

# Fermi – Dirac correlations in $Z^0 \rightarrow ppX$ at LEP

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- Introduction
- New results on Fermi-Dirac correlations
  - for  $\bar{p}\bar{p}$  from DELPHI
  - for  $pp$  and  $\bar{p}\bar{p}$  from ALEPH
- Bose-Einstein correlations for  $K_S^0 K_S^0$  from ALEPH
- Possible explanations for the observed  $R(m)$  dependence
  - using the Heisenberg uncertainty relation (G. Alexander et al.)
  - based on the Björken-Gottfried condition (A. Bialas, K. Zalewski)
- Conclusions

# Introduction

## Hanbury Brown-Twiss (HBT) interferometry

- identical particle interferometry usually identified with HBT (intensity interferometry)
- for some distributions of sources  $\rho(x,t)$  interference pattern observed in momentum (wavelength) space

$$C_2(\vec{r}) = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \sim 1 + \cos(\vec{k} \cdot \Delta\vec{r})$$

## HBT in particle physics

- in particle physics :  
→ symmetrisation (BEC) and antisymmetrisation (FDC) of total wave function
- in  $e^+e^-$  applied to  $\pi\pi$ ,  $KK$ ,  $K^0\bar{K}^0$ ,  $\Lambda\Lambda$ ,  $pp$
- correlation function

$$C_2(q_1, q_2) = C_2(Q) = \frac{\rho_2(Q)}{\rho_2(Q)^{REF}}, \quad Q^2 = -(q_1 - q_2)^2$$

## Goldhaber parametrisation

- $R$  is the radius of a sphere describing source shape
- $\lambda$  is chaoticity parameter (0 – completely coherent source, 1 – entirely chaotic case)
- $N$  – normalisation factor
- long range correlations' term sometimes added
- other parametrisations also used, e.g.

$$C_2(Q) = N (1 \pm \lambda e^{-Q^2 R^2})$$

BEC

FDC

$$C_2(Q) \sim e^{-QR}$$

## DATA

$Z^0 \rightarrow$  hadrons

2.1 M events (the best ident. information)

180 K ev. with  $\geq 2$  protons

## MC

JETSET 7.4 (no F-D corr. simulated)

6.2 M  $Z^0 \rightarrow$  hadrons

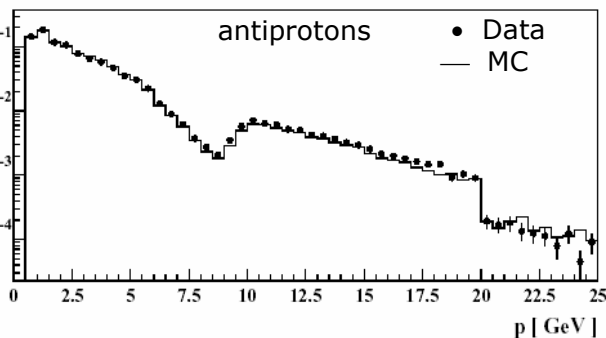
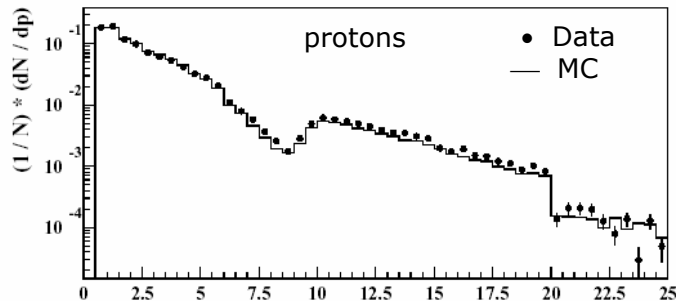
560 K ev. with  $\geq 2$  protons

## Selection criteria

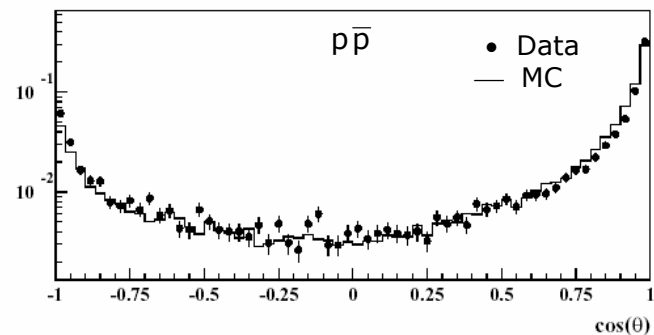
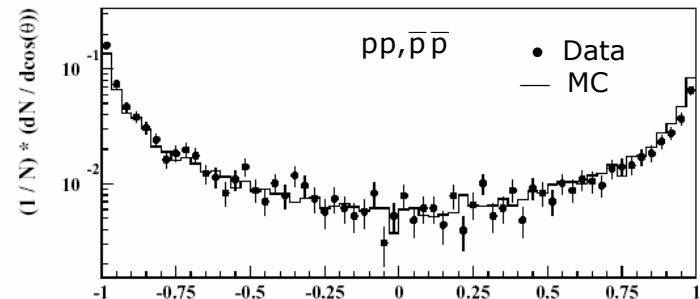
- Standard DELPHI hadronic event selection
- $p_{\text{LAB}} > 0.7$  GeV (good MC-data agreement)
- proton candidate's track measured in the silicon VD
- track well identified as a proton (combining RICH, dE/dx in TPC and VD)
- $\Lambda^0$  decay products excluded
- at least 1 pair ( $pp$ ,  $\bar{p}\bar{p}$ ,  $p\bar{p}$ ) in the event

single proton purity  $\sim 85\%$   
efficiency  $\sim 70\%$

proton-proton purity  $\sim 50\%$   
antiproton-antiproton purity  $\sim 70\%$



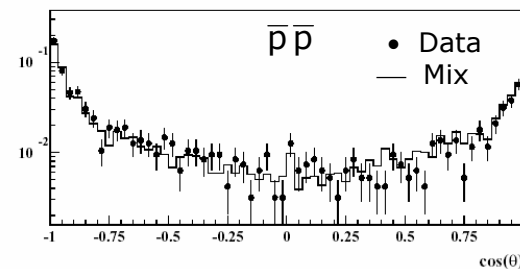
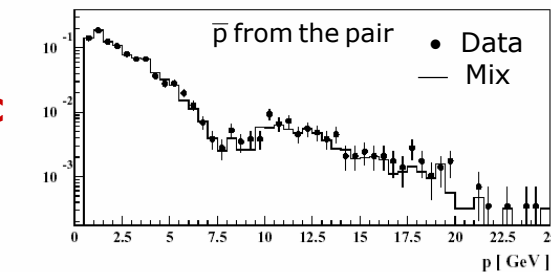
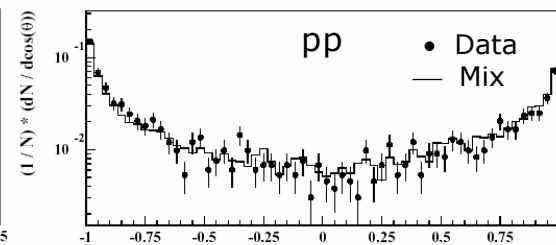
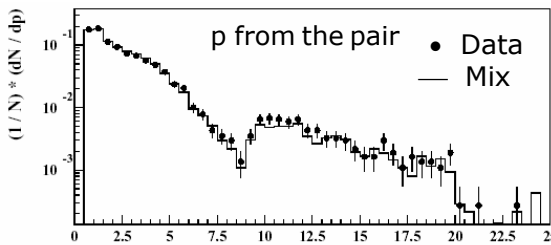
good agreement data-MC observed



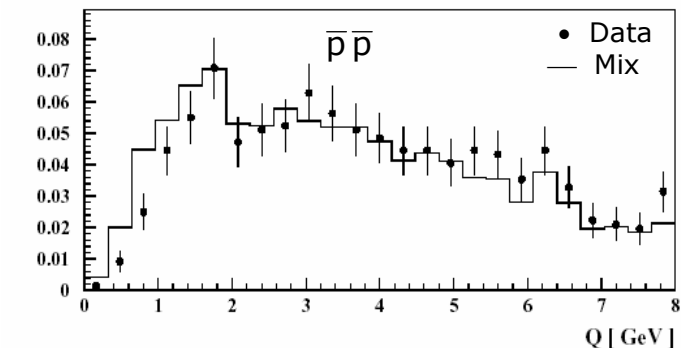
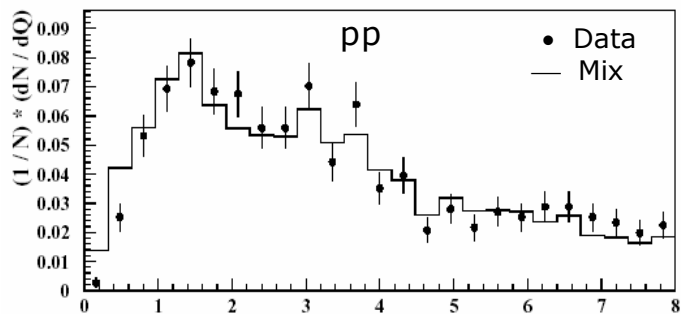
good agreement data-mix

### Mixing recipe

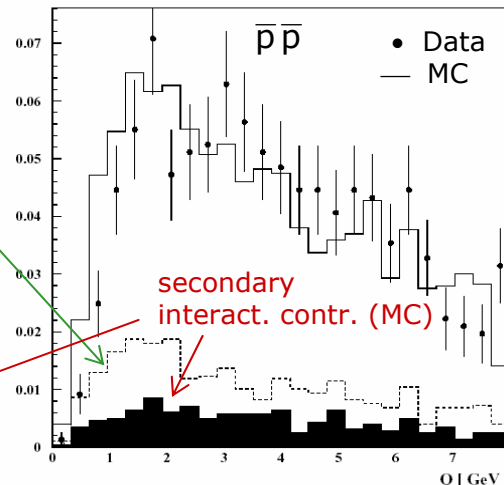
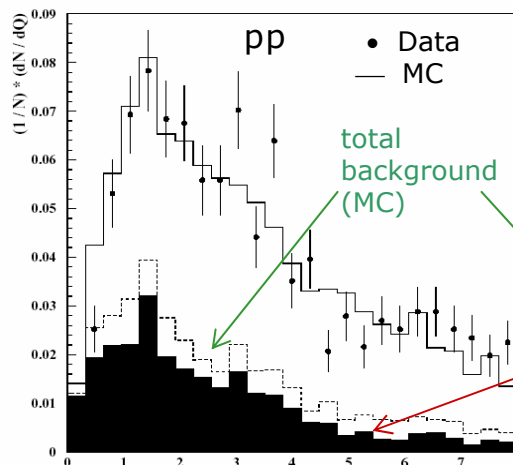
- momentum vectors transformed to the thrust reference frame
- for each  $\bar{p}_1\bar{p}_2$  ( $p_1p_2$ ), momentum vectors' pairs  $\bar{p}_1\bar{p}_{MIX,2}$ ,  $\bar{p}_{MIX,1}p_2$  are constructed



### Q distributions in Data vs Mix and MC



more backgrd in pp (sec. int.) → the main result for  $\bar{p}\bar{p}$



## Correlation function

$$C_2(Q) = \frac{N(Q)^{DATA}}{N(Q)^{REF}}, \quad REF = mix, MC, unlike$$

double ratio needed

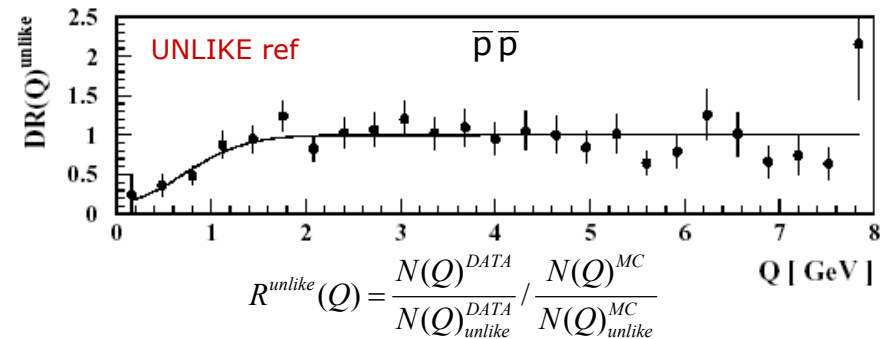
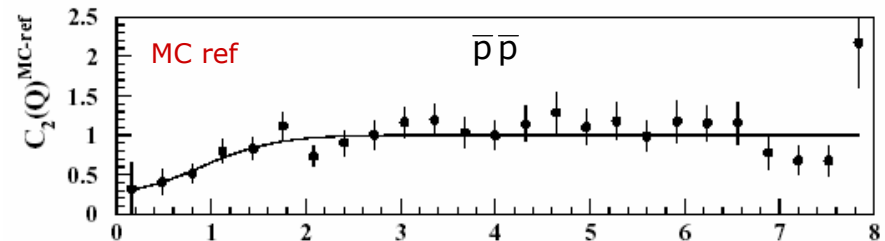
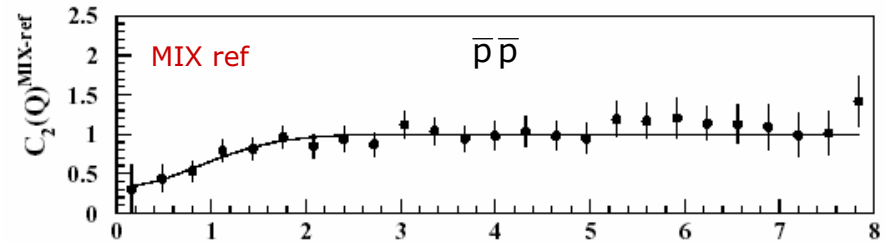
## Goldhaber parametrisation

$$C(Q) = N(1 - \lambda e^{-R^2 Q^2})$$

- double ratio used (whenever appropriate)
- the main result for  $\bar{p}\bar{p}$ , mixed-ref.
- others samples used for x-checks and syst. error determination

List of systematic errors

systematic error source	$\Delta R$	$\Delta \lambda$
identification ; XPRNET $\in (0.90 - 0.97)$	0.017	0.141
opening angle $\theta > 2^\circ$	0.008	0.023
background subtraction	0.004	0.052
different reference samples	0.013	0.068
double ratio	0.002	0.011
$C_2(Q)$ parametrisation	0.011	0.022
fit range ; $Q_{MAX} \in (6.0 - 9.0) GeV$	0.003	0.005
$Q$ bin size ; $\Delta Q \in (0.25 - 0.35) GeV$	0.011	0.063
total systematic error	0.028	0.180



$$R = 0.16 \pm 0.04(stat) \pm 0.03(syst) \text{ fm,}$$

$$\lambda = 0.67^{+0.19}_{-0.17} (stat) \pm 0.18(syst)$$

MIX

# ALEPH analysis for (anti)protons

ALEPH  
preliminary

## DATA

3.9 M  $Z^0 \rightarrow$  hadrons

## MC

JETSET 7.4 (no B-E or F-D corr.)

6.5 M  $Z^0 \rightarrow$  hadrons

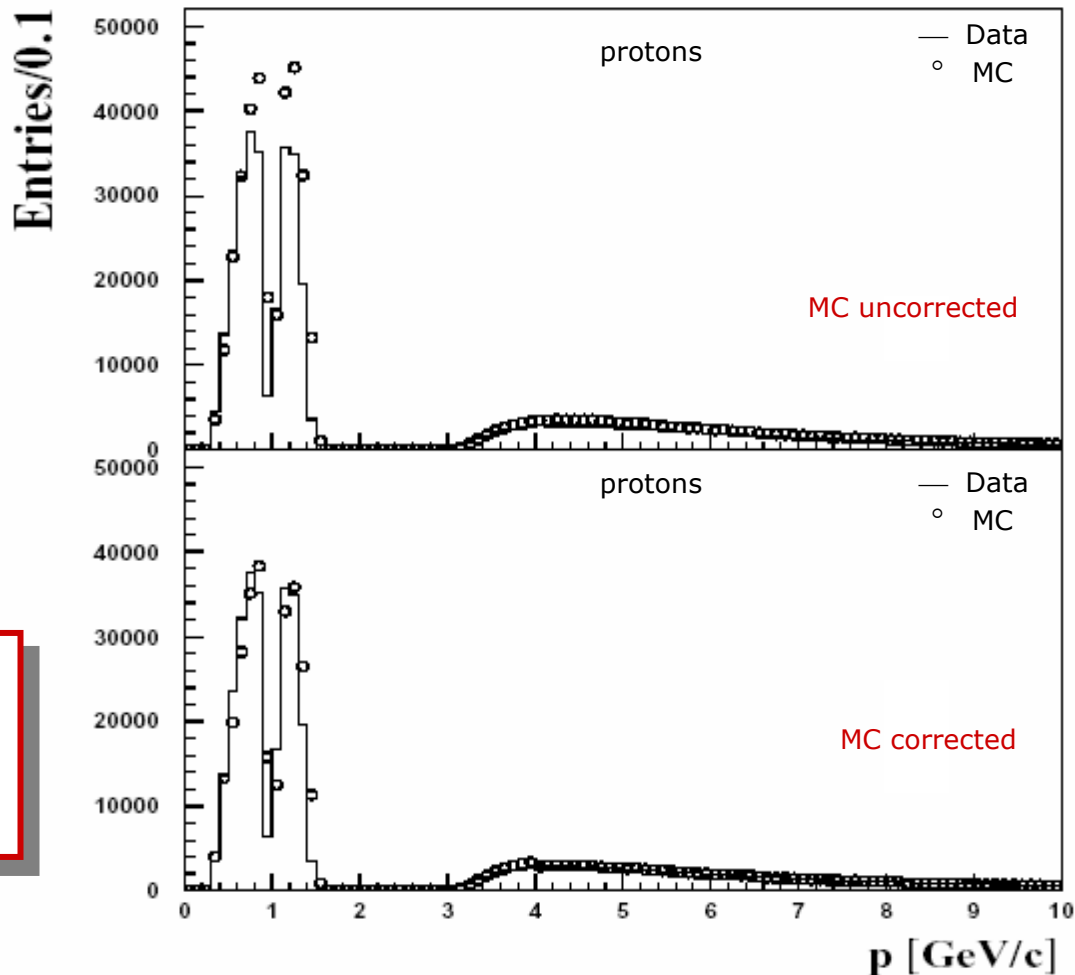
## Data selection

- number of charged tracks  $> 5$
- $E_{\text{tot}} > 0.1 \sqrt{s}$
- $p_{\text{LAB}} > 0.4 \text{ GeV}$
- four TPC hits
- impact parameter in  $z < 10 \text{ cm}$
- impact parameter in  $R\phi < 2 \text{ cm}$
- $|\cos\theta| < 0.95$

- corrections for  $0.5 < p < 1.3 \text{ GeV}$   
(to have agreement data-MC)
- only protons from primary vertex  
(by using  $\chi^2$  criterium)

## Identification

- $p, \bar{p}$  identification  $\rightarrow dE/dx$
- $1.3 < p_{\text{LAB}} < 3.0 \text{ GeV}$  excluded



single proton purity  $\sim 70\%$   
(anti)proton-(anti)proton purity  $\sim 74\%$  (0-5 GeV)

# Correlation radius

ALEPH  
preliminary

## Goldhaber parametrisation

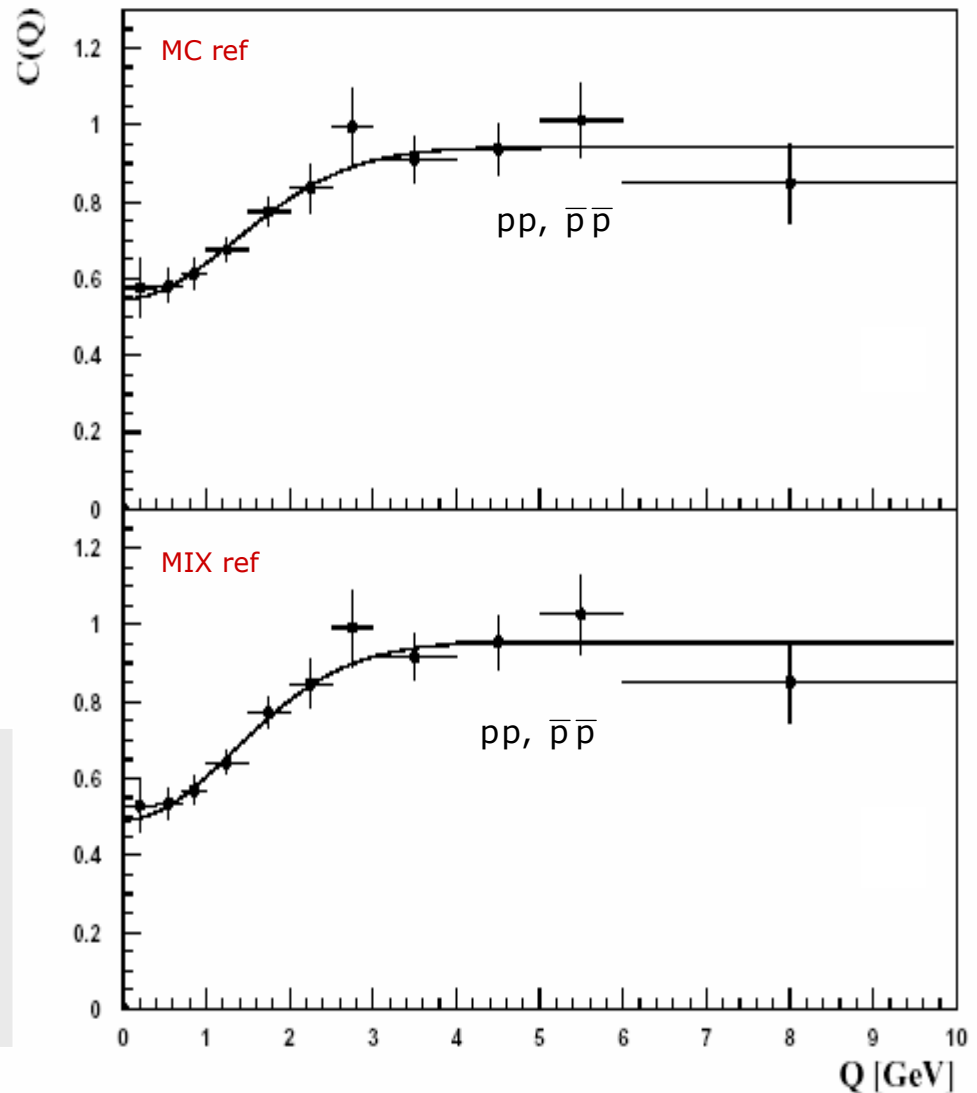
$$C(Q) = N(1 - \lambda e^{-R^2 Q^2})$$

$$R = 0.10 \pm 0.02(\text{stat}) \pm 0.02(\text{syst}) \text{ fm},$$
$$\lambda = 0.46 \pm 0.04(\text{stat}) \pm 0.11(\text{syst})$$

MIX

## Main contributions to systematics

- Coulomb corrections  
(Gamov fact., 12% corr. in the 1st bin)
- different reference sample
- reweighting MC



# Summary for pp analyses

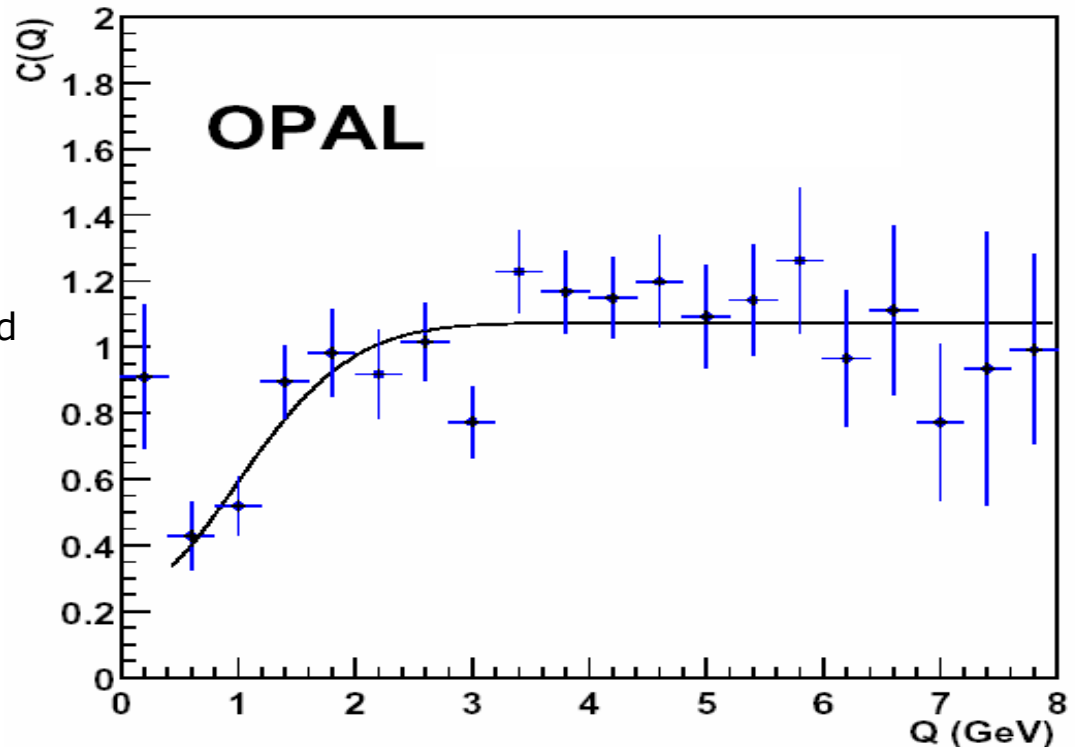
	$R$ [fm]	$\lambda$	ref. sample	
<i>ALEPH</i>	$0.10 \pm 0.02$ (stat) $\pm 0.02$ (syst)	$0.46 \pm 0.04$ (stat) $\pm 0.11$ (syst)	<i>mix</i>	$p p, \bar{p} \bar{p}$
<i>DELPHI</i>	$0.16 \pm 0.04$ (stat) $\pm 0.03$ (syst)	$0.67^{+0.19}_{-0.17}$ (stat) $\pm 0.18$ (syst)	<i>mix</i>	$\bar{p} \bar{p}$
<i>OPAL</i>	$0.14 \pm 0.04$ (stat) $\pm 0.05$ (syst)	$0.76 \pm 0.16$ (stat) $\pm 0.29$ (syst)	<i>MC</i>	$\bar{p} \bar{p}$

## Results consistent within errors

OPAL Coll.,  
CERN-EP/2001, OPAL-PN486

- Event-mixing ref.,
- MC ref.,
- UNLIKE ref. samples studied

• MC for the main result





## Data selection

- pion's momentum > 0.15 GeV
- both tracks from the same secondary vertex
- $\chi^2$  test of energy-momentum conservation
- cuts on the impact parameter of the sec. tracks from  $V^0$
- $K^0 \rightarrow \pi\pi$  accepted if  $\chi^2$  for pion hypothesis > 0.005

kaon-kaon purity  $\sim 96\%$   
efficiency  $\sim 27\%$

216 K ev. with 2 kaons

- enhancement seen at  $Q < 0.5$  GeV
- rise of  $C(Q)$  for  $Q > 0.8$  GeV in MC-ref (imperfect MC simulation)
- region of  $f_0(1710)$  excluded from the fit

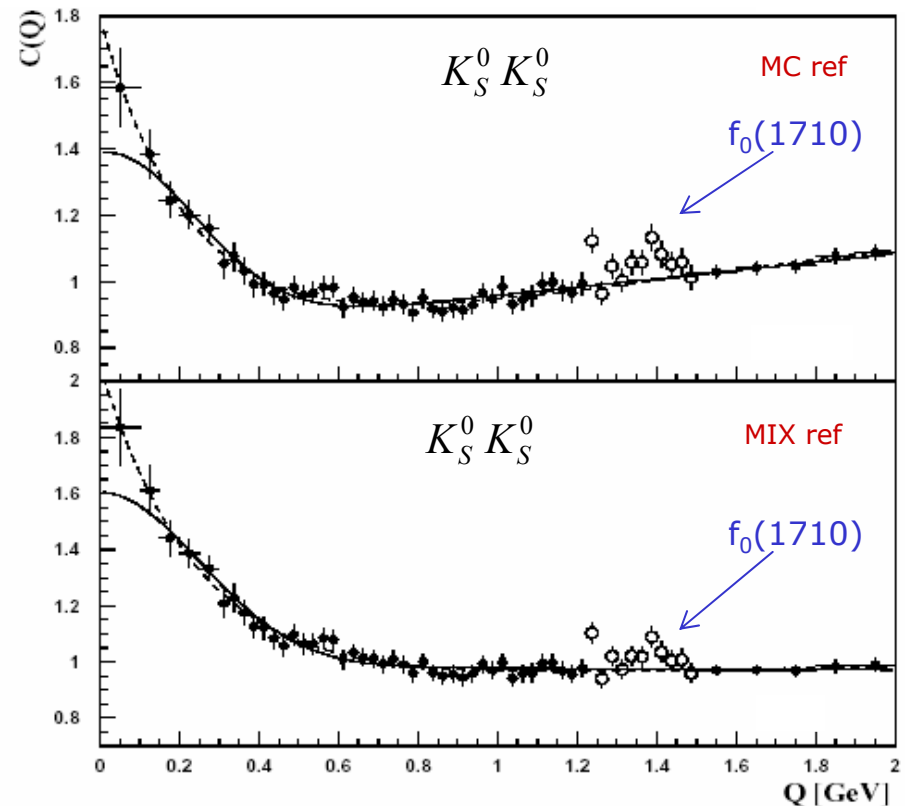
$f_0(980) \rightarrow K_S^0 K_S^0$  signal included in MC  
replaced by contr. from Breit-Wigner distr.

## Parametrisation

$$C(Q) = N(1 + \lambda e^{-R^2 Q^2})(1 + \alpha Q)$$

$$R = 0.57 \pm 0.04(\text{stat}) \pm 0.06(\text{syst}) \text{ fm},$$

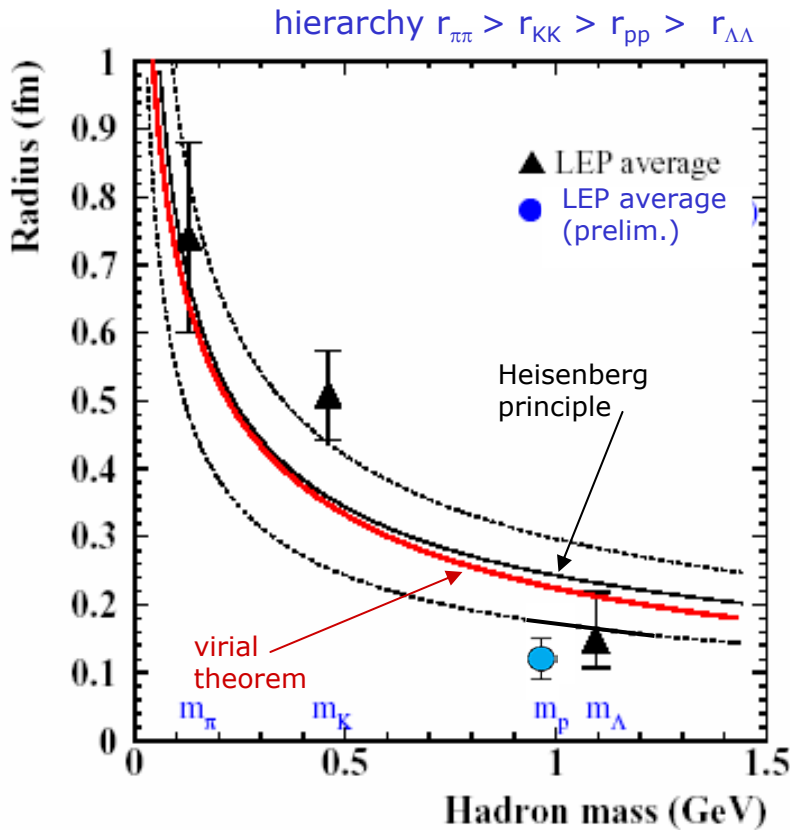
$$\lambda = 0.63 \pm 0.06(\text{stat}) \pm 0.06(\text{syst})$$



MIX, Gaussian-like par.,  $\alpha = 0.005 \pm 0.03$

# Models based on Heisenberg principle

G. Alexander, I. Cohen, E. Levin,  
Phys. Lett. **B452** (1999) 159.



- heavier particles are emitted first
- $\Delta t$  does not depend on the mass
- $\Delta E$  depends only on kinetic energy
- $\Delta r \sim r$

from the Heisenberg uncertainty principle

$$r(m) = \frac{\hbar c / \sqrt{\hbar / \Delta t}}{\sqrt{m}} = \frac{c \sqrt{\hbar \Delta t}}{\sqrt{m}}$$

fit :  $r(m) = A / \sqrt{m}$ ,  $A = 0.243 \text{ fm GeV}^{1/2}$ ,  
 $\Delta t = 10^{-24} \text{ sec}$

upper dashed curve:  
 $\Delta t = 1.5 \times 10^{-24} \text{ sec}$   
lower dashed curve:  
 $\Delta t = 0.5 \times 10^{-24} \text{ sec}$

QCD model using virial theorem

$$r^2 \langle \vec{r} \cdot \vec{\nabla} V(r) \rangle \sim \frac{(\hbar c)^2}{m}$$

general QCD potential

$$V(r) = \kappa r - \frac{4}{3} \frac{\alpha_s \hbar c}{r}$$

$$\kappa = 0.14 \text{ GeV}, \quad \alpha_s = 2\pi \ln(2 + 1.87/r)$$

# Model based on Björken-Gottfried condition

A. Bialas, K. Zalewski,  
Acta Phys. Polon. **B30** (1999) 359.

## Björken-Gottfried condition

$$p_\mu = \lambda x_\mu$$

$$\tau_0^2 = t^2 - z^2$$

$$p_\mu = \frac{M_\perp}{\tau_0} x_\mu$$

source density matrix

$$\rho(q, q') = \int d^4 X e^{iQX} S(P, X)$$

$$P = (q + q')/2, \quad Q = q - q'$$

## source function

$$S(P, X) = W(\vec{P}, \vec{X}) \delta(X_0 = 0) = F(\tau) S_\parallel S_\perp$$

$$S_\parallel = \exp \left[ \frac{1}{2\delta_\parallel^2} \left( P_+ - \frac{M_\perp}{\tau} X_+ \right) \left( P_- - \frac{M_\perp}{\tau} X_- \right) \right],$$

$$S_\perp \sim \exp \left[ -\frac{1}{2\delta_\perp^2} \left( \vec{P}_\perp - \frac{M_\perp}{\tau} X_\perp \right)^2 \right]$$

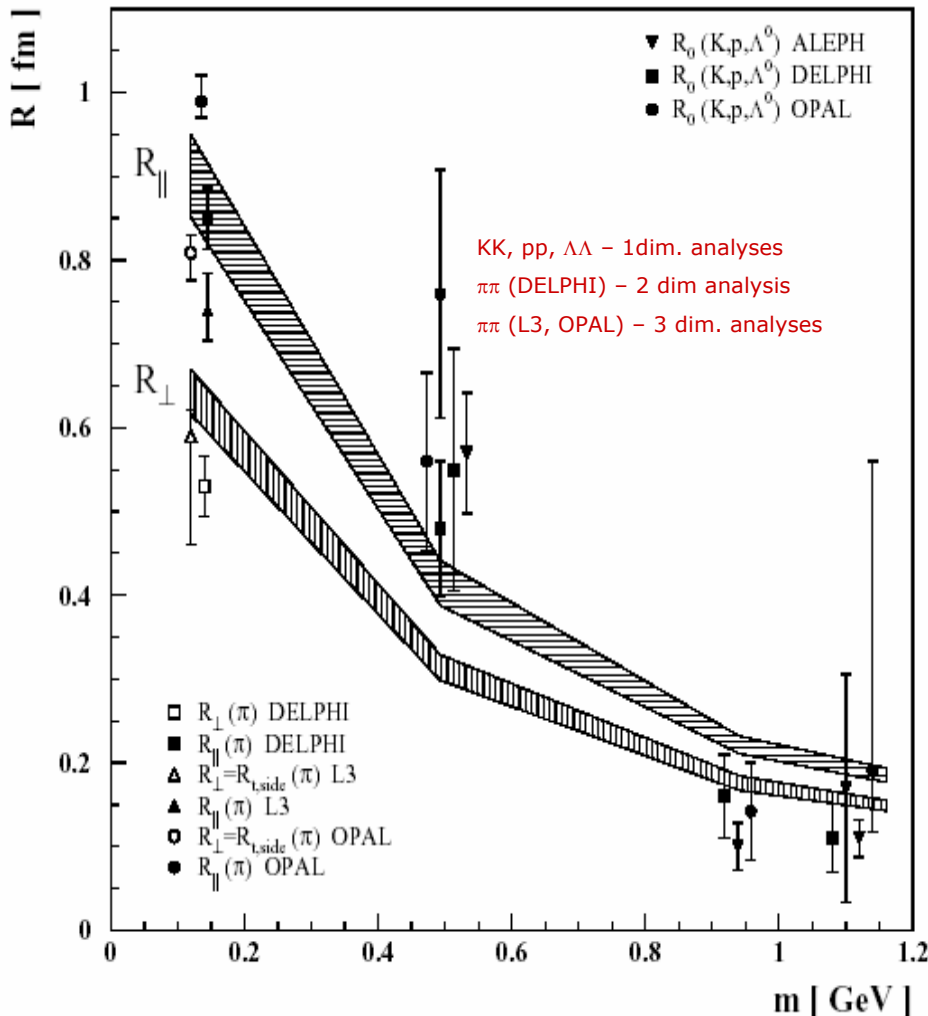
$$X_\pm = t \pm z, \quad P_\pm = P_0 \pm P_z, \quad M_\perp^2 = P_+ P_-, \quad \tau = X_+ X_-$$

## one-particle distribution

$$\rho(p) \equiv \frac{dn}{dy d^2 p_\perp} \sim r_\perp^2 \delta_\perp^2 \exp \left[ \frac{m_\perp^2}{\delta_\parallel^2} \right] K_0 \left( \frac{m_\perp^2}{\delta_\parallel^2} \right) \exp \left[ -\frac{q_\perp^2}{2\Delta^2} \right]$$

# Model based on Björken-Gottfried condition (cont.)

A. Bialas, M. Kucharczyk, H. Palka, K. Zalewski,  
 Phys. Rev. **D62** (2000) 114007.  
 Acta. Phys. Polon., **B32** (2001) 2901.



## Free parameters

- $F(\tau) \rightarrow \tau = 0.9 \text{ fm}$
- $\delta_{||}, \delta_{\perp}$  parametrise the correlation range
- $v^2 = r^2 / \tau^2$
- $\Delta$  represents width of Gaussian-like distr. of  $p_{\perp}$

$v$	$\delta_{\perp} = \delta_{  } \text{ [GeV]}$	$\Delta \text{ [GeV]}$
0.94	0.233	0.421

**x-p correlations**  $\rightarrow$  **apparent source**

$$\frac{1}{R_{HBT}} \approx \frac{1}{R} + \frac{M_{\perp}^2}{\tau_0^2 \delta^2} \Rightarrow R_{HBT} < R$$

all particles emitted at the same radius

## Summary and conclusions

- **New measurements of the hadronic source radius by Fermi-Dirac correlations:**
  - **ALEPH :**  $R = 0.10 \pm 0.02 \pm 0.02 \text{ fm}$
  - **DELPHI :**  $R = 0.16 \pm 0.04 \pm 0.03 \text{ fm}$
- **New result for  $K_S^0 K_S^0$  :**  $R = 0.57 \pm 0.04 \pm 0.06 \text{ fm}$  (BEC, ALEPH)
- **Together with the previous ALEPH, DELPHI, L3, OPAL measurements the  $R(m)$  is established**
- **Do we measure the real radius or apparent?**  
(the real radius  $R_p \sim 0.1 \text{ fm}$  implies the enormous energy density  $> 100 \text{ GeV} / \text{fm}^3$ )
- **The model based on Heisenberg relations and virial theorem gives acceptable description of  $R(m)$**
- **Quantum-mechanical approach based on the Björken-Gottfried condition explains the mass dependence and longitudinal elongation of the source and allows to extract the information of space-time evolution of the hadronic source**