Measurement of the $\pi\pi$ scattering length from a new structure in the K --> 3π Dalitz plot

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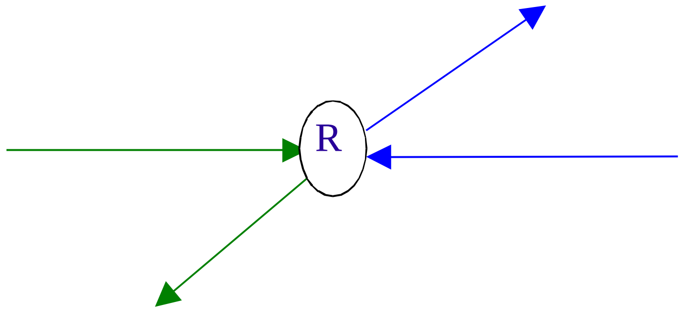
NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

What is the ππ scattering length ?



If $kR << 1$ (small momentum compared to interaction range)
S-wave is the dominant component to the total cross section
Bose statistics allows Isospin: $I = 0, 2$

Scattering matrix

$$S|\,\pi\pi> = \exp(2i\delta)|\,\pi\pi>$$

Phases

$\delta_0 = a_0\, k$

$\delta_2 = a_2\, k$

--> a_0, a_2 are called <u>Scattering Lengths</u>

Thanks to Prof. Cesare Rossetti from Università di Torino who forced me to study a 150 pages book on Scattering Theory when I was a student :-)

Theoretical predictions

Weinberg (1966)
Effective field theory for
strong interaction at low E

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Most recently
Colangelo et al. (2001)
pt -theory two loops
Ref: hep- ph/0103088

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

- 2% level of accuracy is quite unusual for hadronic physics
  experiments have not yet reached the same level of accuracy

Experimental Status

1977: measurement by Genève/Saclay experiment @ 20% accuracy

2003: BNL E865 extracts a_0 at 5% accuracy by measuring the form Factors of the decay K --> π πeν with 400,000 events

$$a_0 \; m = 0.216 \pm 0.013(stat.) \pm 0.002(syst.) \pm 0.002(theor.)$$

Ref. Pislak et al. (2003) hep-ex/0301040

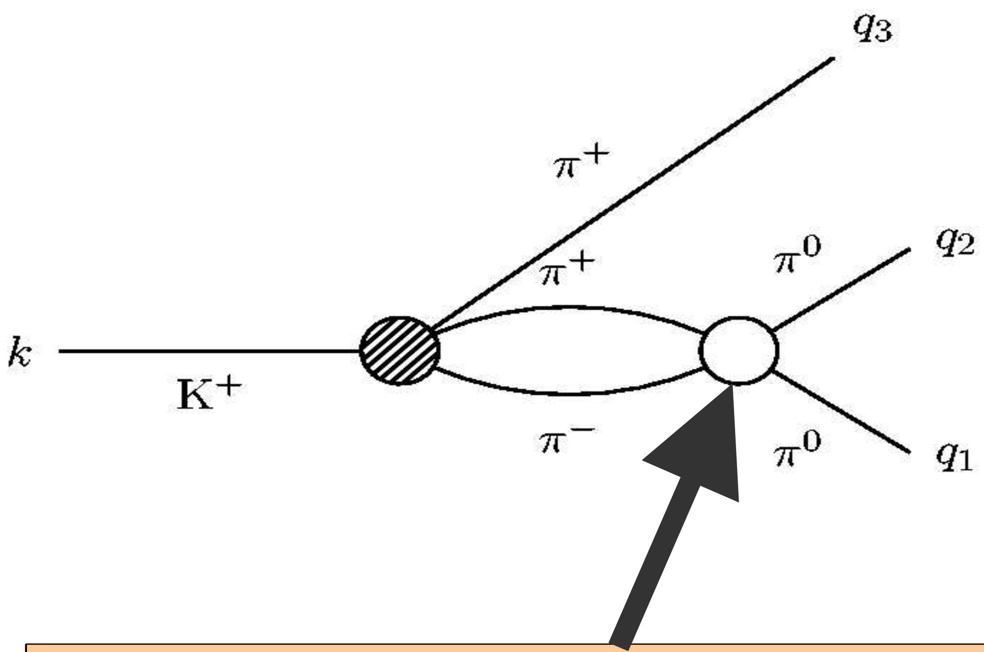
Running: Cern experiment DIRAC with a sophisticated technique aims to measure the pionium lifetime @ 10% accuracy

$$\tau \sim 40 \cdot (a_0 - a_2)^2 \cdot 10^{-15} \text{ sec}$$

Pionium is the atom-like electromagnetic bound state $(\pi^+ \pi^-)$

What does it matter to Kaons ?

A diagram contributing to $K^+ \to \pi^+ \pi^0 \pi^0$



- Strong rescattering + charge exchange with effective coupling constant $(a_0 - a_2)$
- Small Pionium formation also expected

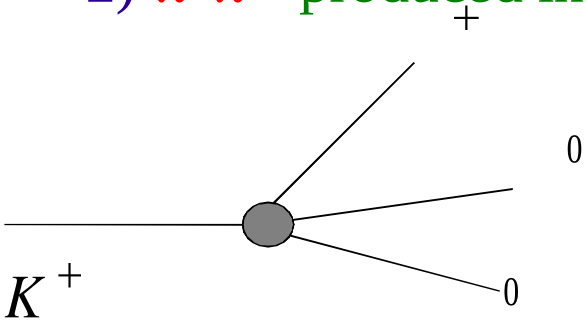
$(a_0 - a_2)$ in $K^+ \to \pi^+ \pi^0 \pi^0$ decays

Two processes contribute to $K^+ \to \pi^+ \pi^0 \pi^0$

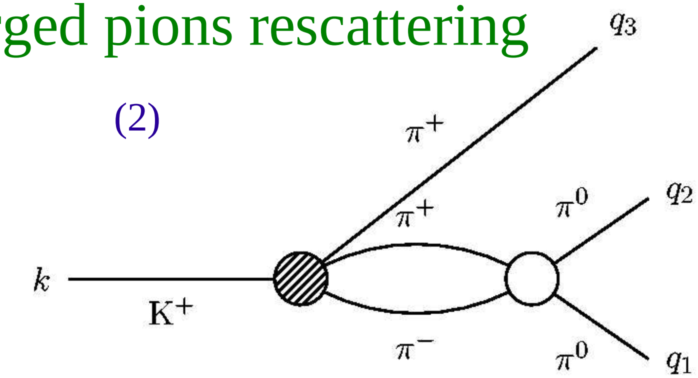
1) Direct emission of $\pi^+ \pi^0 \pi^0$

2) $\pi^0 \pi^0$ produced in charged pions rescattering

(1)



(2)



$\mathcal{M}_0 = 1 + gu/2$

$\mathcal{M}_1 \propto (a_0 - a_2)$

$u = 2m_K (m_K/3 - E^*_{odd})/m_\pi^2$

Dalitz variable

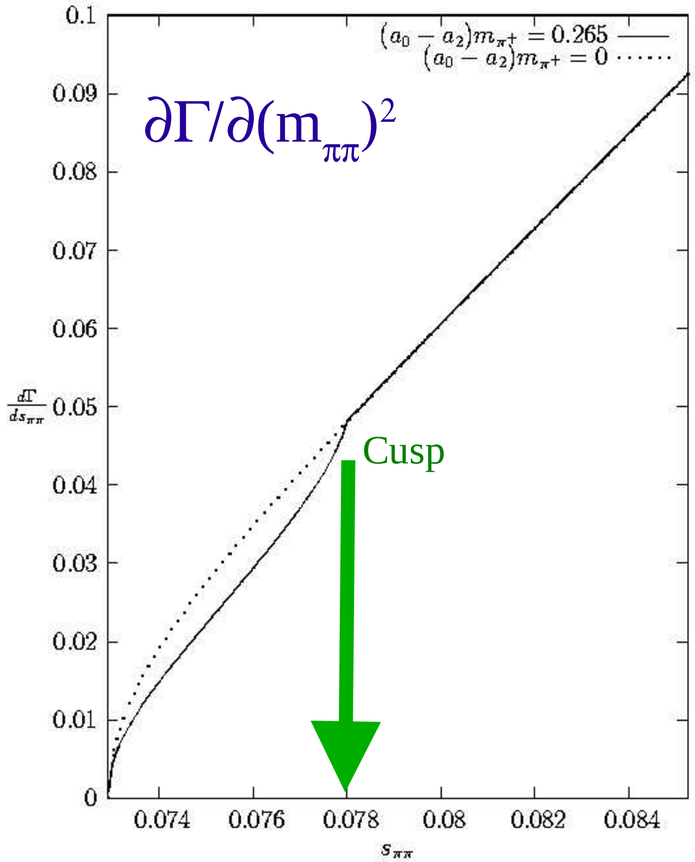
G = 0.638 ± 0.020

(present PDG value)

$$d\Gamma/dm_{\pi\pi} \propto |\mathcal{M}_0 + \mathcal{M}_1|^2$$

Interference is expected

Cusp-like effect in $K^+ \to \pi^+\pi^0\pi^0$



$(m_{\pi\pi})^2$ GeV^2

Nicola Cabibbo (2004), hep-ph/0405001

One loop calculation predicts a cusp located at $m_{\pi\pi} = 2m_{\pi+}$

Where the looping $\pi^+ \pi^-$ pair flips from off-mass to on mass shell

(Remind: $m_{\pi 0}$ = 0.135 GeV , $m_{\pi+}$ = 0.139 GeV)

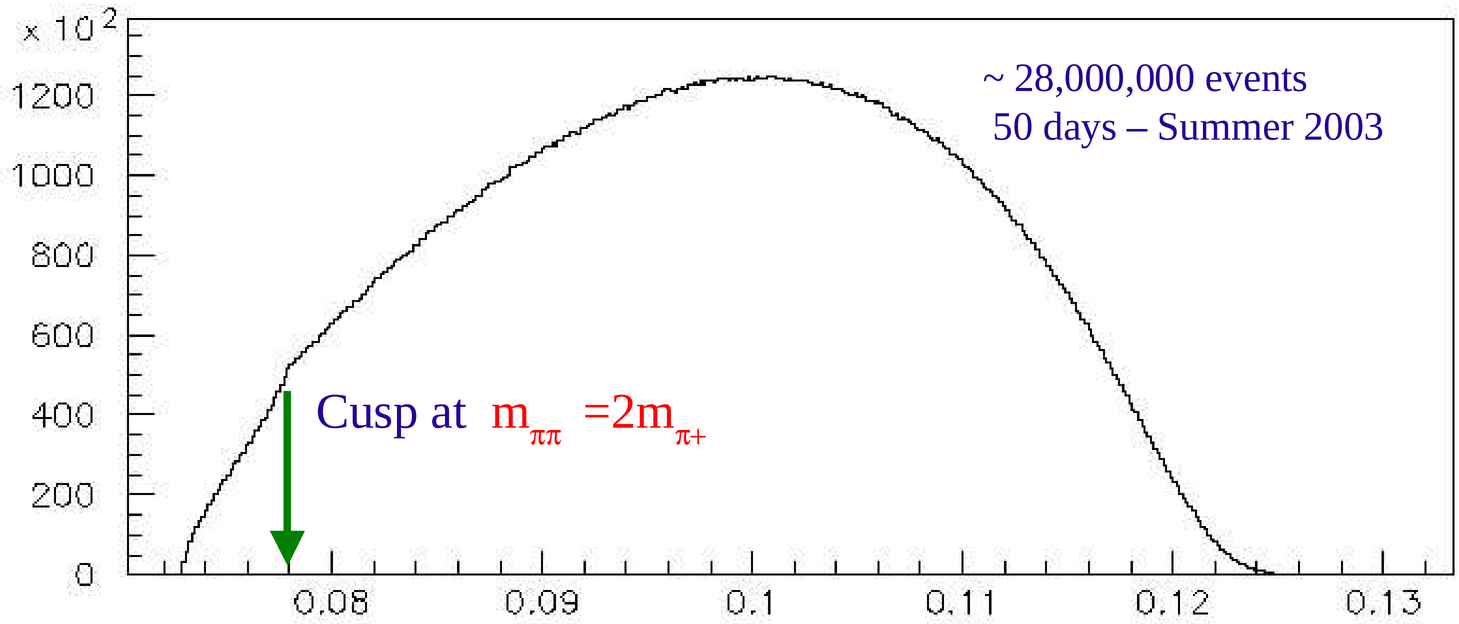
Data collected by NA48 experiment stimulated theoreticians to develop a two loops calculation

N. Cabibbo and G. Isidori hep-ph/0502130

$$\partial\Gamma/\partial(m_{\pi\pi})^2 = \mathcal{F}(a_0 - a_2,\ a_2,\ g,\ h,\ m_{\pi\pi})$$

Indicating a strategy to extract the Scattering lengths from the experimental $m_{\pi\pi}$ spectrum

NA48/2 data $K^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0$



The cusp was first seen by I. Mannelli who was looking for the Pionium and N. Cabibbo realized that it was a clean and beautiful example of a general cusp-like behaviour of cross sections next to threshold for new channels (Wigner 1948)

Why does NA48/2 collect so many $K^{\pm}$?

Direct CP -violation program

1997-2001 NA48/1 $\mathrm{Re}(\varepsilon'/\varepsilon) = (14.7 \pm 2.2)\times 10^{-4}$ Neutral Kaon system

2003-2004 NA48/2 $\Delta g = (0.5 \pm 3.8)\times 10^{-4}$ Charge Kaon system

Future NA48/3 intent to measure $BR(K^{+} \to \pi^{+}\nu\nu)$

$K^{\pm} \to 3\pi$

$\Delta\Gamma/du = 1 + gu$

$\Delta g = (g^{+} - g^{-})/(g^{+} + g^{-})$

looking for difference in the Dalitz plot distribution

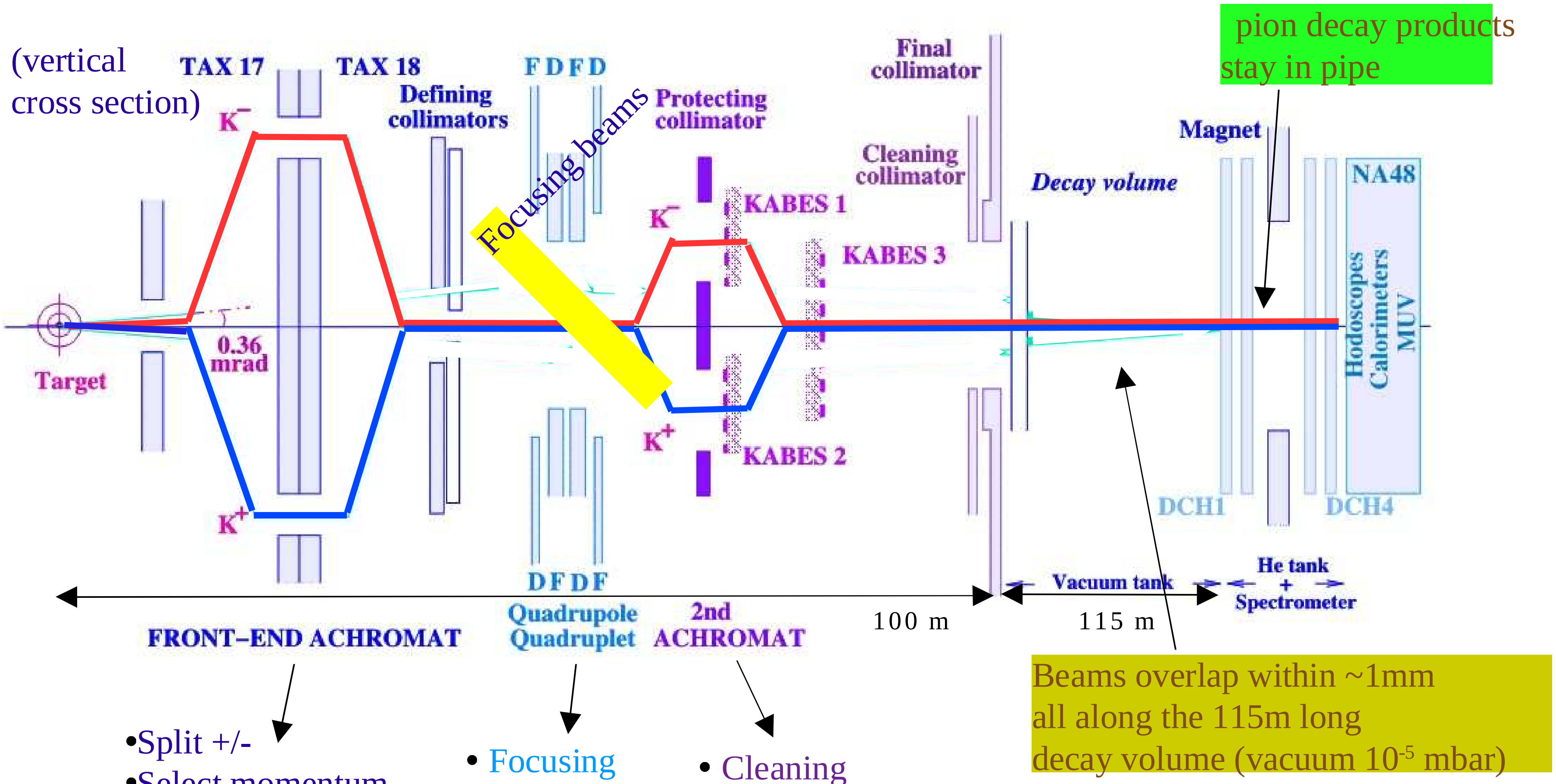
Statistics Collected 2003-2004

$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ $\sim 2\cdot 10^{8}$

$K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ $\sim 4\cdot 10^{9}$

(loose selection cuts)

NA48 simultaneous unseparated beams



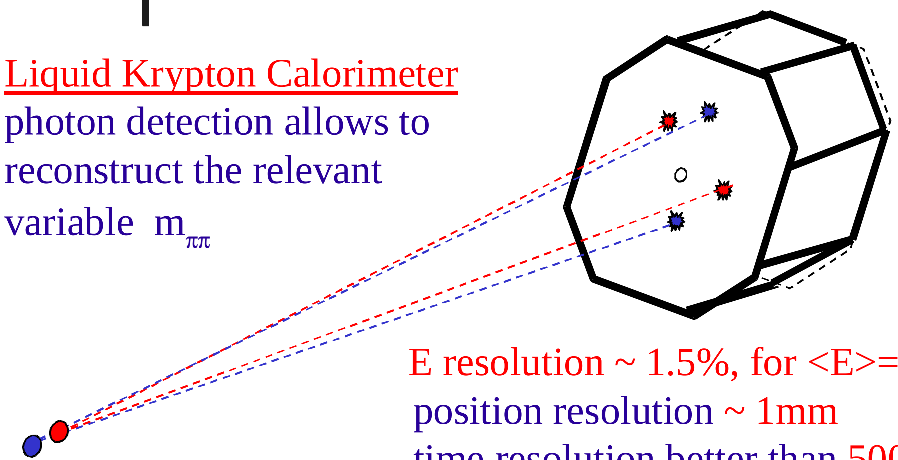
- Split +/-
- Select momentum
- Recombine +/-

- Focusing
- μ sweeping

- Cleaning
- Beam spectrometer (resolution 0.7 %)

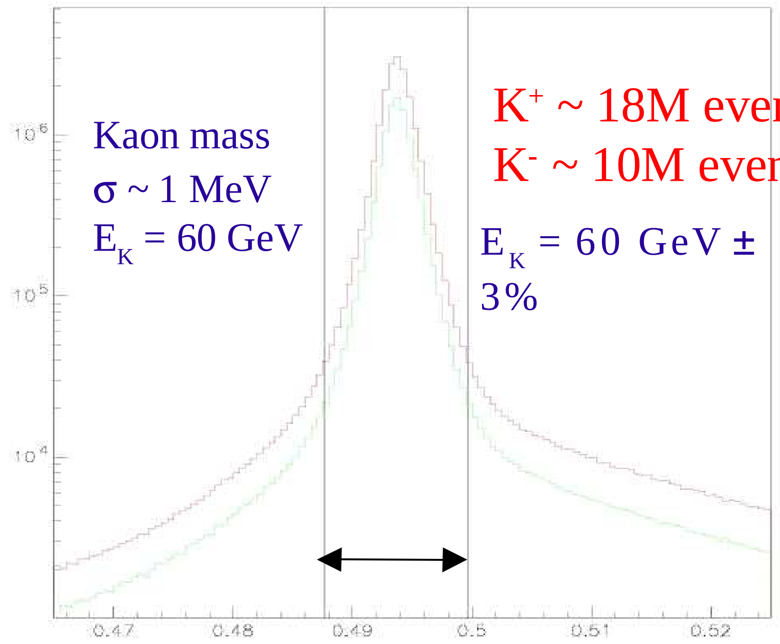
$K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$ Reconstruction

Liquid Krypton Calorimeter
photon detection allows to reconstruct the relevant variable $m_{\pi\pi}$



$\pi^0 \longrightarrow \gamma\gamma$
$\pi^0 \longrightarrow \gamma\gamma$

E resolution ~ 1.5%, for <E>=10 GeV
position resolution ~ 1mm
time resolution better than 500ps
non linearity < 0.1%
(very stable over 8 years)



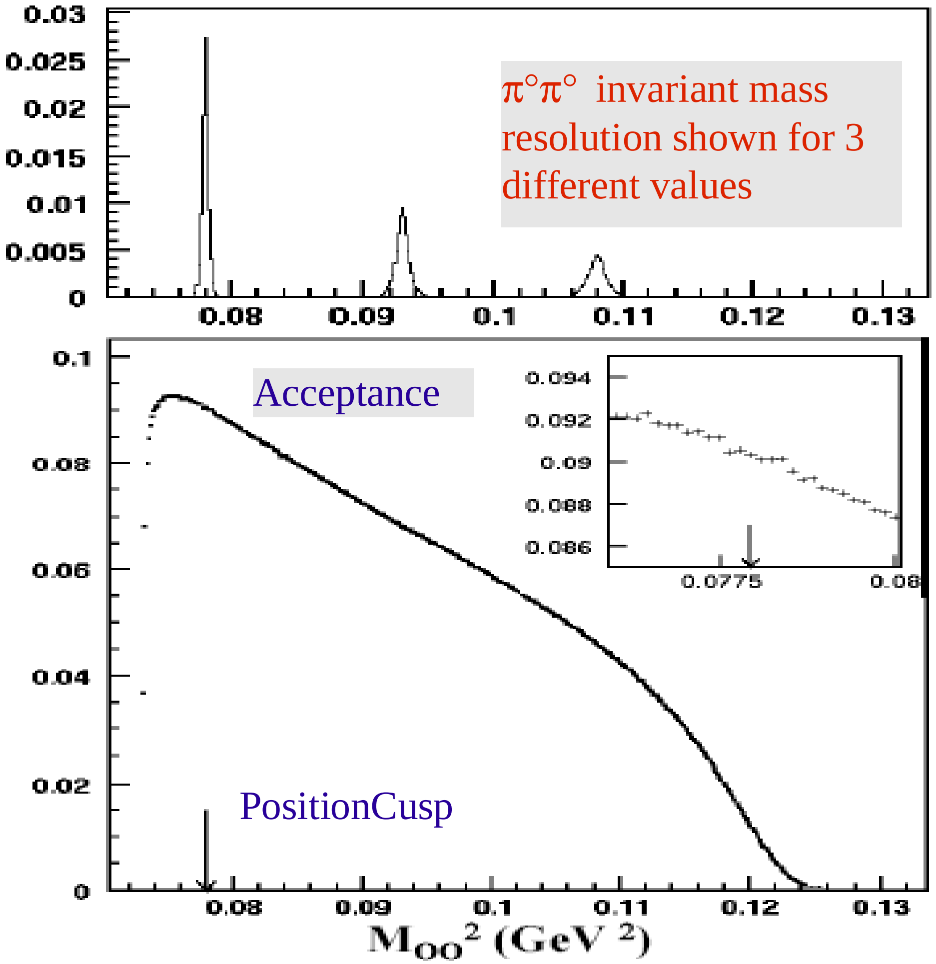
$$\frac{M_{oo}^2}{4m_o^2} = \frac{\sum_{i \; j="1,4} E_i E_j d_{ij}^2}{\left(\sqrt{\sum E_1 \; E_2 \; d_{12}} + \sqrt{\sum E_3 \; E_4 d_{34}}\right)^2} \quad 1$$

$\pi^{\pm} \rightarrow \mu^{\pm}\nu$ makes no effect since charged pion track parameters are not involved in $m_{\pi\pi}$ computation

2 π^0 mass constraints
Z from 2 vertices average

Background free

Acceptance and $m_{\pi\pi}$ resolution



0.0031 GeV^2 resolution on $(\mathbf{m}_{\pi\pi})^2$ @cusp

0.0012 ÷ 0.0120 range elsewhere

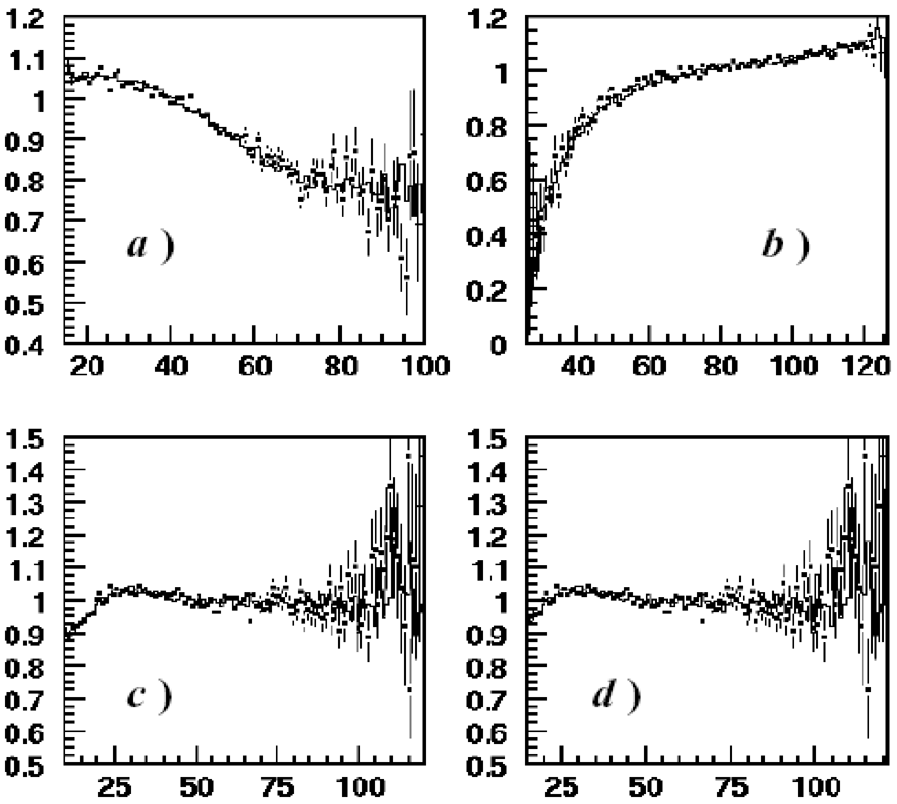
$\pi^0\pi^0$ mass constraints and LKR resolution

$$\frac{(E)}{E} = \frac{0.090}{E} \quad \frac{0.032}{\sqrt{E}} \quad 0.0042$$

The acceptance is linearly varying especially around the cusp

The extraction of (a_0-a_2) is <u>Montecarlo dependent</u> because of the geometrical acceptance

Is our Montecarlo good enough ?



distance (cm)

Test example:

Fig. (a) shows the ratio of the **min γ distance from axis** @ the calorimeter plane between events above and below the observed cusp.

The Montecarlo well reproduces the data

Fig. (b), (c), (d) show other tests based on other distances

***a*) min γ distance from axis ; *b*) max γ distance from axis;**

***c*) min γ–γ distance; *d*) min γ – $\pi^{\pm}$ distance**

Ready to fit the $m_{\pi\pi}$ spectrum

Cabibbo-Isidori 2 loops calculation

$$\partial\Gamma/\partial(m_{\pi\pi})^2 = N \times F\,(a_0 - a_2, a_2, g, h, m_{\pi\pi})$$

4 free parameters + Norm

Scattering legth $a_0 - a_2$

Scattering length a_2

Linear slope g

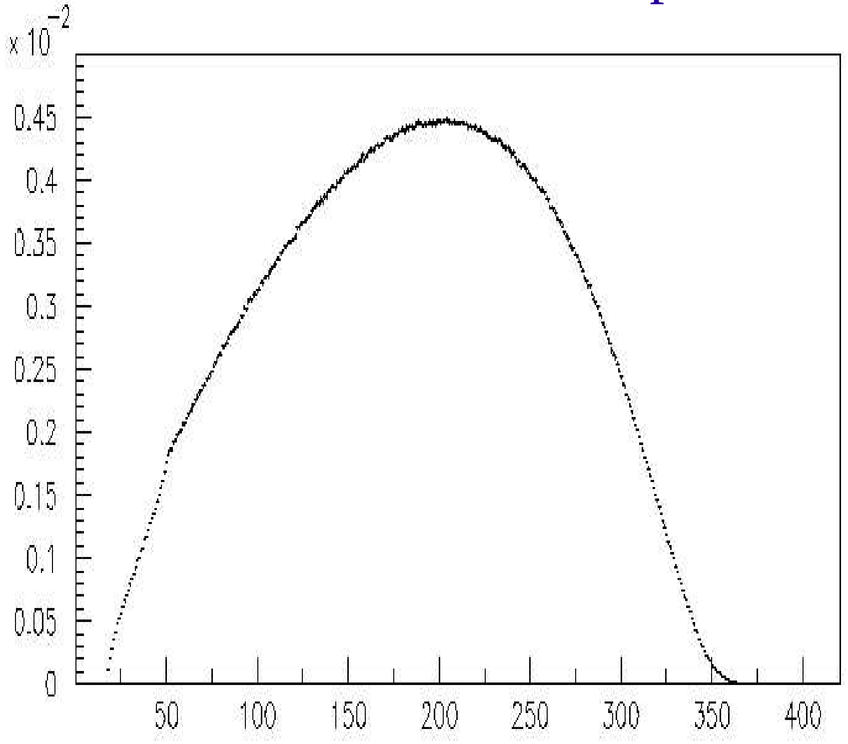
Quadratic slope h

The parameter g and h are equivalent to the PDG parametrisation of the matrix element for $K^\pm \to \pi^\pm \pi^0 \pi^0$

$$|\mathcal{M}|^2 = (1 + gu + hu^2)$$

Our h is defined as $h = h_{PDG} - g^2/4$

Our Data + MC for acceptance

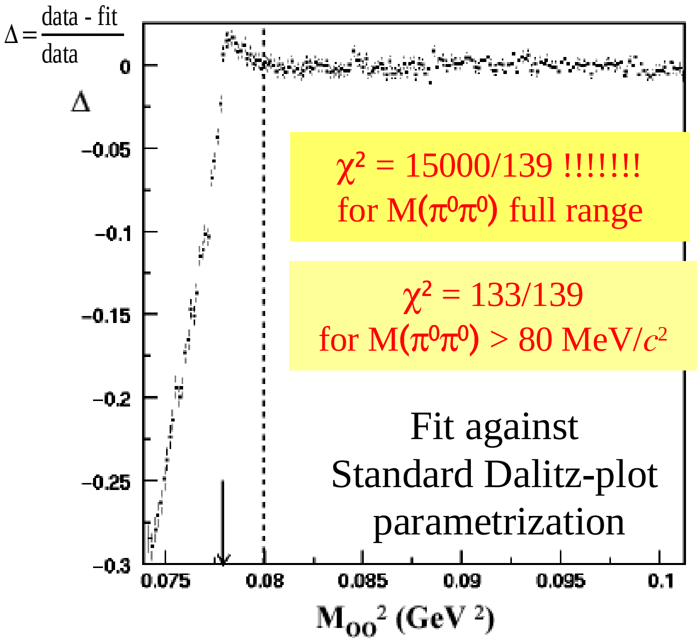


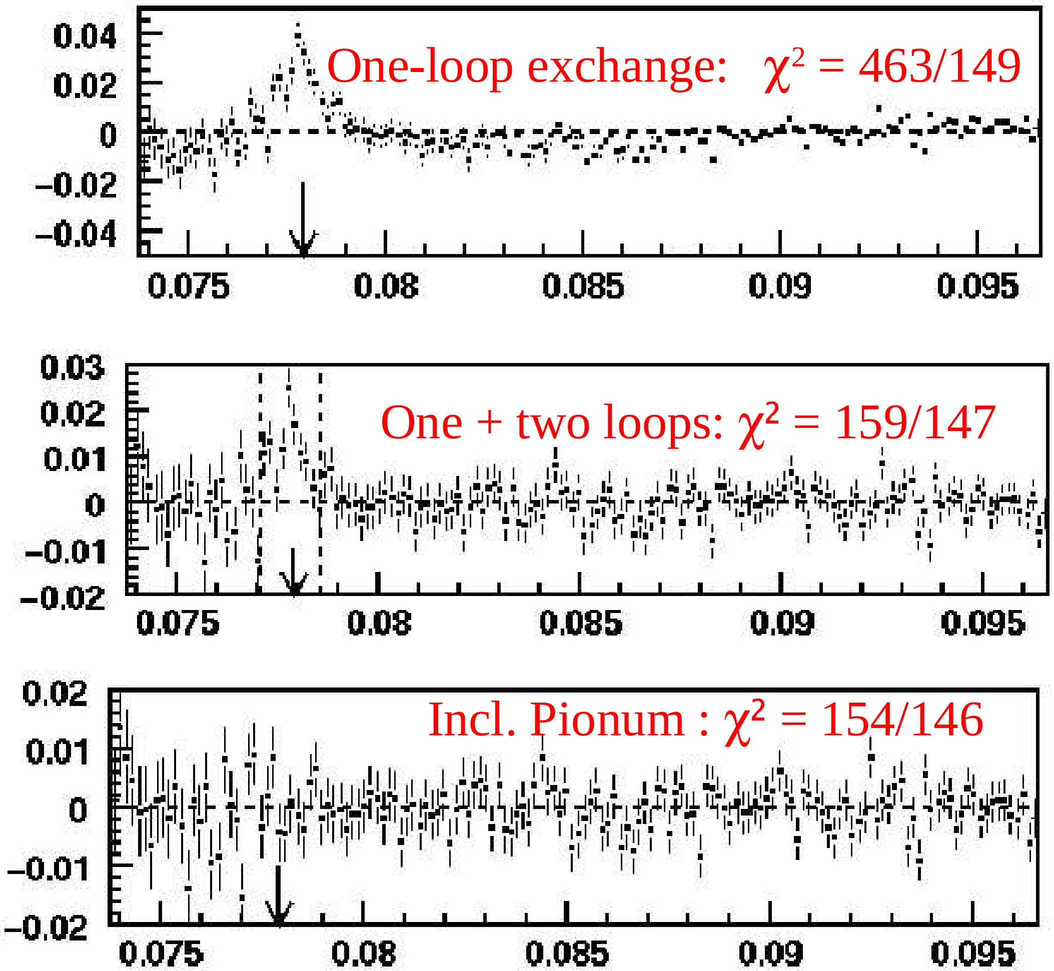
Data in 420 bins

bin width = 0.0015 GeV^2 = half the resolution

Cusp located at 50th bin

Fits against various models

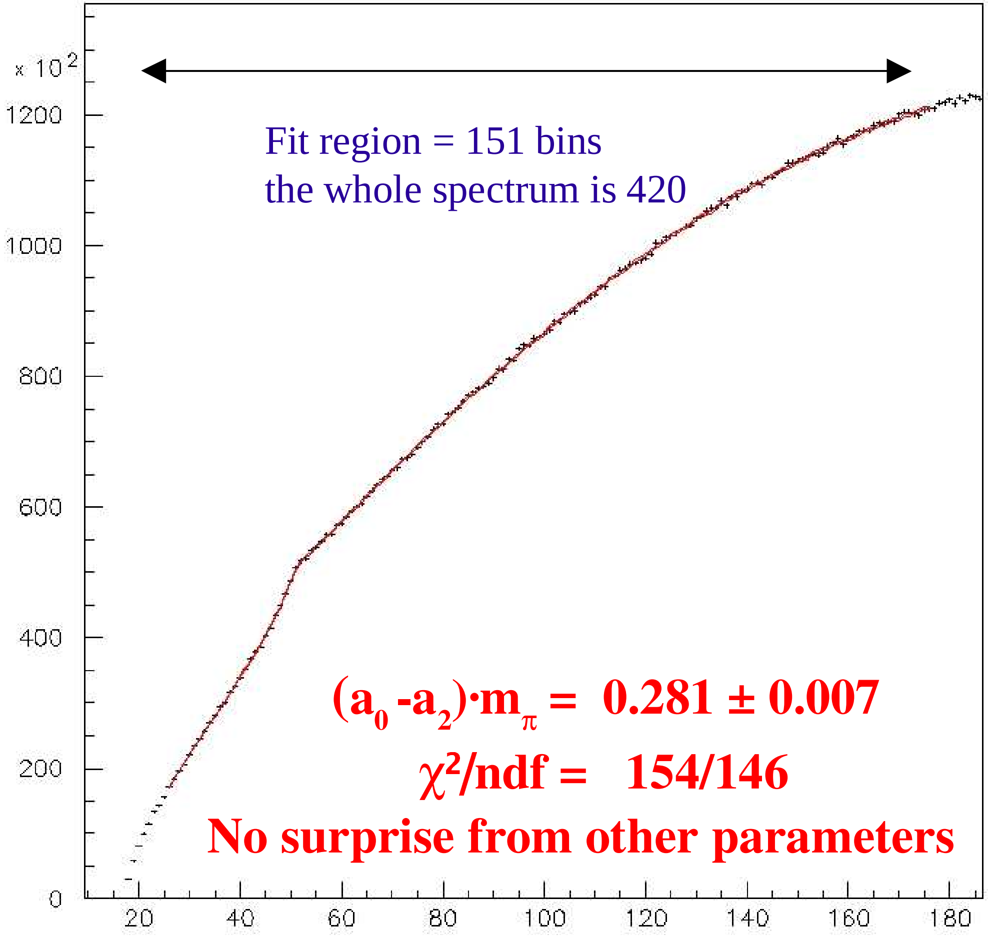




The best fit is obtained with the two loops calculation but a small amount of pionium is to be added in bin 50th to improve the Chi2

N.B. Cabibbo-Isidori model holds not too far from the cusp --> Not all the spectrum is fitted!

Fit results



The pionium amount has been fixed according to the prediction

$$\frac{BR\left(K^+ \to \;{}^{+}\, pionium\right)}{BR\left(K^+ \to 3\;\;\right)} = 0.8 \cdot 10^{-5}$$

Z.K. Silagadze (1994), hep-ph/9411382

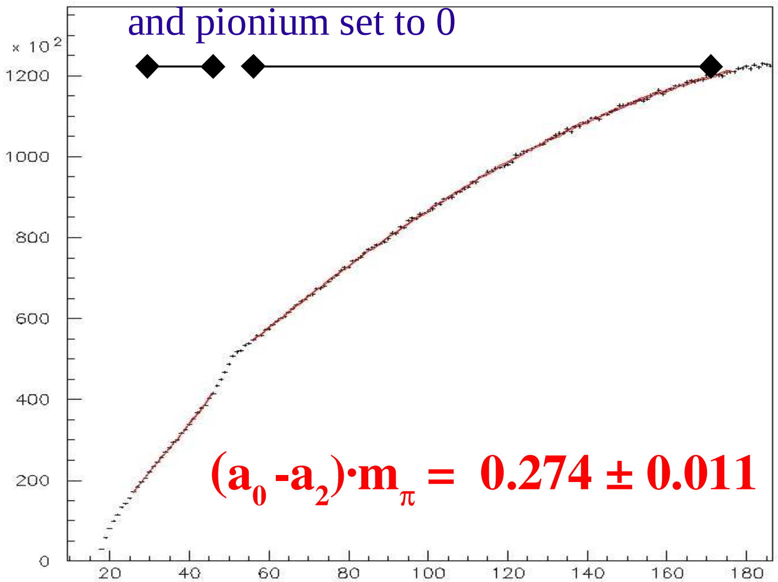
$(a_0 - a_2) \cdot m_\pi$ has low sensitivity to pionium

0.1·σ(BR)/BR

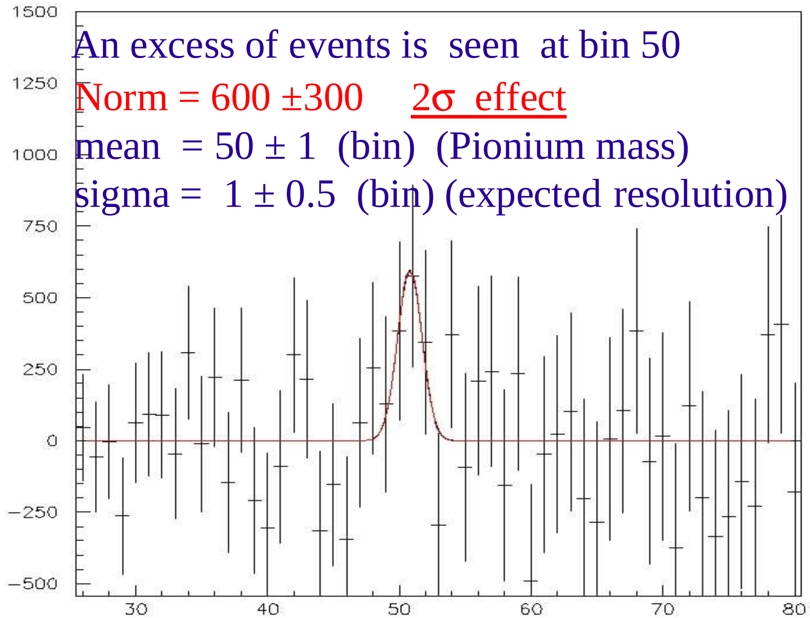
Measured by varying the predicted amount in the range ± 50%

Systematics Check: fitting region

Fit excluding 10 bins around the cusp and pionium set to 0



Data – Fit



Choice of the fitting region

systematic on $(a_0 - a_2)\cdot m_\pi$ ± 0.008

If the excess is interpreted as Pionium formation data agree with Silagadze's prediction within the (large) error

Check on photon isolation and Z vertex

Cut on transverse distance @ the calorimeter to avoid photon energy mis-measurement

min(γ–γ distance) > 10 cm

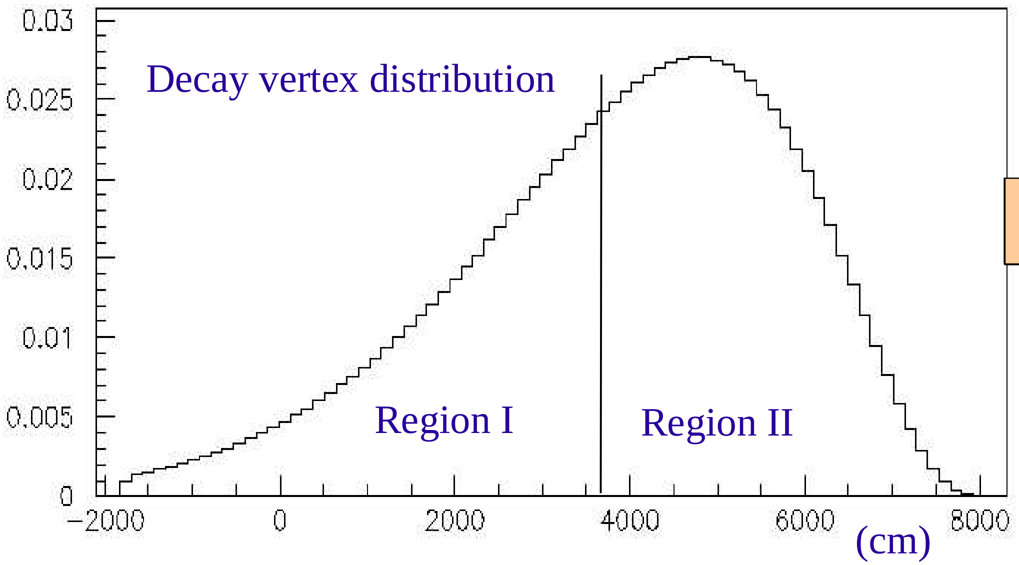
min (γ – $\pi^\pm$ distance) > (10 + d) cm

If $\pi^\pm$ interacts with Kr, energy may be deposited even at large distance from the impact point

Default cut d=5 cm 95% of the extra-shower associated to the $\pi^\pm$ is contained

Trying d=5,10,15 cm

systematic on $(a_0 - a_2) \cdot m_\pi$ ± 0.004



Measurement from two different decay regions

systematic on $(a_0 - a_2) \cdot m_\pi$ ± 0.009

Conclusions on systematics checks

● Excluding pionium from fit region	0.008
● From Cut on track-photon min. distance	0.004
● From dependence on vertex Z coordinate	0.009
● From K+/K- difference	0.006
TOTAL (adding in quadrature)	0.014

Preliminary result on $(a_0 - a_2) \cdot m_\pi$

NA48/2 collaboration performed 3 independent analysis on this subject
two based on a toy (...but not so toy!) Montecarlo
the third exploiting a professional GEANT-based Montecarlo

The 3 central values found are compatible within an error of 0.001

$$(a_0 - a_2) \cdot m_\pi = 0.281 \pm 0.007\ (\text{stat.}) \pm 0.014\ (\text{syst.}) \pm 0.014\ (\text{theor.})$$

The theoretical error quoted refers to the Cabibbo-Isidori model and it has been suggested by the authors

Conclusions

- By including the full 2003 – 04 data sample we expect an increase in statistics from the ~ 30 M events used in the present analysis by a factor of at least 4

- The study of the systematic uncertainties is well under way, (detailed Monte Carlo, additional systematic effects)

- The data quality calls for additional theoretical effort in order to extract precise values of the pion – pion scattering parameters

- A study of the corresponding effects in $K_L \rightarrow 3\pi^0$ from the 2002 NA48 data has been started

  ... the PDG pages related to the K Dalitz plot description will be deeply modified ...

- Data have also been collected by NA48/2 on K+e4 decays, which will provide independent information on pion – pion scattering

NA48 Detector

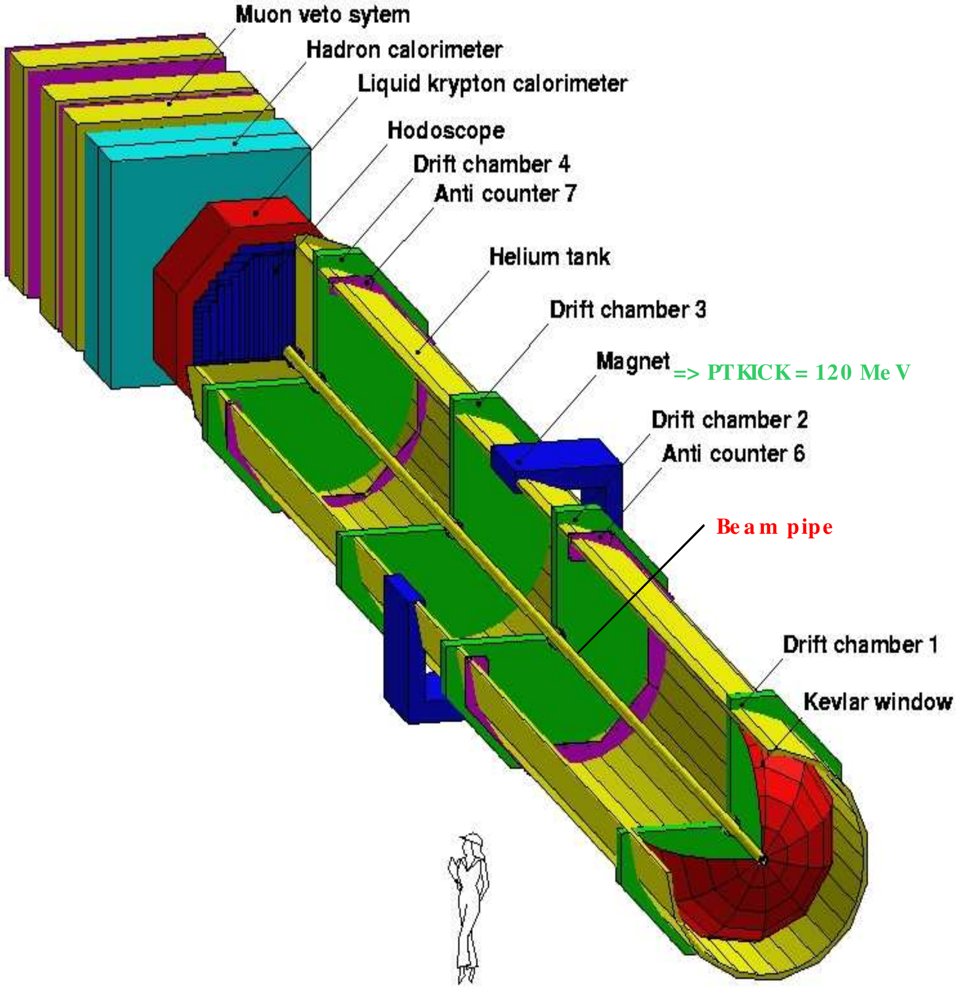
Main detector components:

• Magnetic spectrometer (4 DCHs):
redundancy high efficiency;
$\Delta p/p = 1.0\% + 0.044\%*p$ [GeV/c]

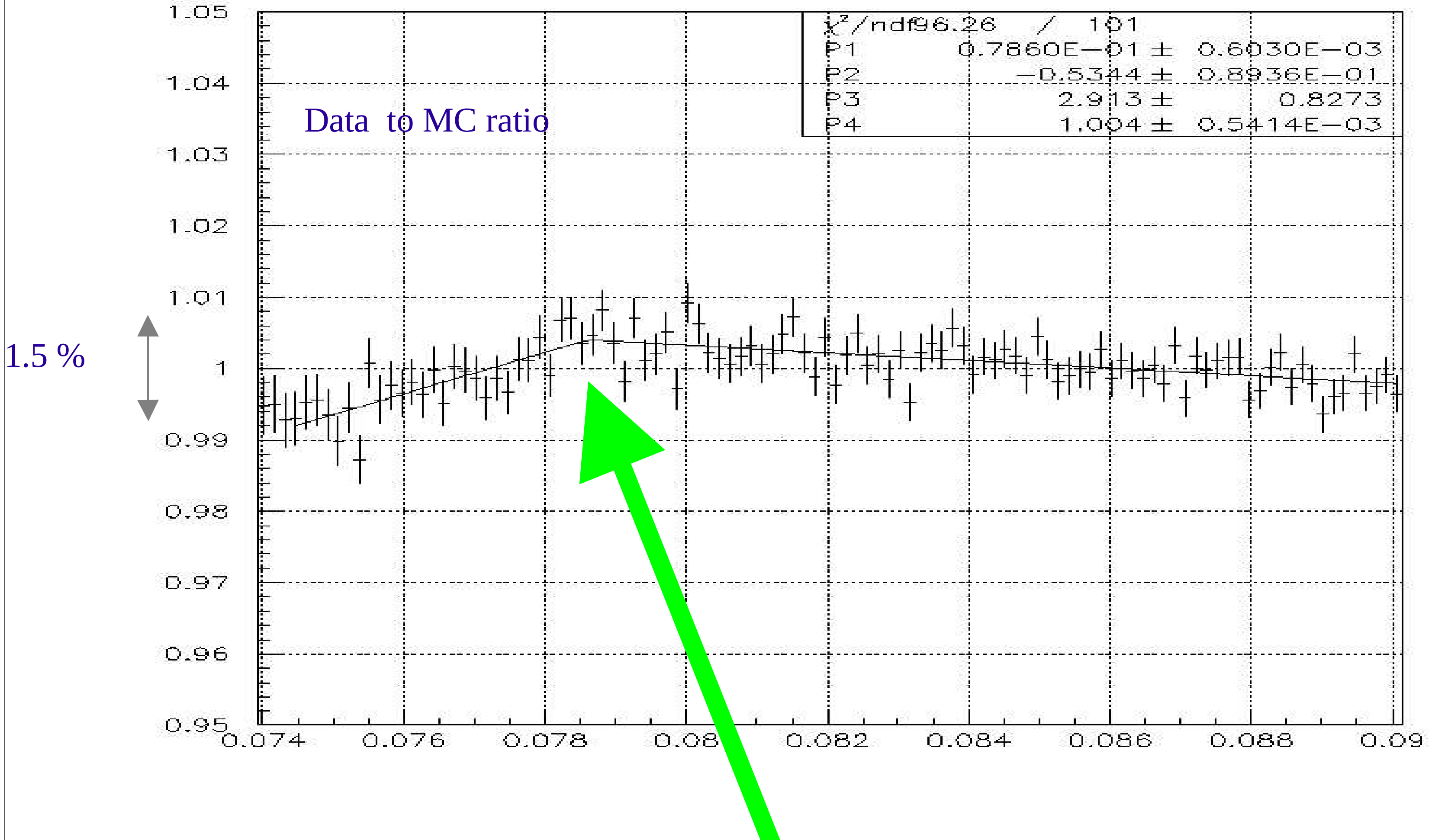
• Hodoscope
fast trigger;
precise time measurement (150ps).

• Liquid Krypton EM calorimeter (LKr)
High granularity, quasi-homogeneous;
$\Delta E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$ [GeV].

• Hadron calorimeter, muon veto counters, photon vetoes.



Cusp in K_L --> $3\pi^0$



Fitted CUSP position at (0.0786 +- 0.0006) GeV**2

Consistent with the expected value 0.07814