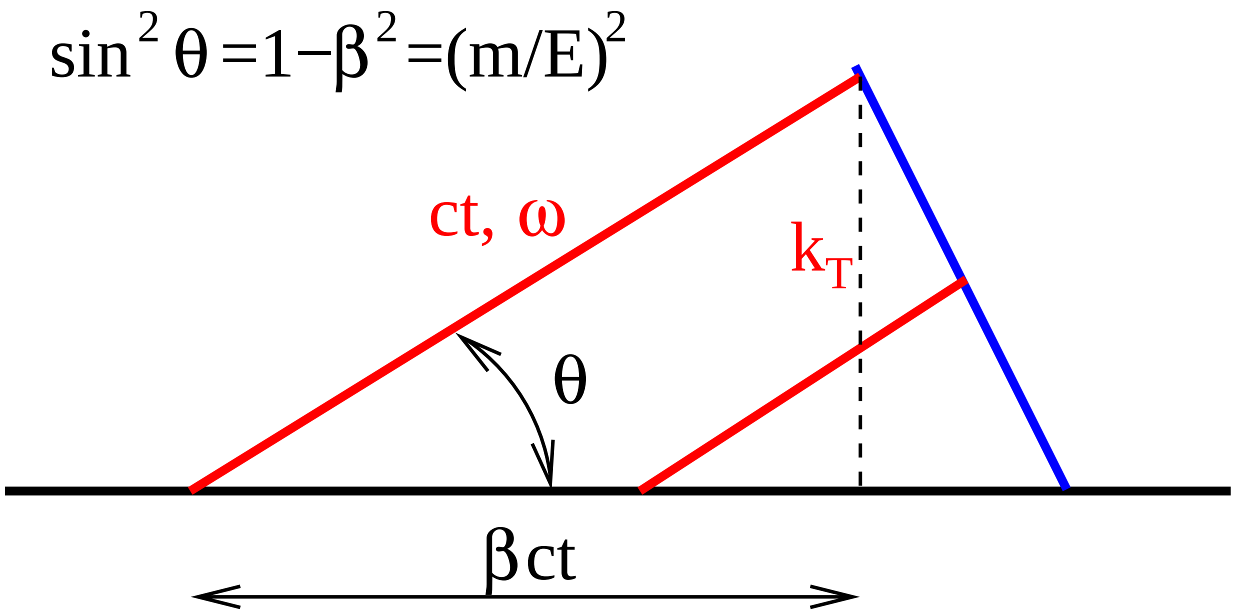
Dead cone factor

$$\frac{1}{\mathbf{k}_\perp^2} \rightarrow \frac{1}{\mathbf{k}_\perp^2}\left[\frac{\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2+\left(\frac{m\omega}{E}\right)^2}\right]^2 \equiv \frac{1}{\mathbf{k}_\perp^2} F\left(\mathbf{k}_\perp^2, \frac{m\omega}{E}\right).$$



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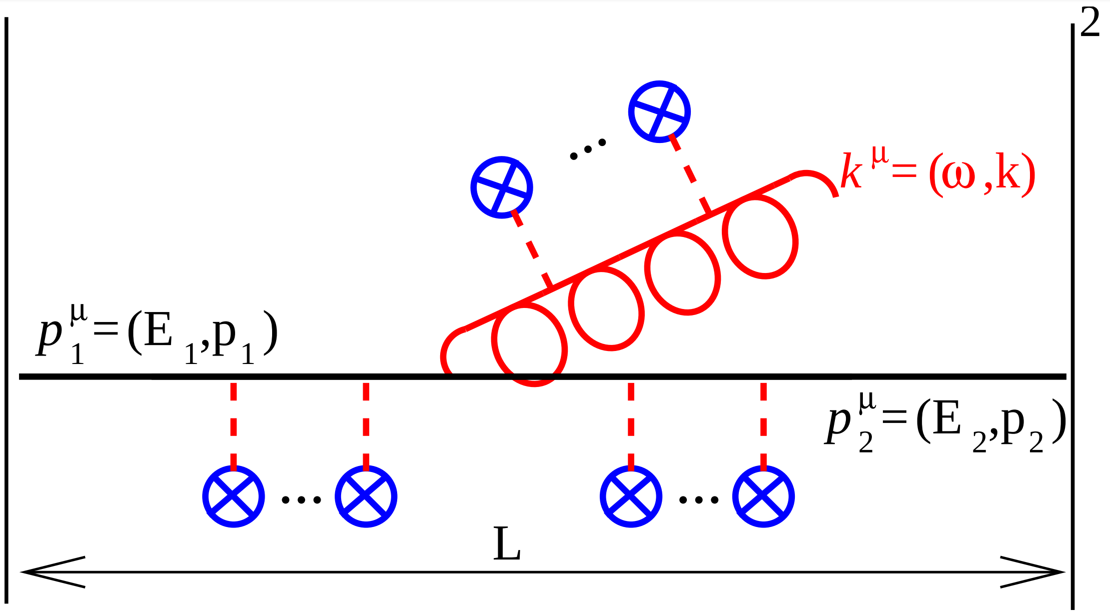
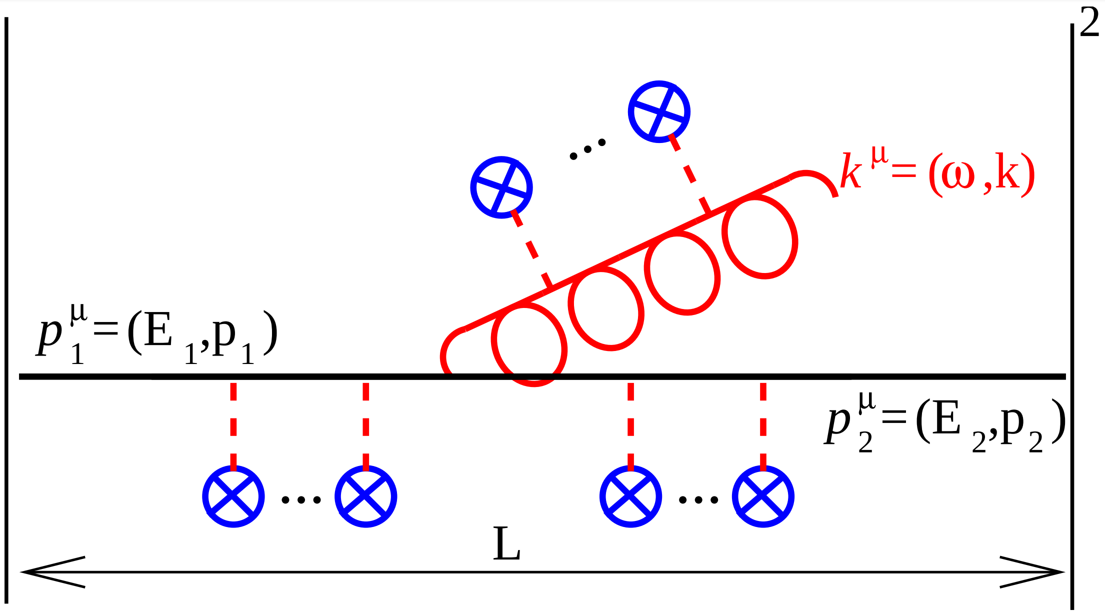
- Dokshitzer and Kharzeev (PLB519(01)199) proposed that medium-induced gluon radiation is reduced by the same effect. In this first exploratory study:

$$\omega\frac{dI^{m>0}_{\rm medium}}{d\omega} = \omega\frac{dI^{m=0}_{\rm medium}}{d\omega}\, F\left(\langle\mathbf{k}_\perp^2\rangle, \frac{m\omega}{E}\right).$$



$$\omega \frac{dI}{d\omega\, d\mathbf{k}_\perp} = \frac{\alpha_s C_F}{(2\pi)^2 \omega^2} 2\mathrm{Re} \int_0^\infty dy_l \int_{y_l}^\infty d\bar{y}_l\, e^{i\bar{q}(y_l - \bar{y}_l)} \int d\mathbf{u}\, e^{-i\mathbf{k}_\perp \cdot \mathbf{u}}$$

$$\times\ e^{-\frac{1}{2}\int_{\bar{y}_l}^\infty d\xi\, n(\xi)\, \sigma(\mathbf{u})} \frac{\partial}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{u}} \int_{\mathbf{y}=0=\mathbf{r}(y_l)}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r} \exp\left[i \int_{y_l}^{\bar{y}_l} d\xi \frac{\omega}{2} \left(\dot{\mathbf{r}}^2 - \frac{n(\xi)\sigma(\mathbf{r})}{i\,\omega} \right) \right] .$$



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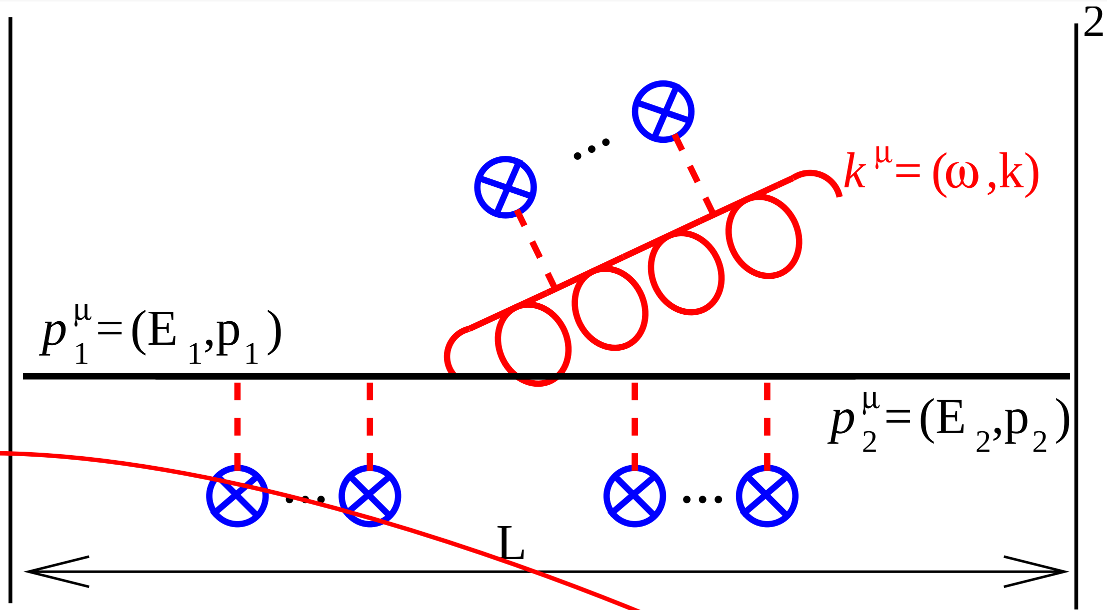
- Information about the medium contained in $n(\xi)\sigma(\mathbf{r}) \simeq \frac{1}{2}\, \hat{q}(\xi)\mathbf{r}^2$.

The only model parameter is $\hat{q}(\xi) \simeq \langle \mathbf{q}_\perp^2 \rangle_{\mathrm{medium}} / \lambda_{\mathrm{mfp}}$.



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- All mass effects: $\bar{q} = p_1 - p_2 - k \simeq \frac{x^2\, m^2}{2\omega}$, $x = \omega/E \ll 1$.

1) Vacuum term: dead cone,

$$\omega \frac{dI_{\rm vacuum}}{d\omega\, d\mathbf{k}_\perp} \propto \frac{1}{\mathbf{k}_\perp^2}\, F\left(\mathbf{k}_\perp^2, \frac{m\omega}{E}\right) = \frac{1}{\mathbf{k}_\perp^2}\left[\frac{\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2 + \left(\frac{m\omega}{E}\right)^2}\right]^2 .$$

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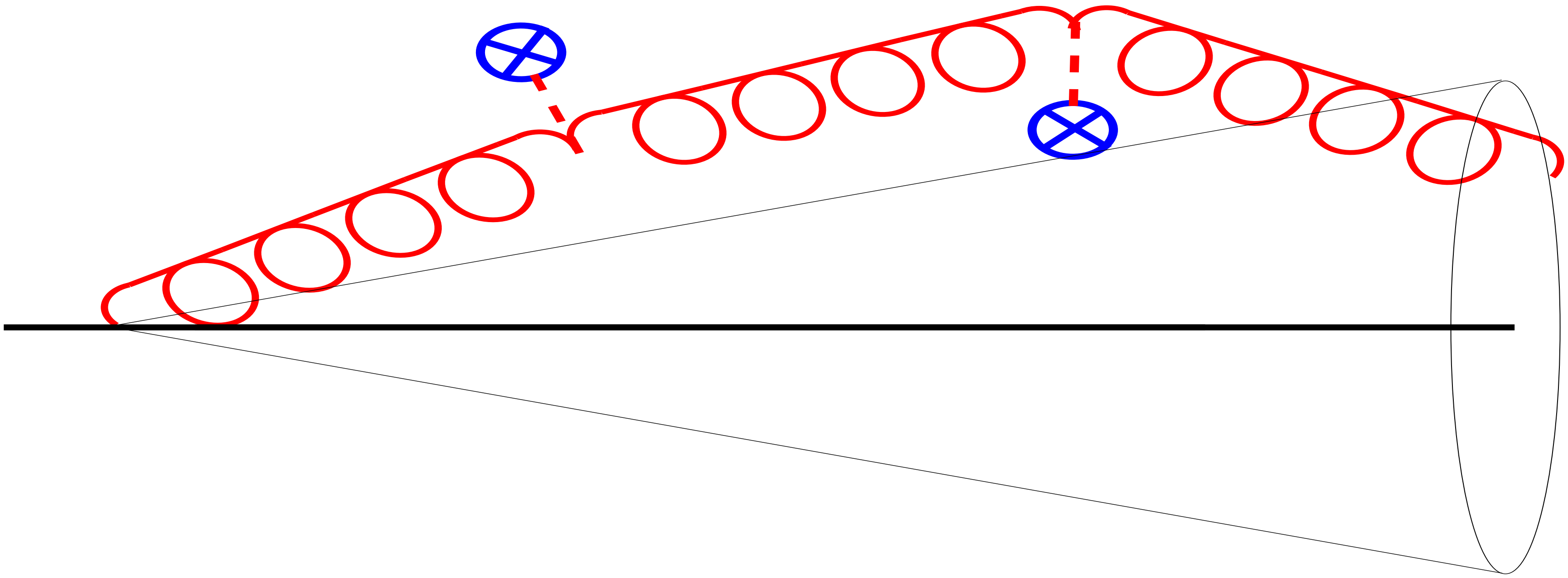
2) Medium term:

Naive: gluon moves into the dead cone due to multiple scattering (Brownian motion) $\Rightarrow$ the dead cone may be filled.



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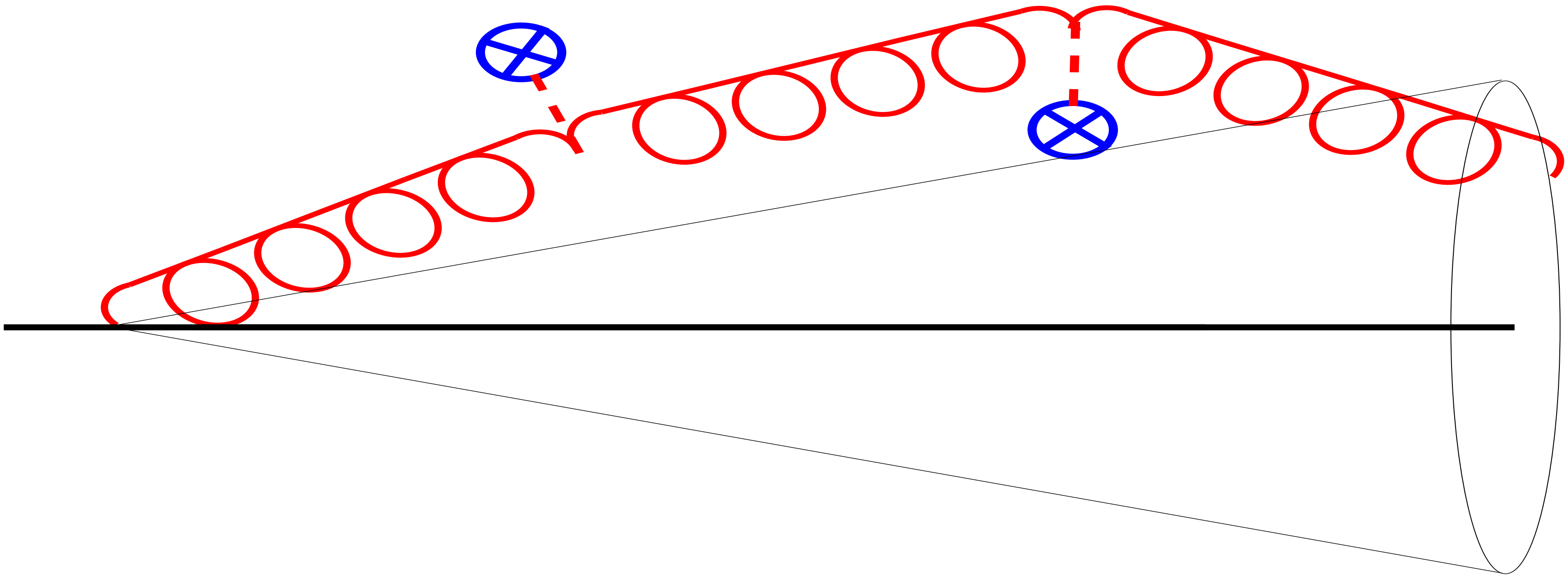
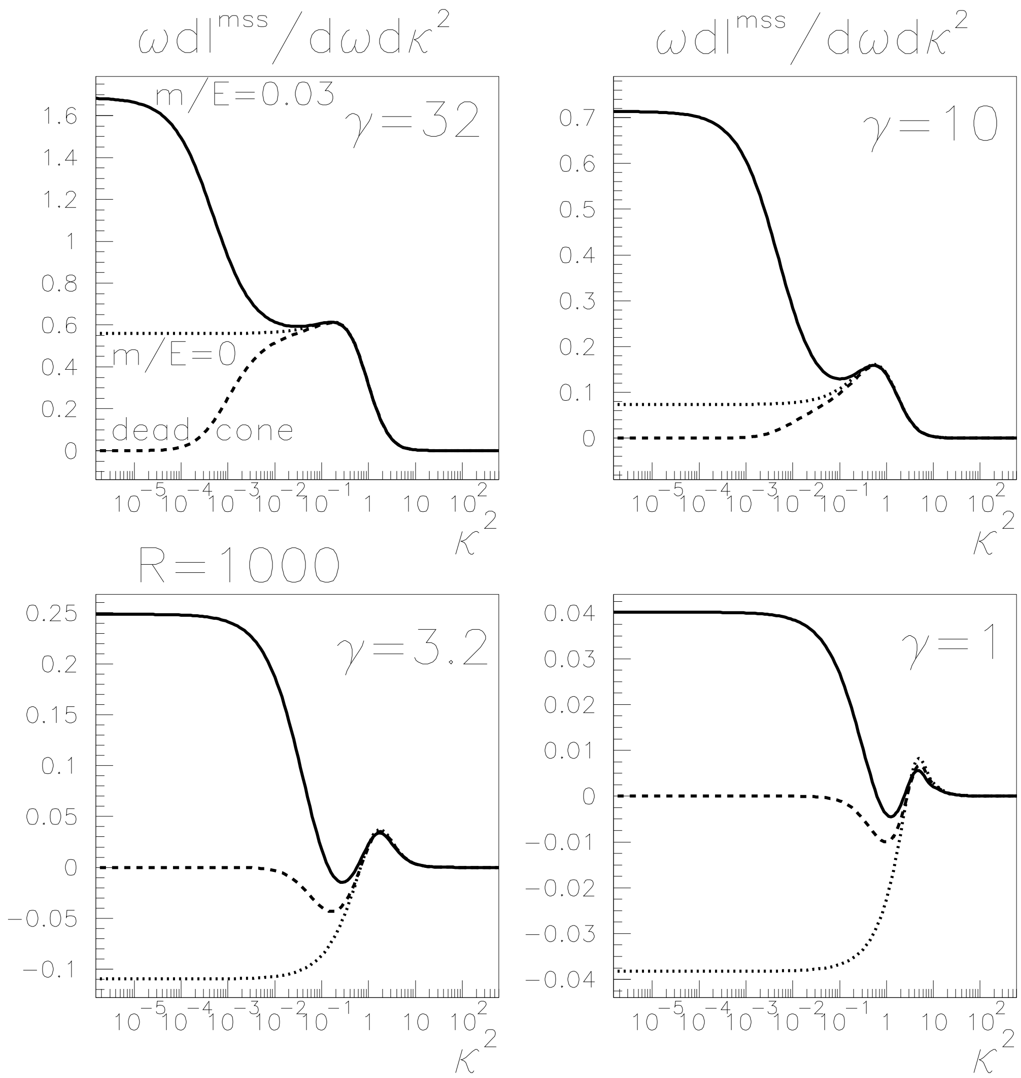


Technically: competition between interference and rescattering $\Rightarrow$ look at the numerical results.



- $\kappa^2 = \frac{\mathbf{k}_\perp^2}{\hat{q}L}$, $\omega_c = \hat{q}L^2/2$, $R = \omega_c L$, $\gamma = \omega_c/\omega$, $M^2 = \frac{x^2 m^2}{\hat{q}L}$.
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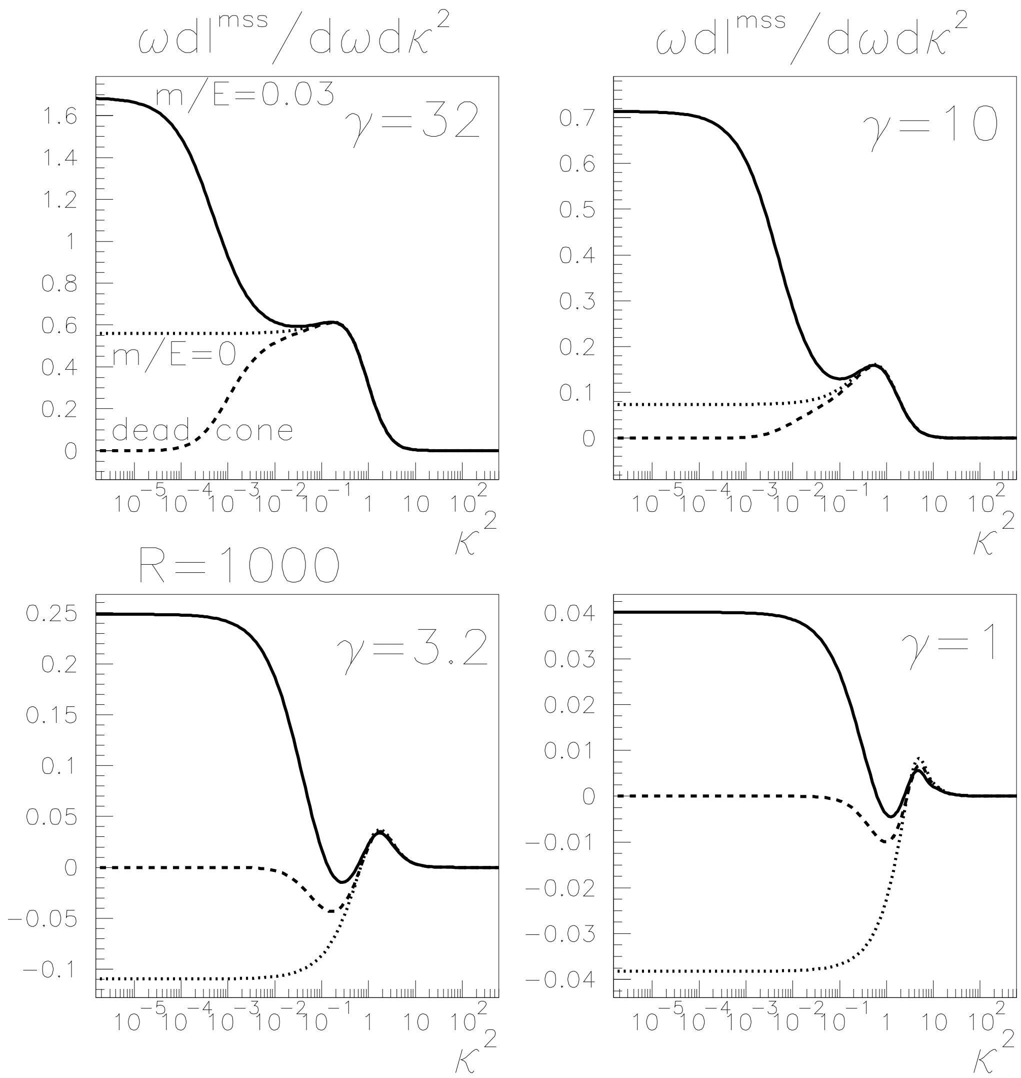
2. Results (I) – the dead cone is filled:



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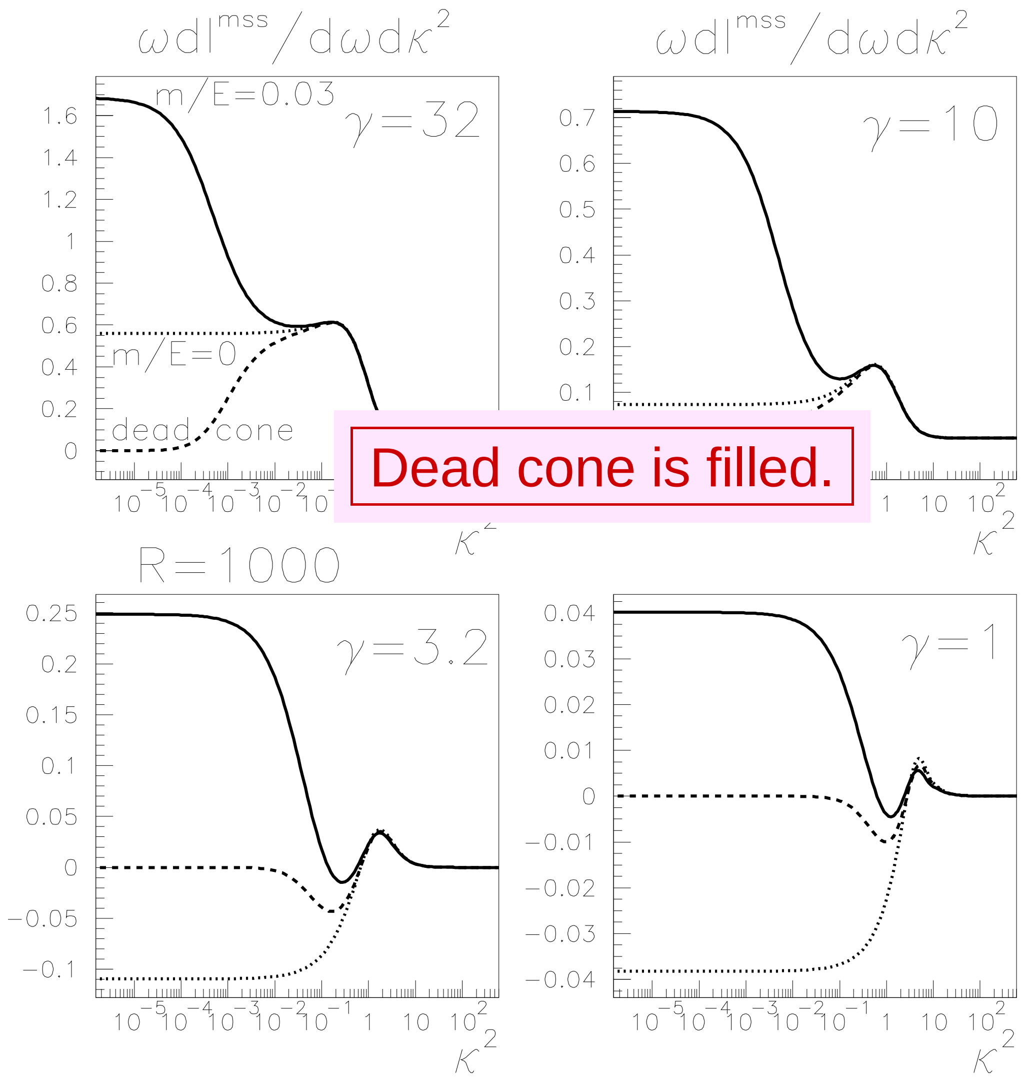
- $I_{\rm med} + I_{\rm vac} \geq 0$.
- For $m > 0$, $I_{\rm med} \geq 0$.



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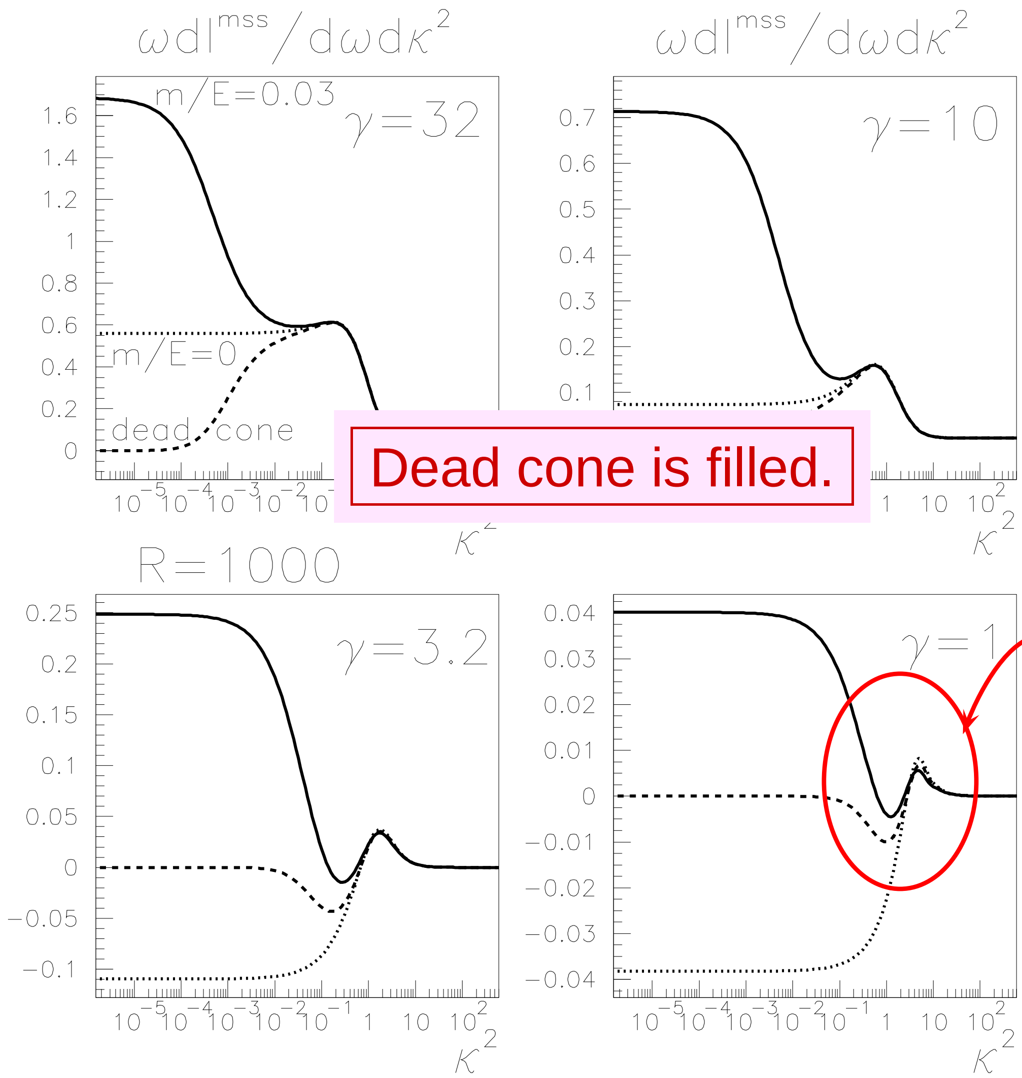

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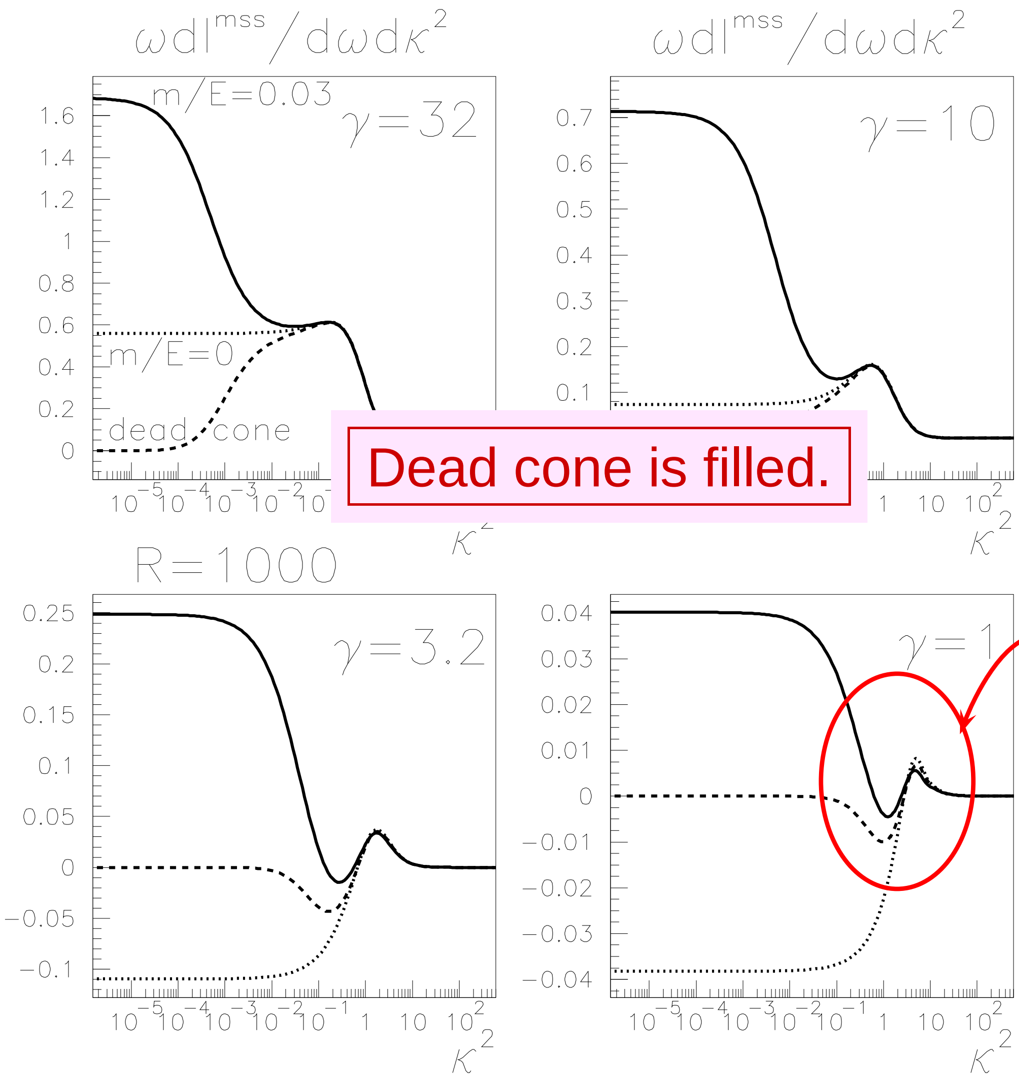

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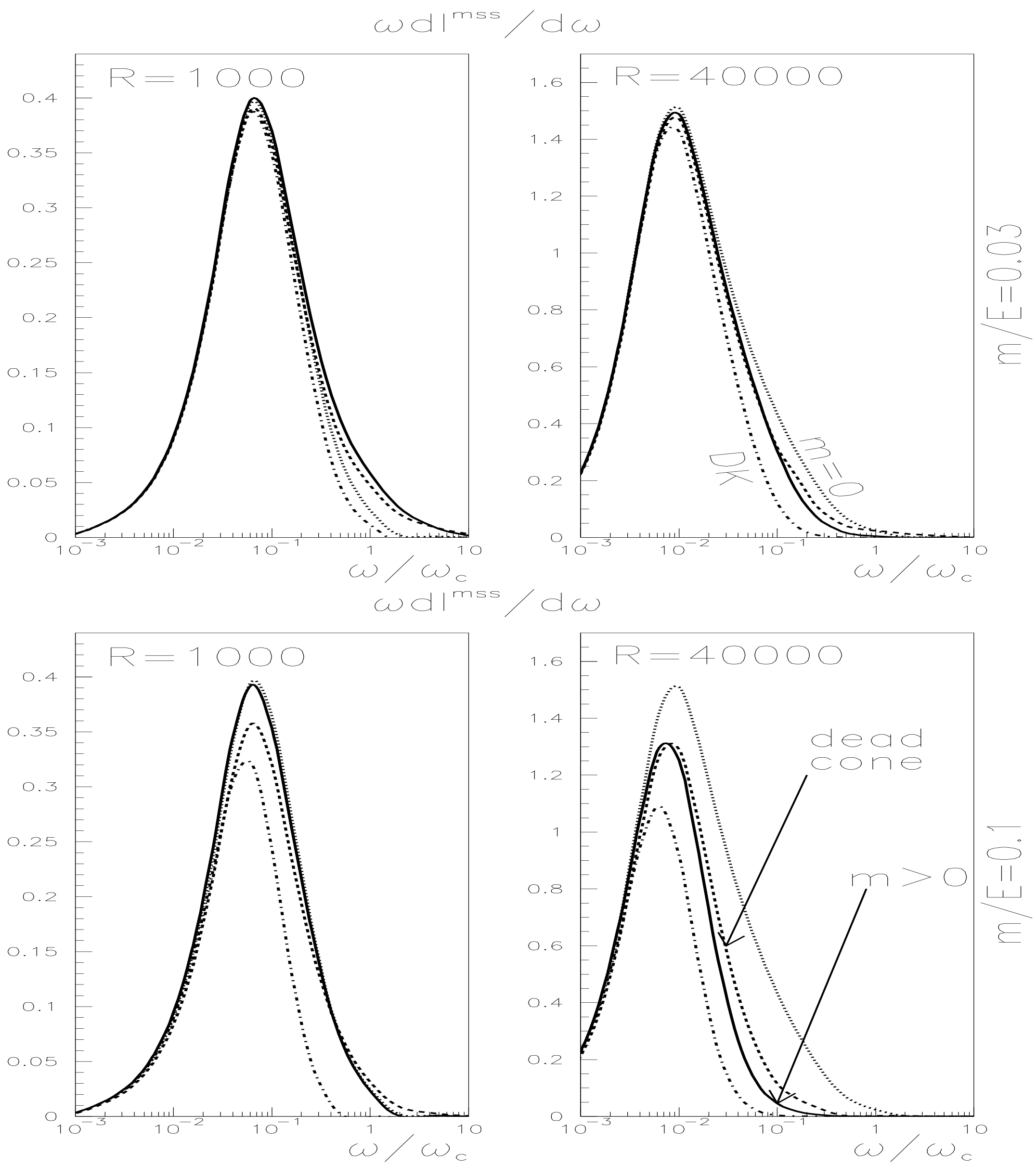
- Kinematically, the dead cone is only a small region of the available phase space.



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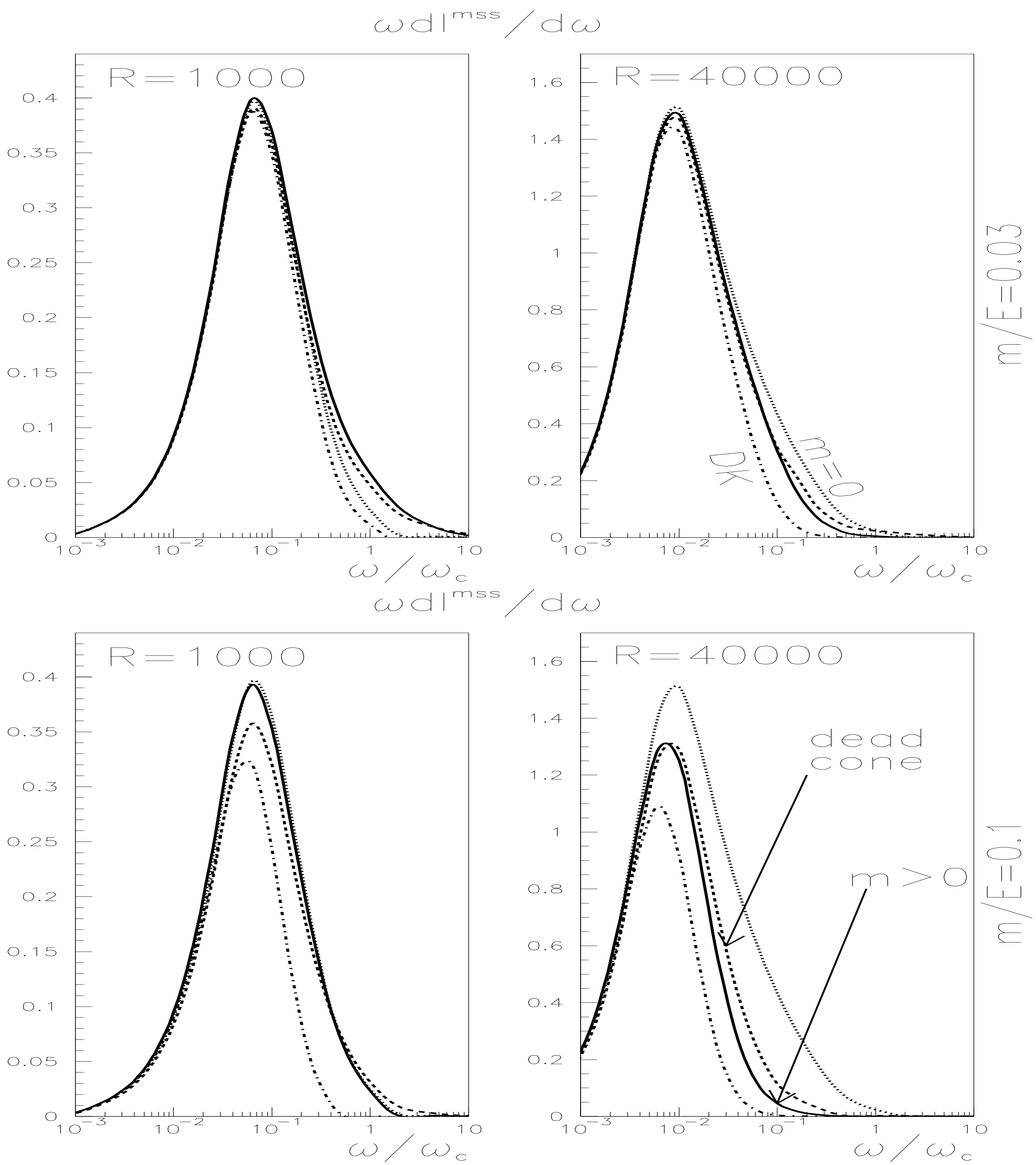

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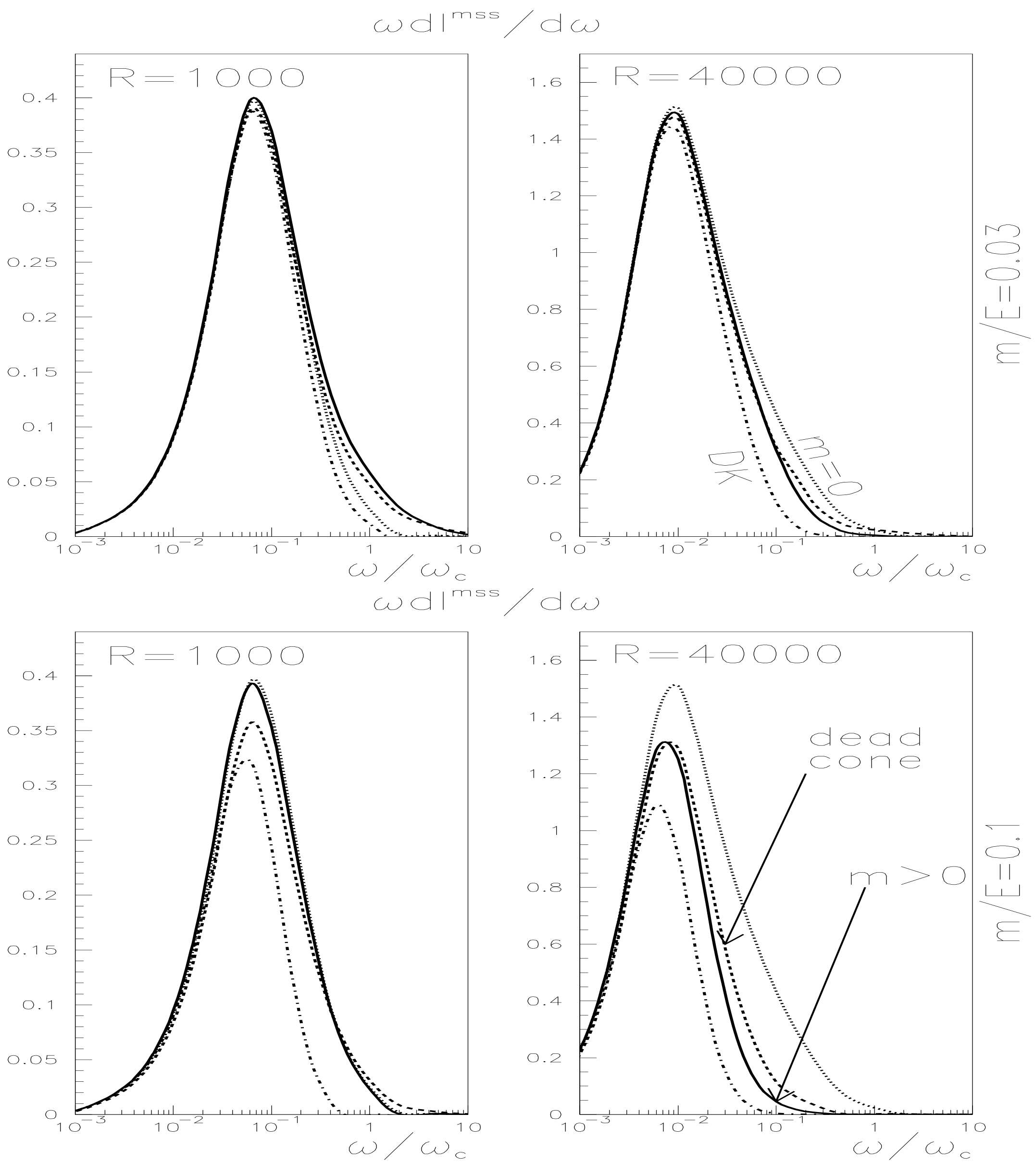

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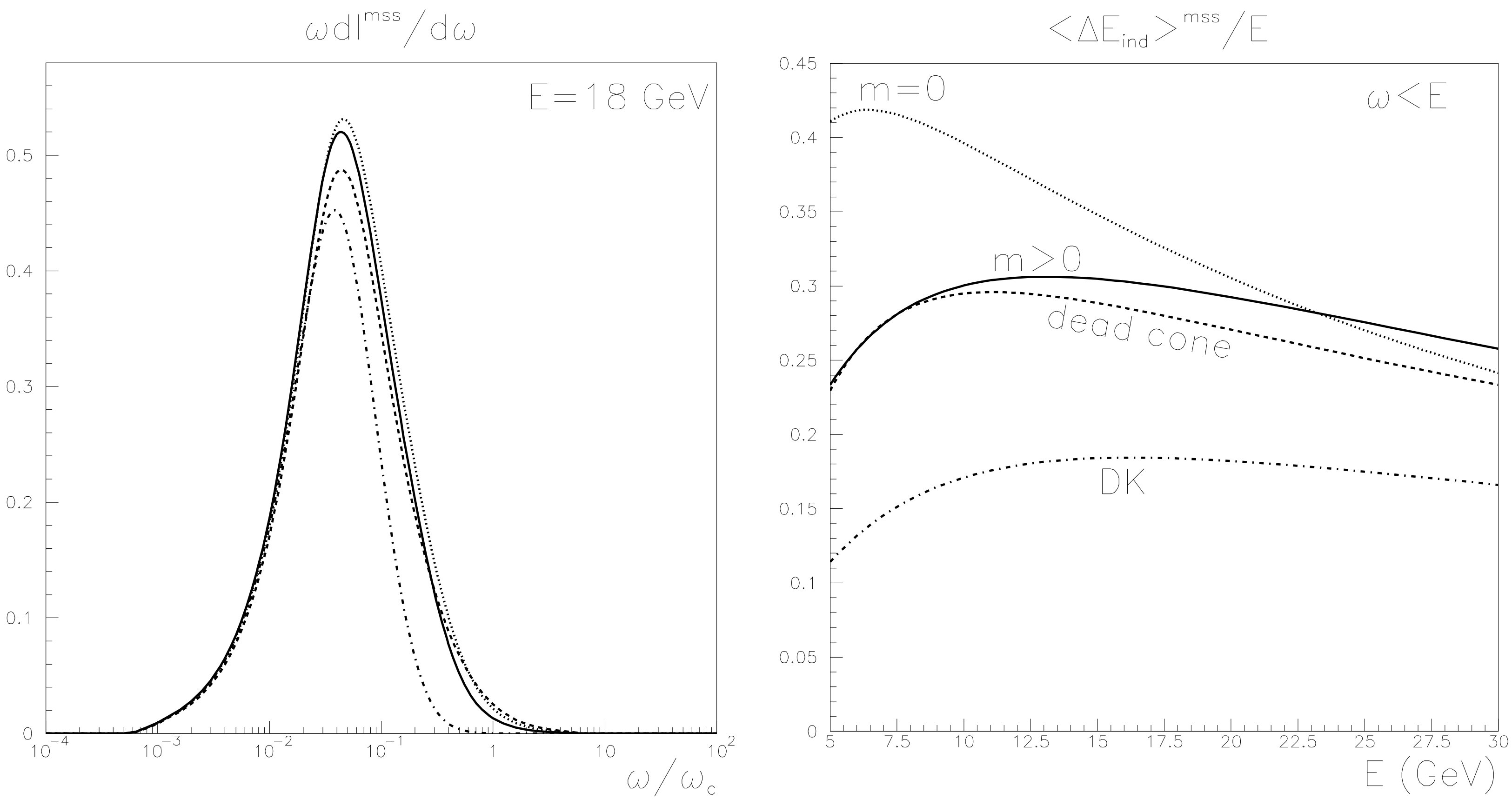
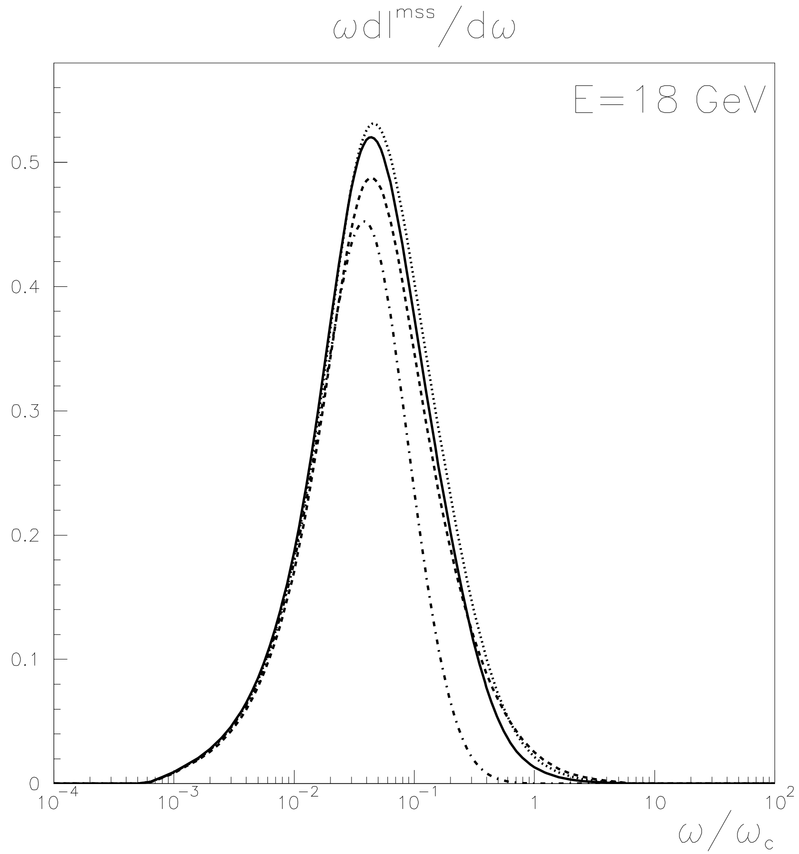
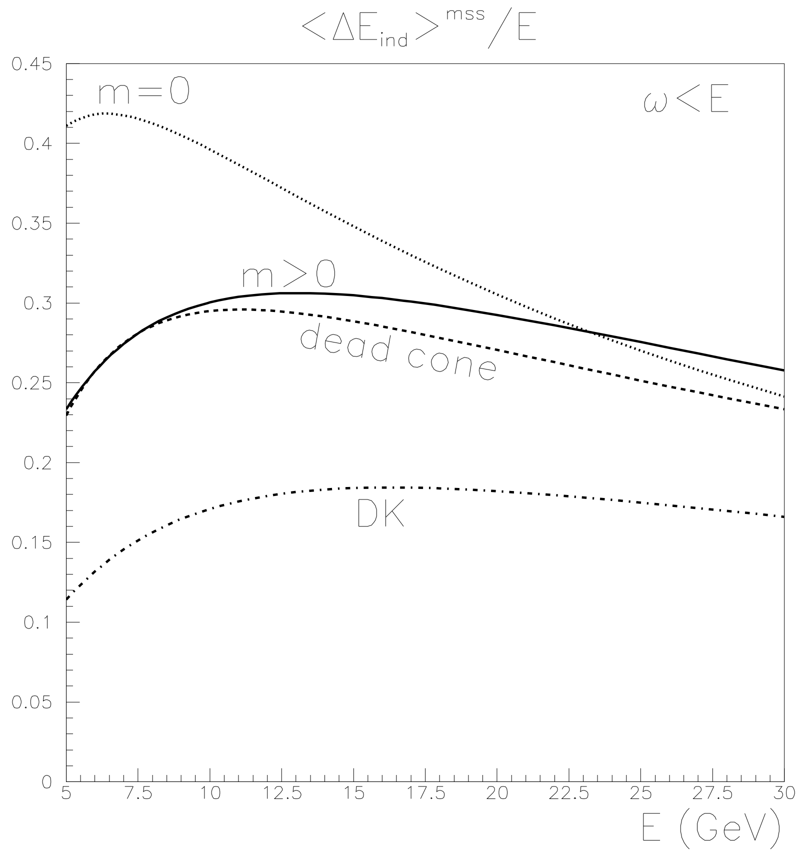
- $F(\kappa, M) = \left(\frac{\kappa^2}{\kappa^2+M^2}\right)^2$.
- $F_{\rm DK}(M) = F(\mathbf{k}_\perp^2 = \sqrt{\hat{q}\omega}, M)$ (Dokshitzer, Kharzeev, PLB519(01)199) underestimates quantitatively the emission.



- $\langle \Delta E_{\rm ind} \rangle = \int_0^{E} d\omega\, \omega \frac{dI_{\rm medium}}{d\omega}$; $L = 6$ fm, $\hat{q} = 0.8$ GeV^2/fm ($R = \omega_c L = 2000$, $\omega_c = \hat{q} L^2/2 = 67.5$ GeV) (Salgado, Wiedemann, PRD68(03)014008) and $m = 1.5$ GeV.





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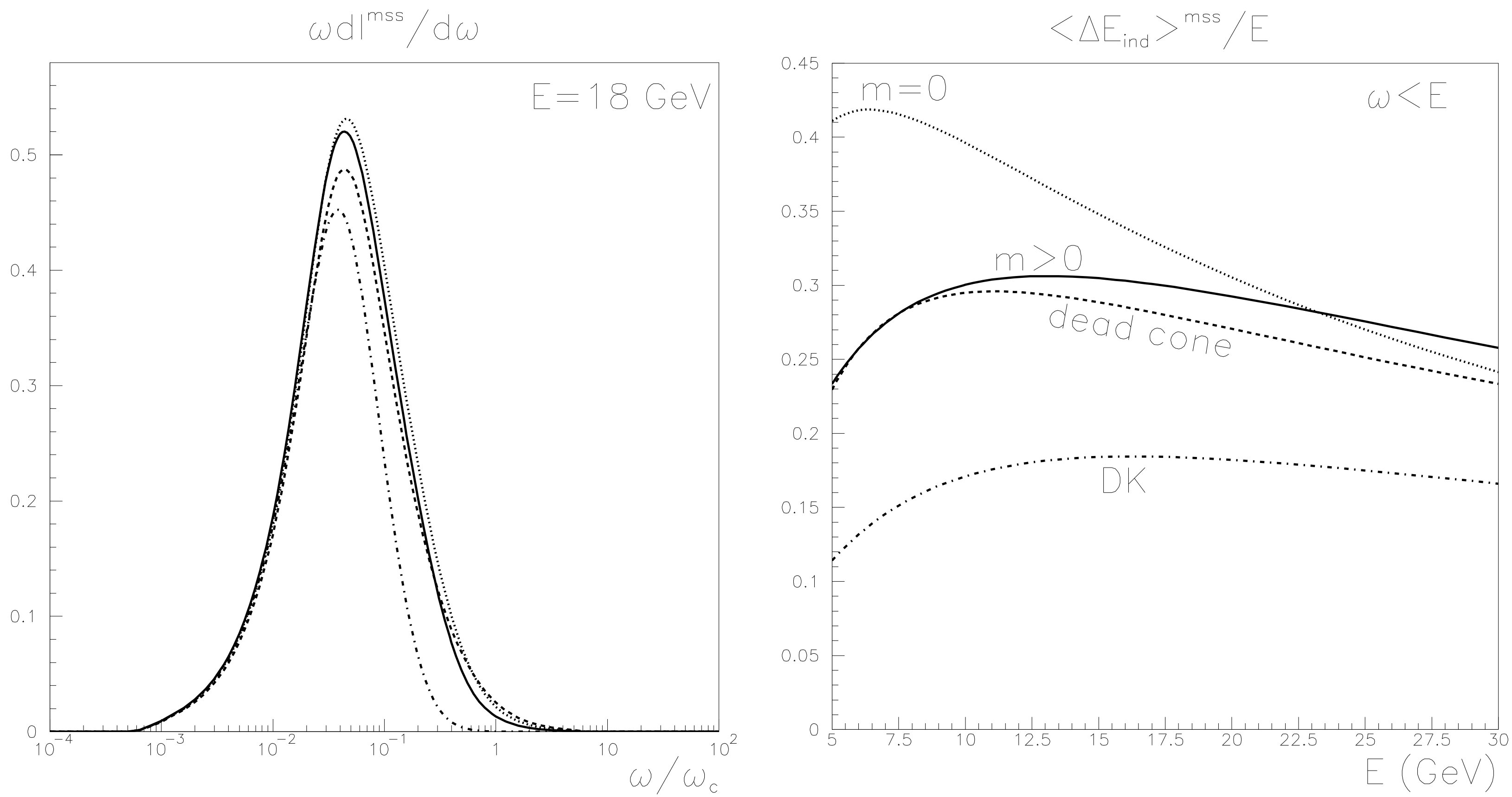
- Energy loss for charm at RHIC turns out to be a factor ~ 2 smaller than for light quarks, but should still be observable.



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- Energy loss for charm at RHIC turns out to be a factor ~ 2 smaller than for light quarks, but should still be observable.
- Caveat: uncertainties in all computations are significant . . .

3. Conclusions:

♣ Medium-induced gluon radiation off massless and massive quarks is treated in the same formalism. Ours is the first $\mathbf{k}_\perp$-differential result (consistent with available $\mathbf{k}_\perp$-integrated ones (Djordjevic, Gyulassy, PLB560(03)37; NPA733(04)265; Zhang, Wang, Wang, nucl-th/0309040)).

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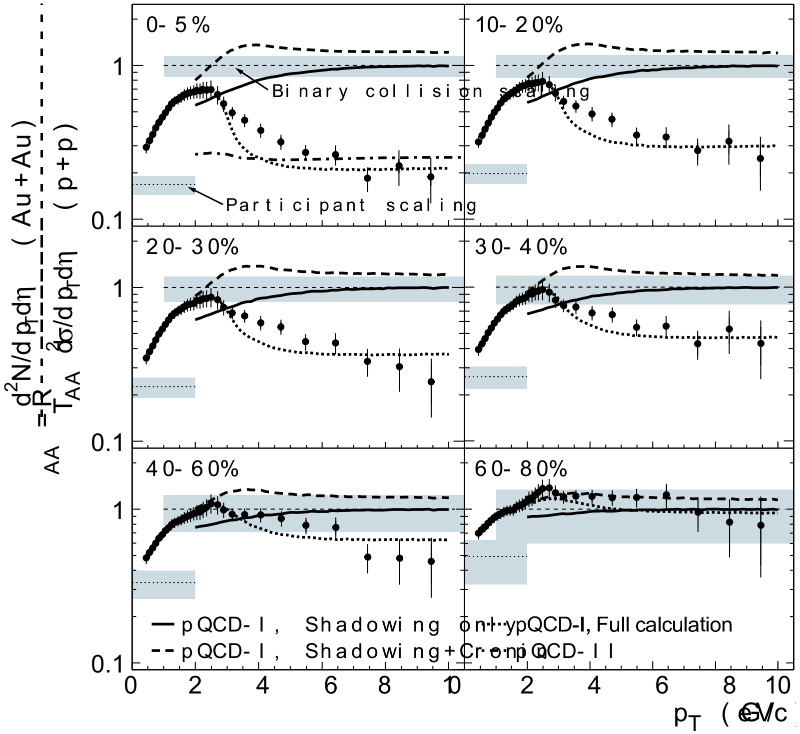
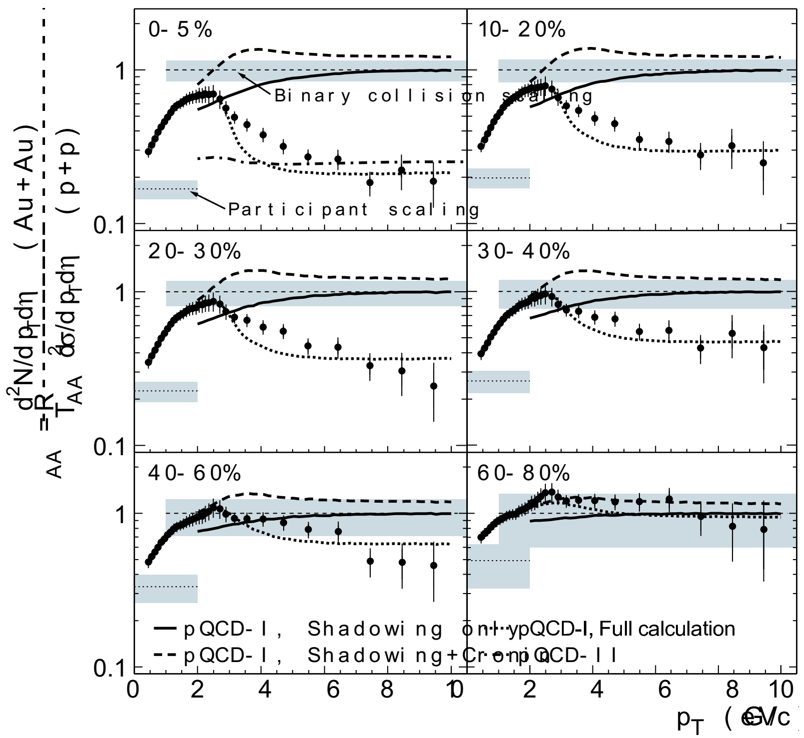
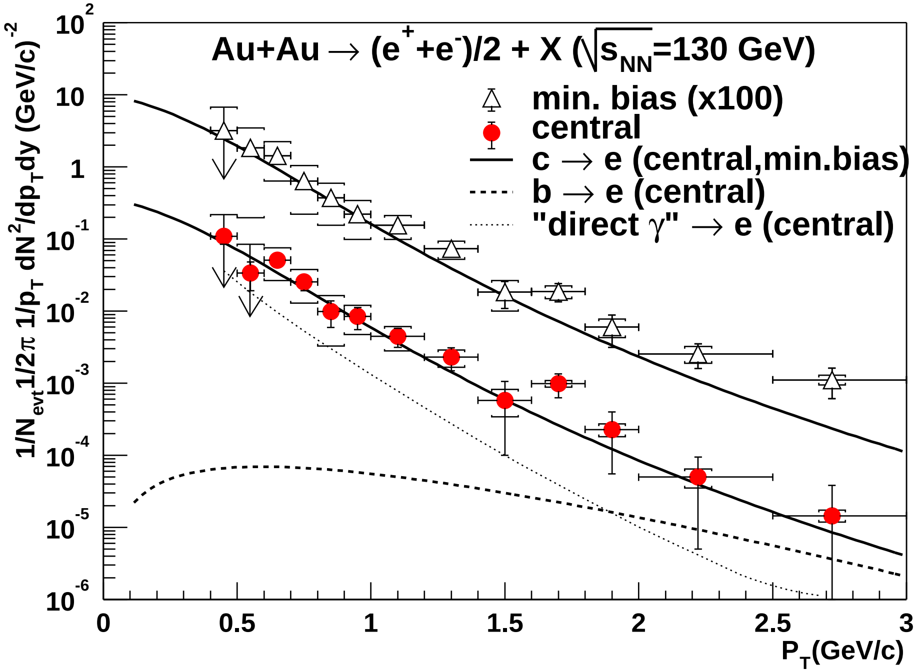
- Uncertainties motivate the computation of $1/E$ corrections.



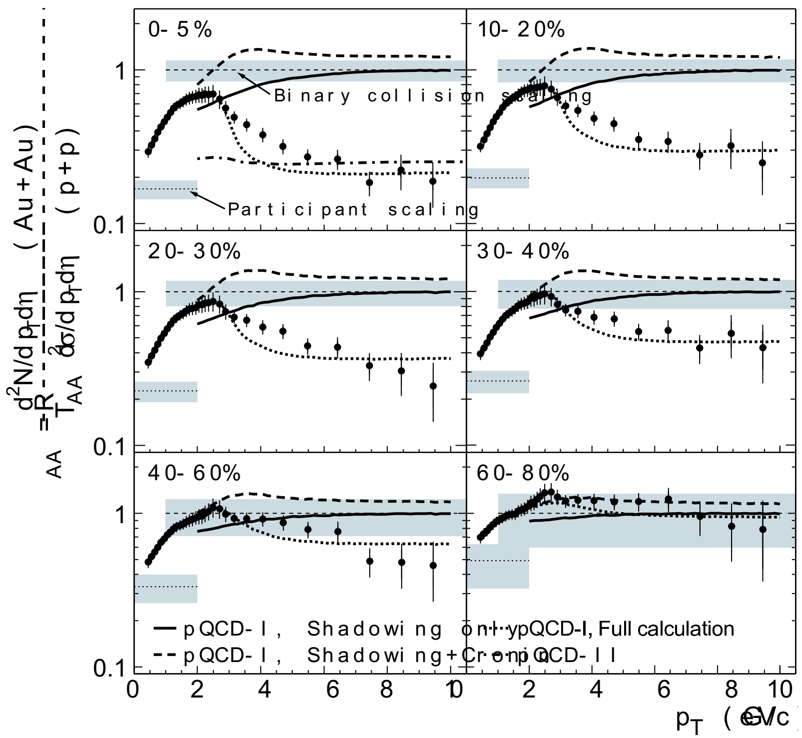
STAR: PRL91(03)172302, AuAu@200 GeV, charged.



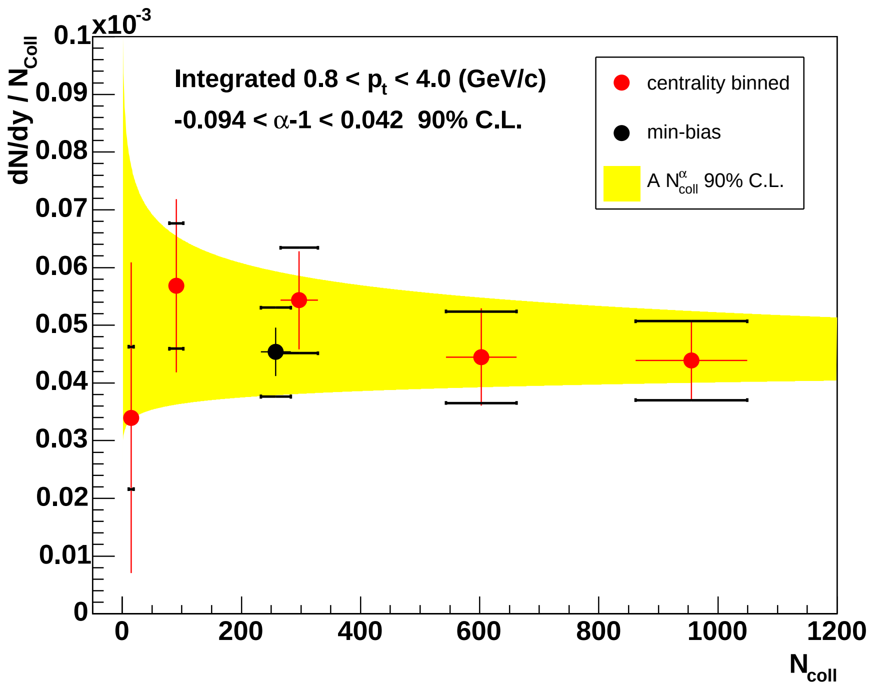
STAR: PRL91(03)172302, AuAu@200 GeV, charged.



PHENIX: PRL88(02)192303.

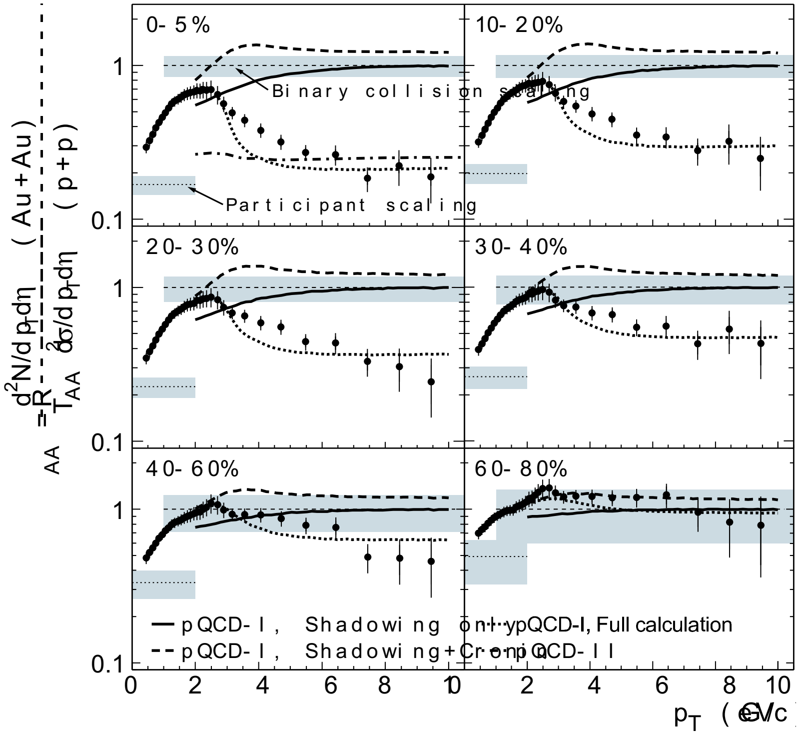


STAR: PRL91(03)172302, AuAu@200 GeV, charged.

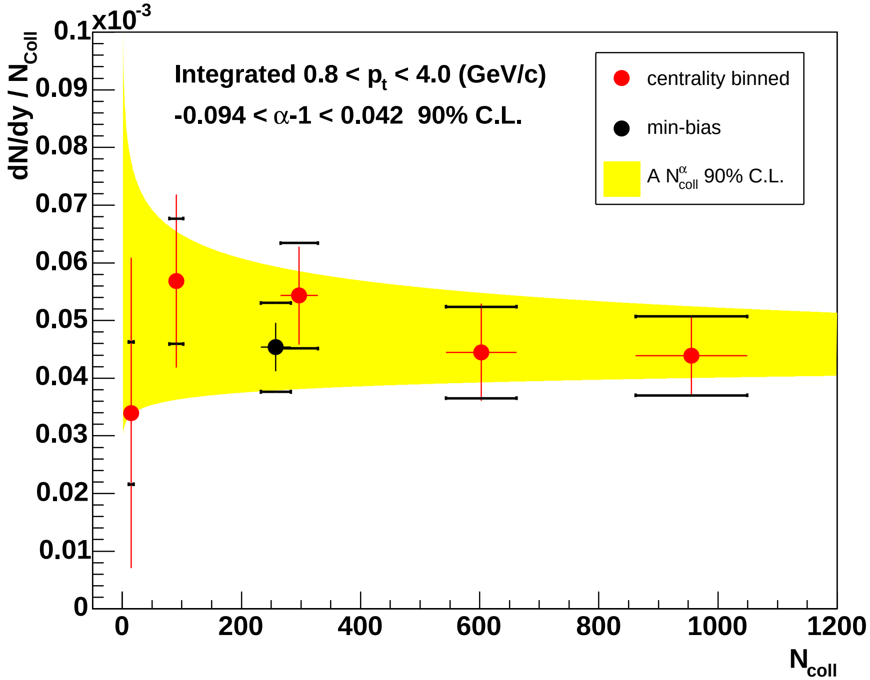


PHENIX: Kelly, nucl-ex/0403057.

Experimental status:



STAR: PRL91(03)172302, AuAu@200 GeV, charged.



PHENIX: Kelly, nucl-ex/0403057.

$\Longrightarrow$ apparently less suppression, if any, for charmed than for lighter hadrons (but single e$^-$ spectra do not constrain energy loss significantly (Batsouli, Kelly, Gyulassy, Nagle, PLB557(03)26), and $p_\perp$ are small so hadronization effects have to be taken into account).