Hadronic effects in the muon anomalous magnetic moment

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Presentation is based on:

A. Czarnecki, W. J. Marciano and A. V.	hep-ph/0212229
A. V.	hep-ph/0212231
K. Melnikov and A. V.	hep-ph/0312226

Outline

Not B physics – A physics, $a = \frac{g-2}{2}$

- Decomposition of the muon anomalous magnetic moment
- Hadrons in polarization operator
- Hadrons in light-by-light
- Hadronic effects in electroweak corrections
- Summary

Muon anomalous magnetic moment

Definition: $E = -\vec{\mu}\vec{B}\,, \quad \vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s}\,.$

From the Dirac equation $g_\mu = 2$. Deviations are due to radiative corrections,

$$g_\mu = 2(1 + \tfrac{\alpha}{2\pi} + \ldots)$$ Schwinger '48

The anomalous magnetic moment of muon is measured with a very high precision

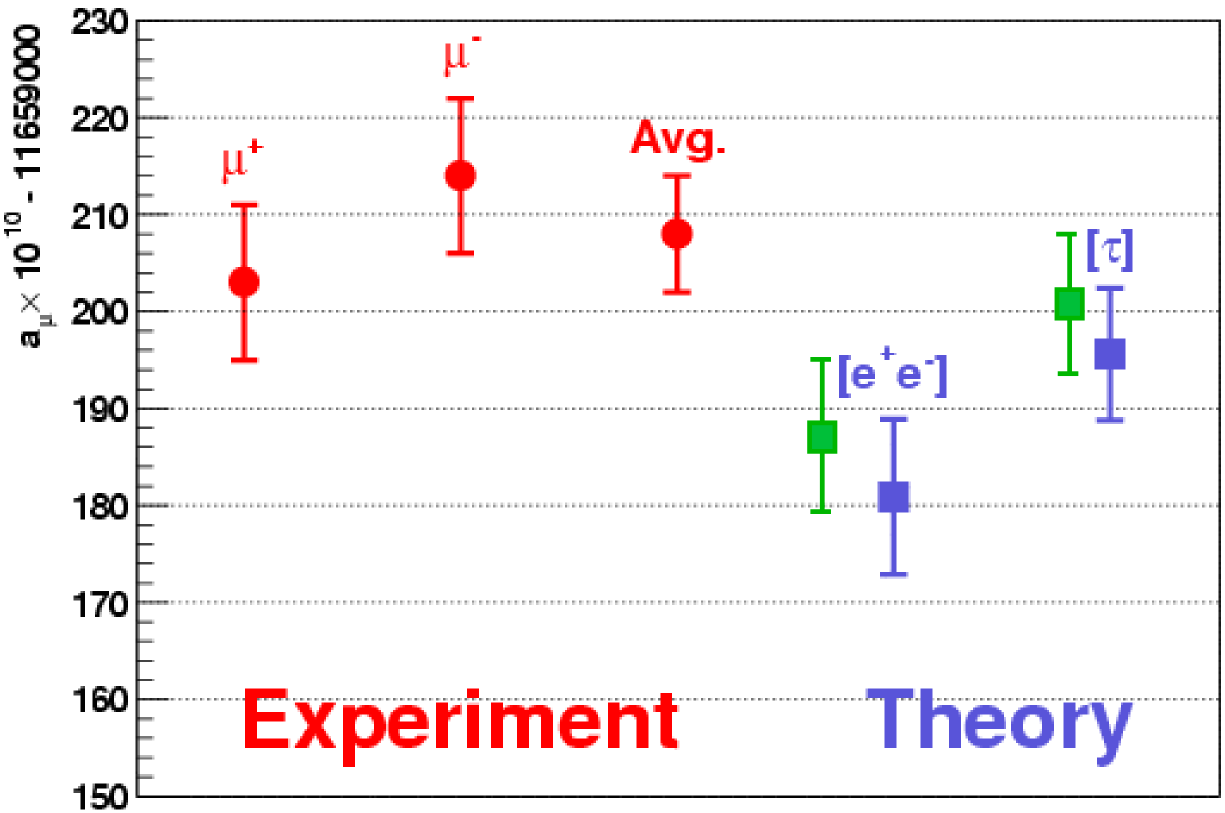
$$a^{\rm exp}_{\mu^+} = \frac{g_{\mu^+} - 2}{2} = 116\,592\,030(80) \times 10^{-11}$$ '02

E821 at BNL

$$a^{\rm exp}_{\mu^-} = \frac{g_{\mu^-} - 2}{2} = 116\,592\,140(80) \times 10^{-11}$$ '04

Assuming CPT invariance the average is also given

$$a^{\rm exp}_{\mu} = 116\,592\,080(60) \times 10^{-11}$$



Experimental values and theoretical predictions. The green bars are due to the shift in the hadronic light-by-light contribution.

The Standard Model prediction for $\boldsymbol{a}_\mu$ can be represented as a sum

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW}$$

The QED part involving only leptons and photons is the main one,

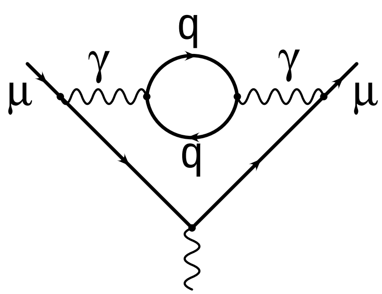
$$a_\mu^{\rm QED} = 116\;584\;706(3) \times 10^{-11}\,.$$

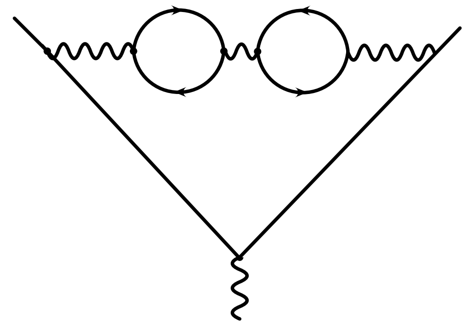
This accounts for one-, two- and three-loop contributions, i.e., up to the α^3 terms. Not calculated yet the four-loop terms are of order $\alpha^4 \sim 10^{-11}$.

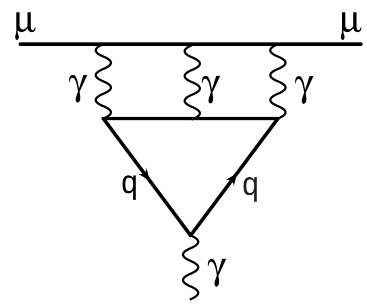
Next is the hadronic contribution.

Hadronic contributions

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm LBL}$$







Lowest order hadronic contribution represented by a quark loop

An example of higher order hadronic contribution $a_\mu^{\rm h,HO} = -100(6)\times 10^{-11}$

Light-by-light scattering contribution $a_\mu^{\rm LBL} = 86(35)\times 10^{-11}$

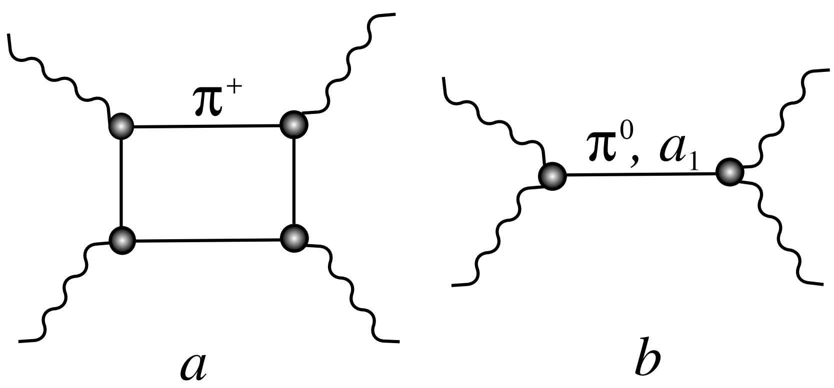
$$a_\mu^{\rm had,LO} = \begin{cases} 6963(62)(36)\times 10^{-11} & e^+e^- \text{ based} \\ 7110(50)(8)(28)\times 10^{-11} & \tau \text{ based} \end{cases}$$

Light-by-light

The $\gamma^*\gamma^* \to \gamma^*\gamma$ amplitude is not accessible experimentally, a challenge for theorists. Parametrically the LbL contribution to a_μ can be presented in the form

$$a_\mu^{\rm Lbl} \sim \left(\frac{\alpha}{\pi}\right)^3 \left[c_1\,\frac{m_\mu^2}{m_\pi^2} + c_2 N_c\,\frac{m_\mu^2}{m_\rho^2}\right]$$

The first, chirally enhanced term, is due to the loops of of charged pion, the second, N_c-enhanced, term is due to exchanges of π^0 and heavier resonances. A similar similar expansion for the leading contribution, hadronic polarization.

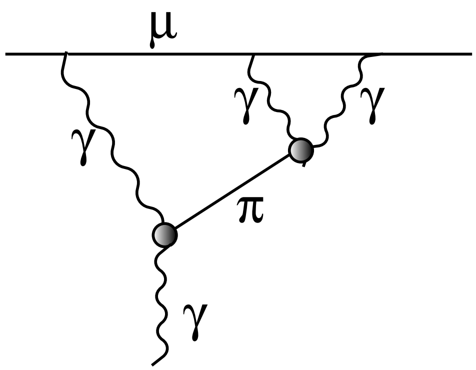


The chirally enhanced contribution does not result in large number, it is actually rather small, Kinoshita et al, Melnikov

$$a_\mu^{\rm LbL}(\text{pion box}) \approx -4\times 10^{-11}$$

(similarly to the hadronic polarization case).
The π^0 pole part of LbL contains besides N_c the chiral enhancement in the logarithmic form,

$$a_\mu^{\rm LbL}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c\,\frac{m_\mu^2\,N_c}{48\pi^2 F_\pi^2}\,\ln^2\frac{m_\rho}{m_\pi} + \ldots$$



Knecht, Nyffeler,
Kinoshita, Hayakawa
Bijens, Prades, Pallante
Blokland, Czarnecki, Melnikov

Numerically

- $$a_\mu^{\rm LbL}(\pi^0) = 56 \times 10^{-11}$$ Knecht, Nyffeler

Using constraints from the Operator Product Expansion (OPE) we show that it is underestimated value. The diifference comes from absence of form factor suppression in $\gamma\gamma^*\pi^0$ vertex.

OPE constraints and hadronic model

$\epsilon_i^{\mu}(q_i)\,, \quad i=1,2,3,4, \quad \sum q_i = 0$

ϵ_4 represents the external magnetic field $f^{\gamma\delta} = q_4^{\gamma}\epsilon_4^{\delta} - q_4^{\delta}\epsilon_4^{\gamma}, \quad q_4 \to 0.$

The LbL amplitude

$$\begin{aligned} \mathcal{M} &= \alpha^2 N_c\, \mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A} = \alpha^2 N_c\, \mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3} f^{\gamma\delta} \\ &= -e^3 \int \mathrm{d}^4x\, \mathrm{d}^4y\, \mathrm{e}^{-iq_1x - iq_2y}\, \epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3} \langle 0| T\,\{j_{\mu_1}(x)\, j_{\mu_2}(y)\, j_{\mu_3}(0)\}\,|\gamma\rangle \end{aligned}$$

The electromagnetic current $j_\mu = \bar{q}\,\hat{Q}\gamma_\mu q, \quad q = \{u, d, s\}$

Three Lorentz invariants: q_1^2, q_2^2, q_3^2

Consider the Euclidian range $q_1^2 \approx q_2^2 \gg q_3^2$

We can use OPE for the currents that carry large momenta q_1, q_2

$$i\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,T\,\{j_{\mu_1}(x), j_{\mu_2}(y)\} =$$

$$\int \mathrm{d}^4z\,\mathrm{e}^{-i(q_1+q_2)z}\,\frac{2i}{\hat{q}^2}\,\epsilon_{\mu_1\mu_2\delta\rho}\,\hat{q}^\delta j_5^\rho(z) + \cdots\,.$$

$\hat{q} = (q_1 - q_2)/2$, the axial current $j_5^\rho = \bar{q}\,\hat{Q}^2\gamma^\rho\gamma_5\,q$ is the linear combination of

$j_{5\rho}^{(3)} = \bar{q}\,\lambda_3\gamma^\rho\gamma_5\,q$ isovector

$j_{5\rho}^{(3)} = \bar{q}\,\lambda_8\gamma^\rho\gamma_5\,q$ hypercharge

$j_{5\rho}^{(3)} = \bar{q}\,\gamma^\rho\gamma_5\,q$ singlet

$$j_{5\rho} = \sum_{a=3,8,0}\frac{\mathrm{Tr}\,[\lambda_a\hat{Q}^2]}{\mathrm{Tr}\,[\lambda_a^2]}\,j_{5\rho}^{(a)}$$

The triangle amplitude

$$T^{(a)}_{\mu_3\rho} = i\,\langle 0|\int \mathrm{d}^4 z\, \mathrm{e}^{iq_3 z} T\{j^{(a)}_{5\rho}(z)\, j_{\mu_3}(0)\}|\gamma\rangle$$

kinematically is expressed via two scalar amplitudes

$$T^{(a)}_{\mu_3\rho} = -\frac{ie\,N_c \mathrm{Tr}\,[\lambda_a \hat{Q}^2]}{4\pi^2}\Big\{w^{(a)}_L(q_3^2)\, q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} + \\ + w^{(a)}_T(q_3^2)\Big(-q_3^2 \tilde{f}_{\mu_3\rho} + q_{3\mu_3} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3}\Big)\Big\}$$

Longitudinal w_L pseudoscalar mesons exchange
Transversal w_T pseudovector mesons exchange

In perturbation theory for massless quarks

$$w_L^{(a)}(q^2) = 2w_T^{(a)}(q^2) = -\frac{2}{q^2}$$

Nonvanishing w_L is the signature of the axial Adler-Bell-Jackiw anomaly. Moreover, for nonsinglet $w_L^{(3,8)}$ it is the *exact* QCD result, no perturbative as well as nonperturbative corrections. So the pole behavior is preserved all way down to small q^2 where the pole is associated with Goldstone mesons $\pi^0,\ \eta$.

Comparing the pole residue we get the famous ABJ result

$$g_{\pi\gamma\gamma} = \frac{N_c \mathrm{Tr}\,[\lambda_3 \hat{Q}^2]}{16\pi^2\, F_\pi}$$

There exists the nonrenormalization theorem for w_T as well but only in respect to pertubative corrections. Higher term in the OPE does not vanish in this case, they are responsible for shif of the pole $1/q^2 \rightarrow 1/(q^2 - m^2)$

Combining we get

$$\begin{aligned}\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta} f^{\gamma\delta} &= \frac{8}{\hat{q}^2}\epsilon_{\mu_1\mu_2\delta\rho}\hat{q}^\delta \sum_{a=3,8,0} W^{(a)} \Big\{ w_L^{(a)}(q_3^2)\, q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3} \\ &+ w_T^{(a)}(q_3^2)\Big(-q_3^2 \tilde{f}^\rho_{\mu_3} + q_{3\mu_3} q_3^\sigma \tilde{f}^\rho_\sigma - q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3}\Big)\Big\} + \cdots\end{aligned}$$

where the weights $W^{(3)} = 1/4,\ W^{(8)} = 1/12,\ W^{(0)} = 2/3.$

The model

$$\mathcal{A} = \mathcal{A}_{\rm PS} + \mathcal{A}_{\rm PV} + \text{permutations},$$

$$\mathcal{A}_{\rm PS} = \sum_{a=3,8,0} W^{(a)} \phi_L^{(a)}(q_1^2, q_2^2)\, w_L^{(a)}(q_3^2)\,\{f_2\tilde{f}_1\}\{\tilde{f} f_3\},$$

$$\mathcal{A}_{\rm PV} = \sum_{a=3,8,0} W^{(a)} \phi_T^{(a)}(q_1^2, q_2^2)\, w_T^{(a)}(q_3^2)\,\Big(\{q_2 f_2\tilde{f}_1\tilde{f} f_3 q_3\}$$

$$+\{q_1 f_1\tilde{f}_2\tilde{f} f_3 q_3\} + \frac{q_1^2+q_2^2}{4}\{f_2\tilde{f}_1\}\{\tilde{f} f_3\}\Big).$$

- For π^0

$$w_L^{(3)}(q^2) = \frac{2}{q^2+m_\pi^2}\,,$$

$$\phi_L^3(q_1^2, q_2^2) = \frac{N_c}{4\pi^2 F_\pi^2}\, F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)$$

$$= \frac{q_1^2 q_2^2 (q_1^2+q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2+q_2^2) + (N_c M_1^4 M_2^4/4\pi^2 F_\pi^2)}{(q_1^2+M_1^2)(q_1^2+M_2^2)(q_2^2+M_1^2)(q_2^2+M_2^2)}$$

Following the form factor analysis Knech, Nyffeler

$M_1 = 769$ MeV, $M_2 = 1465$ MeV, $h_5 = 6.93\ \mathrm{GeV}^4$

They did not fix h_2 and put $h_2 = 0$ for the central value. Actually, it is fixed by the old QCD sum rule analysis Novikov et al '84 $h_2 \approx -10\ \mathrm{GeV}^2$.

It results in

$$a_\mu^{\pi^0} = 76.5 \times 10^{-11}\,, \qquad a_\mu^{\mathrm{PS}} = 114(10) \times 10^{-11}$$

A similar analysis for pseudovector exchange gives

$$a_\mu^{\mathrm{PV}} = 20.5(5) \times 10^{-11}$$

and finally

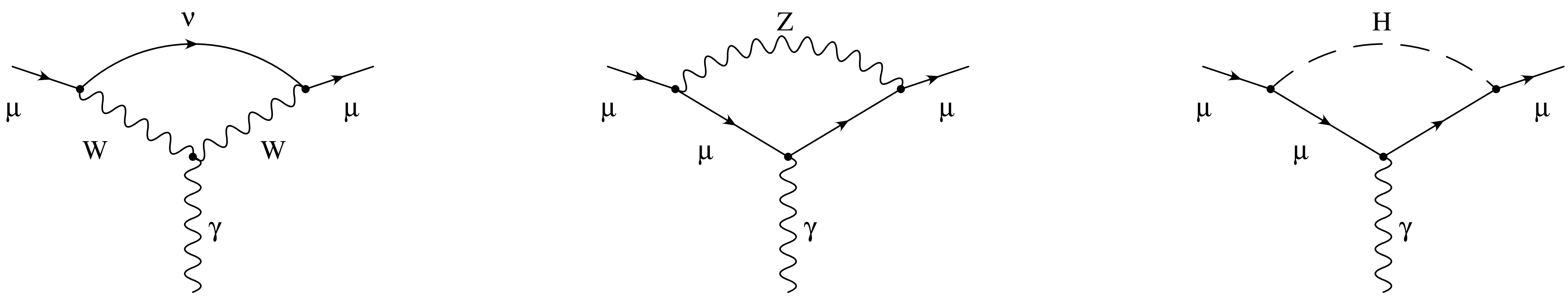
$$a_\mu^{\mathrm{LbL}} = 134(25) \times 10^{-11}$$

Compare with 86(35)$\times 10^{-11}$ from previous estimates.

Electroweak contributions to a_μ

In the Standard Model the one-loop electroweak contributions were calculated about 30 years ago

$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5\,G_\mu\,m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\,(1-4\sin^2\theta_W)^2+\mathcal{O}\left(\frac{m_\mu^2}{m_{W,H}^2}\right)\right] = 194.8\times 10^{-11}$$



One-loop electroweak contributions to a_μ

Two-loop corrections are more involved

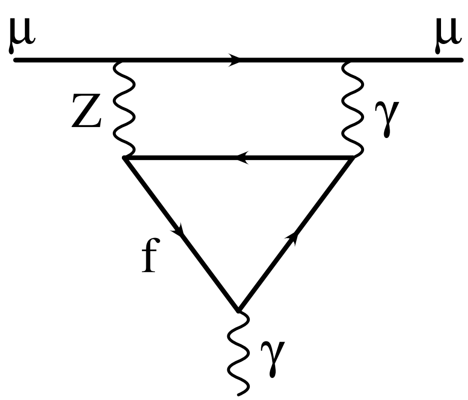
$$a_\mu^{\rm EW}(\text{2-loop})_{LL} = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\cdot\frac{\alpha}{\pi}\left\{-\frac{43}{3}\ln\frac{m_Z}{m_\mu}+\frac{36}{5}\sum_{f\in F}N_f\,Q_f^2\,I_f^3\ln\frac{m_Z}{m_f}\right\}$$

$$\approx -37\times 10^{-11} \qquad F=\tau,\,u\,,d,\,s,\,c,\,b$$

Kukhto, Kuraev, Schiller, Silagadze '92

Peris, Perrottet, Rafael '95

Czarnecki, Krause, Marciano '95

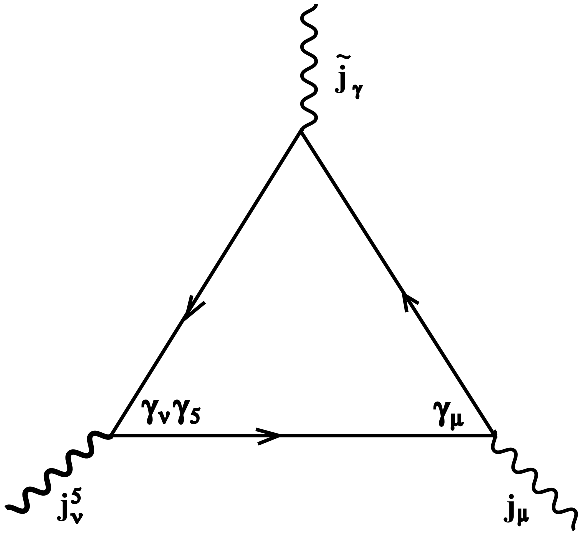


$$m_{u,d}=0.3\ \text{GeV},\ m_s=0.5\ \text{GeV},$$
$$m_c=1.5\ \text{GeV},\ m_b=4.5\ \text{GeV}$$

Fermion triangles ($Z^*\gamma\gamma^*$ vertex)

Total: $a_\mu^{\rm EW}=152(4)\times 10^{-11}$ Czarnecki, Marciano '01

Structure of $Z^*\gamma\gamma^*$ vertex



$$T_{\mu\nu} = i\int \mathrm{d}^4x\, \mathrm{e}^{iqx}\, \langle 0|\, T\{j_\mu(x)\, j^5_\nu(0)\}|\,\gamma(k)\rangle$$

$$j_\mu = \sum_f Q_f \bar{f}\gamma_\mu f\,, \quad j^5_\nu = \sum_f 2I^3_f \bar{f}\gamma_\nu\gamma_5 f$$

Q_f is an electric charge,
I^3_f is a weak isospin projection

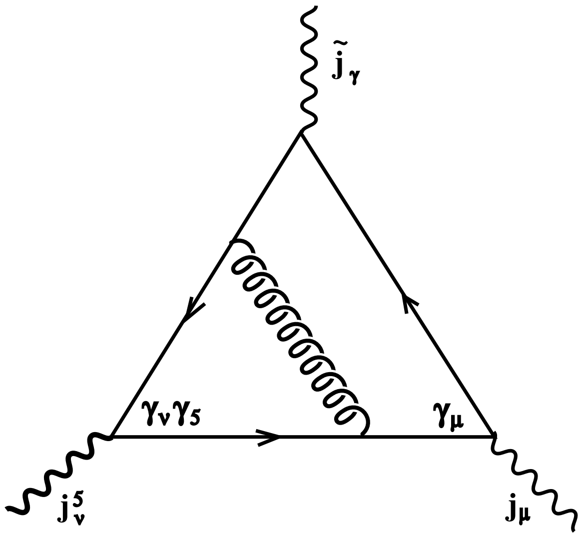
$$\boxed{T_{\mu\nu} = -\frac{ie}{4\pi^2}\left[w_T(q^2)\left(-q^2\tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right) + w_L(q^2)\, q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right]}$$

where $\tilde{f}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}\,,\qquad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu$

$$\boxed{\Delta a^{\mathrm{EW}}_\mu = \frac{\alpha}{\pi}\,\frac{G_\mu\, m^2_\mu}{8\pi^2\sqrt{2}}\int \mathrm{d}Q^2\left(w_L(Q^2) + \frac{m^2_Z}{m^2_Z+Q^2}\, w_T(Q^2)\right)}$$

Hadronic corrections for quark triangles

How good is the one-loop approximation for w_L and w_T? This question pertains to strong interaction effects for quark loops.



First, about perturbative corrections at $Q \gg m_q$. The longitudinal function w_L is protected by Adler-Bardeen nonrenormalization theorem. What about the transversal function w_T? It turn out that the α_s corrections in w_T are also absent at $Q \gg m_q$ due to the new non-renormalization theorem based on

$$w_T[m_q = 0] = \frac{1}{2}\, w_L[m_q = 0]$$

A.V. '02

No α_s corrections in chiral limit ! For heavy quarks perturbative corrections show up at $Q \sim m_q$, they are regulated by small $\alpha_s(m_q)$ in a_μ .

Nonperturbative corrections

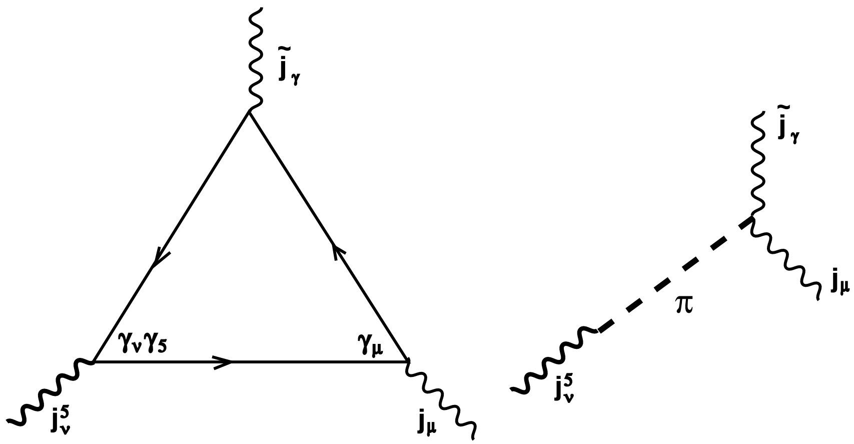
Small for heavy quarks, $\left(\Lambda^2_{\rm QCD}/m_q^2\right)^n$, but 100% important for light quarks, $\left(\Lambda^2_{\rm QCD}/Q^2\right)^n$. Two approaches were used:

- Effective quark masses for light quarks   Czarnecki, Krause, Marciano '95
- Hadron dynamics consistent with OPE

  Knecht, Peris, Perrottet, Rafael '95, '02

While the second approach was an improvement some conceptual mistakes were made in application of the OPE resulting in a change of large Q asymptotic.

First generation



Let us consider hadronic effects for the first generation, e, u and d. In the chiral limit $m_{u,d} = 0$ the quark longitudinal function

$$w_L[u] + w_L[d] = \frac{12 \sum_{u,d} I^3_q \, Q^2_q}{Q^2}$$

acquire no nonperturbative corrections.

In other words the $1/Q^2$ dependence is valid not only at large Q but everywhere. The pole corresponds to the exact duality between the pion and quarks: 't Hooft matching condition. It gives a theoretical prediction for $\pi^0 \to \gamma\gamma$. Deviations from the chiral limit are readily accounted for by a shift $1/Q^2 \to 1/(Q^2 + m^2_\pi)$, For the transversal function $w_T[u,d]$ it is only spin one hadrons, like $\rho(770)$, $\omega(770)$ and $a_1(1230)$, which contribute. Thus, nonperturbative effects shift $1/Q^2$ at $Q\sim m_{\rho,a_1}$. The model of $w_T[u,d]$ consistent with the OPE is

$$w_T[u,d]=\frac{1}{m_{a_1}^2-m_\rho^2}\left[\frac{m_{a_1}^2-m_\pi^2}{Q^2+m_\rho^2}-\frac{m_\rho^2-m_\pi^2}{Q^2+m_{a_1}^2}\right].$$

Together with $w_T[e]=-1/Q^2$ it gives for a_μ

$$\Delta a_\mu^T[e,u,d]=-\frac{\alpha}{\pi}\frac{G_\mu m_\mu^2}{8\pi^2\sqrt{2}}\left\{\ln\frac{m_\rho^2}{m_\mu^2}-\frac{m_\rho^2}{m_{a_1}^2-m_\rho^2}\ln\frac{m_{a_1}^2}{m_\rho^2}+\frac{3}{2}\right\}=-1.32\cdot 10^{-11}$$

The longitudinal part contributes $-0.7\cdot 10^{-11}$. Thus,

$$\Delta a_\mu[e,u,d]=-2.32\cdot 10^{-11}$$

what is about a half of the constituent quark mass value. The accuracy is rather high, about 10-20%, i.e. $(0.2-0.4)\cdot 10^{-11}$ is a theoretical uncertainty.

Summary

- The hadronic light-by-light scattering contribution to a_μ is shown to be larger than previous estimates. We cannot claim any significant reduction in the theoretical uncertainty although believe that the shift $\approx 50 \times 10^{-11}$ in the central value is real,

$$a_\mu^{\rm LbL} = 134(25) \times 10^{-11}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm th} = \begin{cases} (220 \pm 100) \times 10^{-11} & (2.2\,\sigma), \quad e^+e^- \text{ based} \\ (76 \pm 100) \times 10^{-11} & (0.8\,\sigma), \quad \tau \text{ based} \end{cases}$$

- Hadronic effects in electroweak corrections are determined by matching the OPE and hadronic phenomenology. Remaining uncertainty is shown to be very small. In total a small shift in $a_\mu^{\rm EW}$ from the previous value of $152(4) \times 10^{-11}$ to a slightly larger (but consistent) value

$$a_\mu^{\rm EW} = 154(1)(2) \times 10^{-11}$$

where the first error corresponds to hadronic loop uncertainties and the second to an allowed Higgs mass range of 114 GeV $< m_H <$ 250 GeV, the current top mass uncertainty and unknown three-loop effects.