New approach to inclusive decay spectra

Einan Gardi (Cambridge)

Plan of the talk

- Introduction: inclusive B decay spectra — the challenge
- Sudakov resummation with NNLL accuracy
- Divergence of perturbation theory
- The quark distribution in the meson — cancellation of renormalon ambiguities
- Dressed Gluon Exponentiation: renormalon resummation in the Sudakov exponent
- computed $\bar{B} \longrightarrow X_s\gamma$ spectrum — comparison to Belle data

Inclusive B–decay Spectra and Infrared Renormalons

Einan Gardi (Cambridge)

References

- Taming the $\bar{B} \longrightarrow X_s\gamma$ spectrum by Dressed Gluon Exponentiation,
  J.R. Andersen, E. Gardi, [hep-ph/0502159].

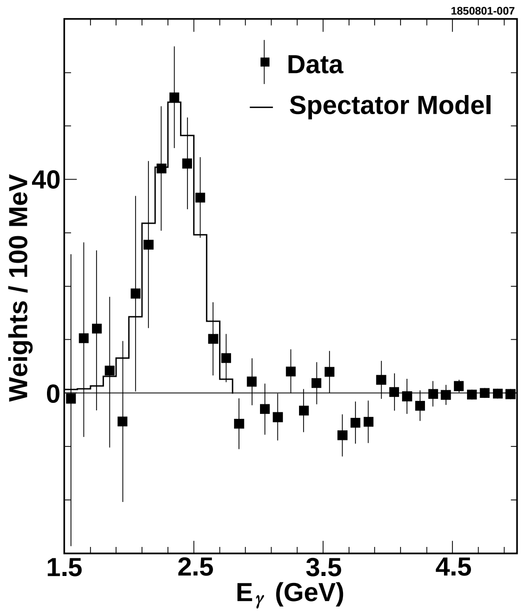
- On the quark distribution in an on-shell heavy quark and its all-order relations with the perturbative fragmentation function,
  E. Gardi, JHEP **0502**, 053 (2005) [hep-ph/0501257].

- Radiative and semi-leptonic B-meson decay spectra: Sudakov resummation beyond logarithmic accuracy and the pole mass,
  E. Gardi, JHEP **0404**, 049 (2004) [hep-ph/0403249].

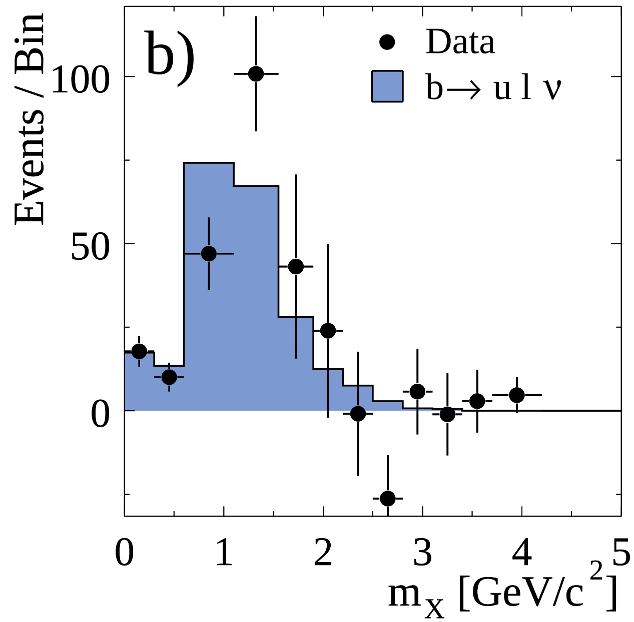
Inclusive B–decay Spectra

radiative decay: $\bar{B} \longrightarrow X_s \gamma$

semi-leptonic decay: $\bar{B} \longrightarrow X_u l \bar{\nu}_l$



CLEO



BABAR

The distribution peaks close to the endpoint $(E_\gamma \longrightarrow \frac{M}{2}; \quad \text{small } m_X)$

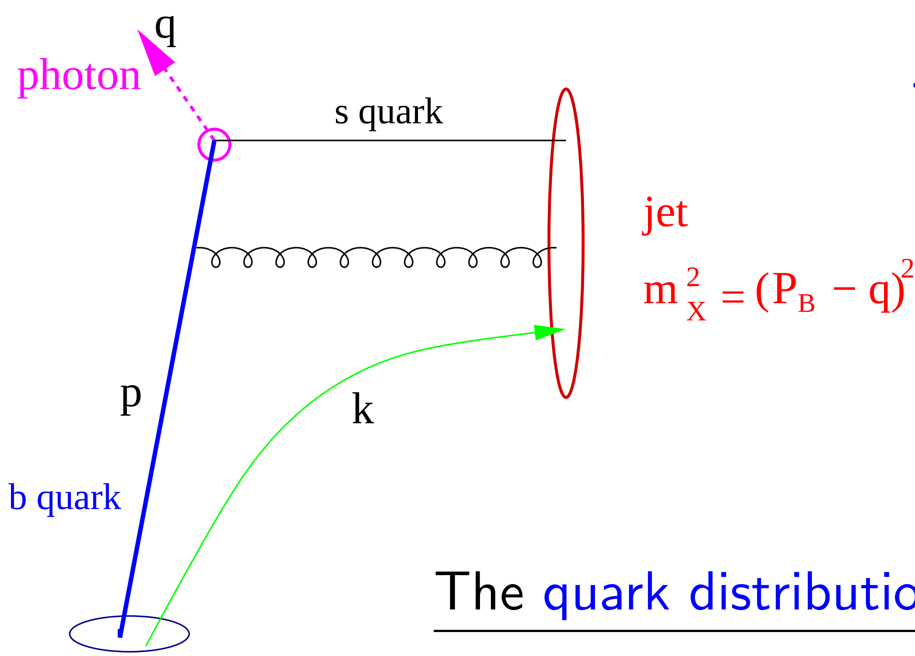
Example: extracting $|V_{ub}|$ from the semi-leptonic decay

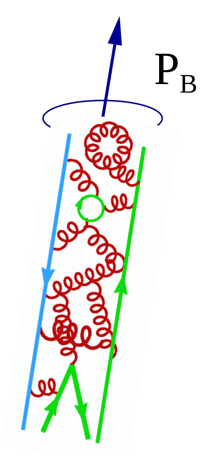
Precise measurements are restricted to the peak region (charm background)

Determination of $|V_{ub}|$ relies on calculation of the spectrum.

Kinematics in $\bar{B} \longrightarrow X_s \gamma$

Most of the B meson momentum is carried by the b quark





$$x \equiv \frac{2E_\gamma}{m_b}; \qquad \frac{1}{\Gamma_{\rm tot}}\frac{d\Gamma}{dx}\bigg|_{\rm LO} = \delta(1-x)$$

Beyond LO: the peak is smeared.

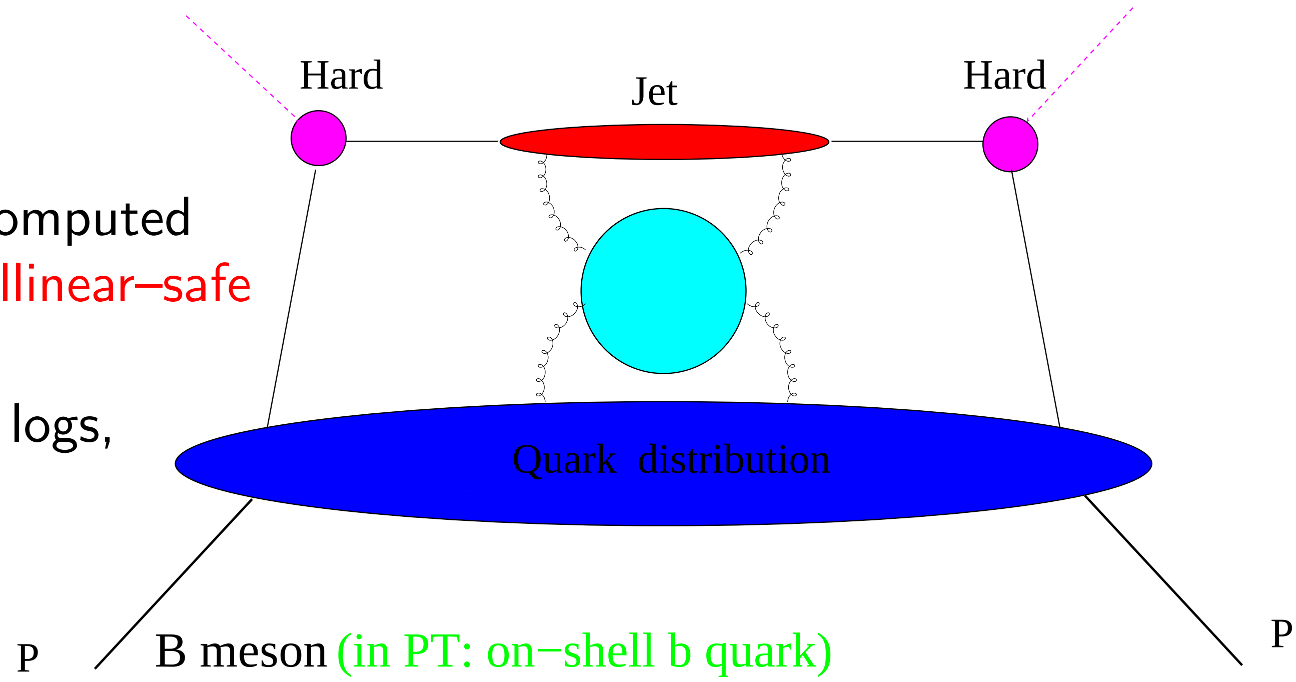
Perturbative endpoint: $x = 1$

The quark distribution in the B meson $f(z;\mu)$

In the B rest frame all components of k are $\mathcal{O}(\Lambda)$
so the quark distribution effect is power suppressed
But near the endpoint $M(1-x) \sim \Lambda$
the effect of $f(z;\mu)$ is $\mathcal{O}(1)$ Neubert (93)
Bigi, Shifman, Uraltsev & Vainshtein (93)

$$m_X^2 = (k+p-q)^2 \simeq (p-q)^2 + 2k\cdot(p-q) = \mathcal{O}(\Lambda M)$$

Large–x factorization in inclusive B decays



The spectrum can be computed in PT: infrared– and collinear–safe

Dominated by Sudakov logs, $\ln(1-x)$

scales:

Hard: m

Jet: $m_X^2 = (P_b - q)^2 \simeq m^2(1-x) \Longrightarrow m^2/N$

Soft: $m(1-x) \Longrightarrow m/N$

Korchemsky & Sterman (94)

Spectral moments:

$$\begin{aligned}\Gamma_N^{\rm PT} &\equiv \int_0^1 dx x^{N-1} \frac{1}{\Gamma_{\rm tot}^{\rm PT}} \frac{d\Gamma^{\rm PT}}{dx} \\ &= H(m) J(m^2/N;\mu) S_{\rm PT}(m/N;\mu) \;+\; \mathcal{O}(1/N) \\ &\equiv H(m)\,{\rm Sud}(N,m) \;+\; \mathcal{O}(1/N)\end{aligned}$$

Coefficients in the Sudakov exponent

$$\mathrm{Sud}(N,m) = \exp\left\{ - \sum_{n=1}^{\infty} \sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\mathsf{MS}}}(m^2)}{\pi} \right)^n \right\}$$

The coefficients $C_{n,k}$ are known **exactly** to NNLL accuracy. For $N_f = 4$ they are:

$k \longrightarrow$

$n \downarrow$

-1.564	0.667	0	0	0	0	0	0
3.837	-0.078	1.389	0	0	0	0	0
?	20.579	6.339	3.376	0	0	0	0
?	?	116.464	33.024	9.042	0	0	0
?	?	?	597.221	138.600	25.955	0	0
?	?	?	?	2859.284	548.170	78.492	0
?	?	?	?	?	13141.289	2129.058	247.233
?	?	?	?	?	?	58941.217	8238.359
?	?	?	?	?	?	?	260391.559

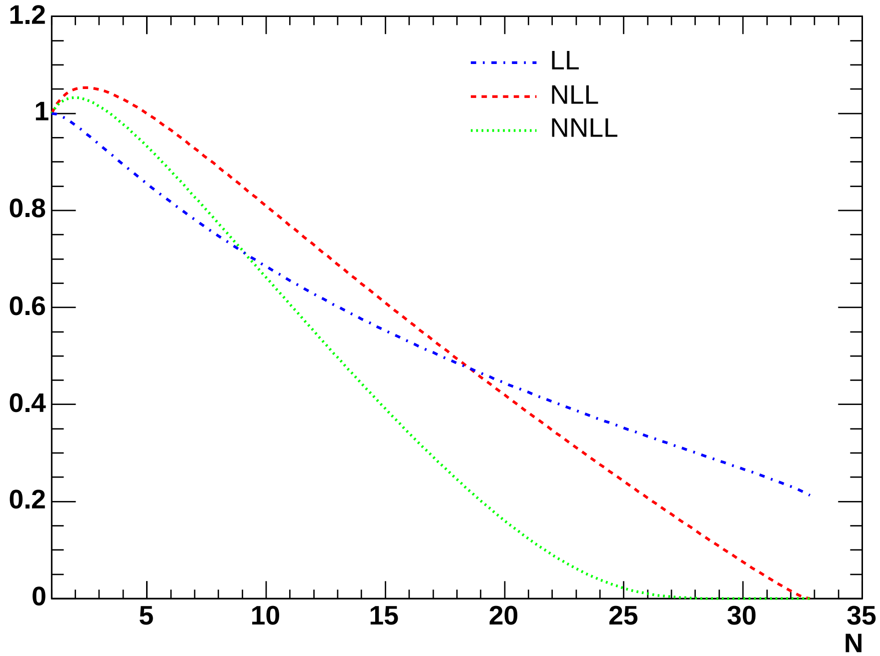
- At a given order in α_s the coefficients of **subleading logs** (lower k) get large...
- Is the fixed–logarithmic–accuracy approximation at LL / NLL / NNLL good?

Conventional Sudakov resummation with NNLL accuracy

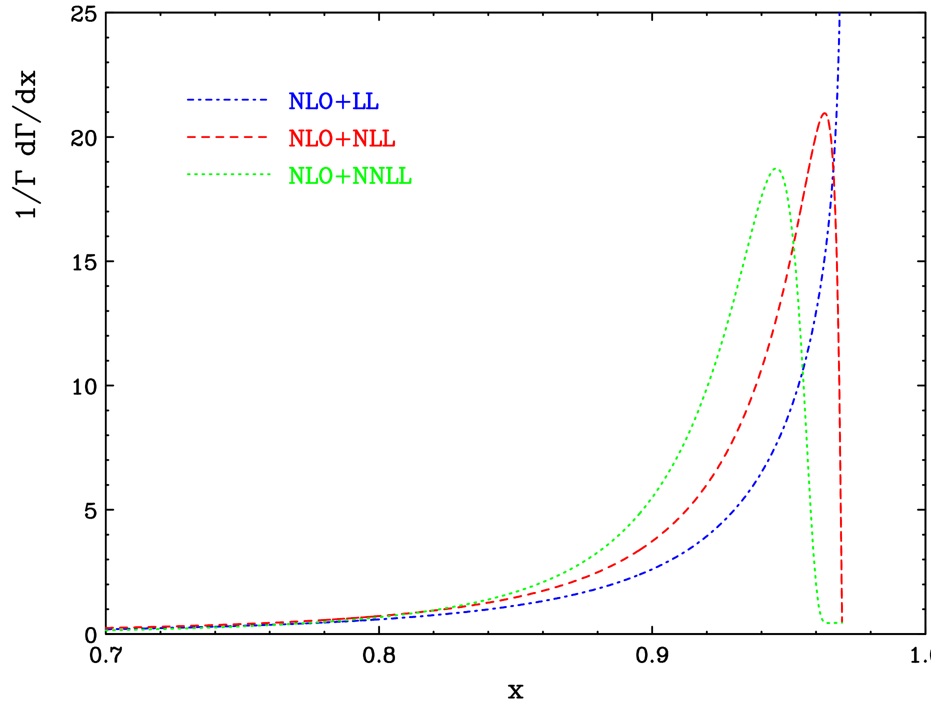
$$\text{Sud}(N,m) = \exp\left\{\sum_{n=0}^{\infty} g_n(\lambda)\left(\frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi}\right)^{n-1}\right\}; \qquad \lambda \equiv \frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi}\beta_0 \ln N$$

$$g_0(\lambda) = \frac{C_F}{{\beta_0}^2}\left[(1-\lambda)\,\ln(1-\lambda) - \frac{1}{2}(1-2\lambda)\,\ln(1-2\lambda)\right]$$

$\text{Sud}(N,m)$



Corresponding spectra



Coefficients in the Sudakov exponent in the large–β_0 limit

$$\mathrm{Sud}(N,m) = \exp\left\{ -\sum_{n=1}^{\infty}\sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}(m^2)}{\pi}\right)^n \right\}$$

The part in $C_{n,k}$ that is proportional to $(\beta_0)^{n-1}$ is known to all orders:

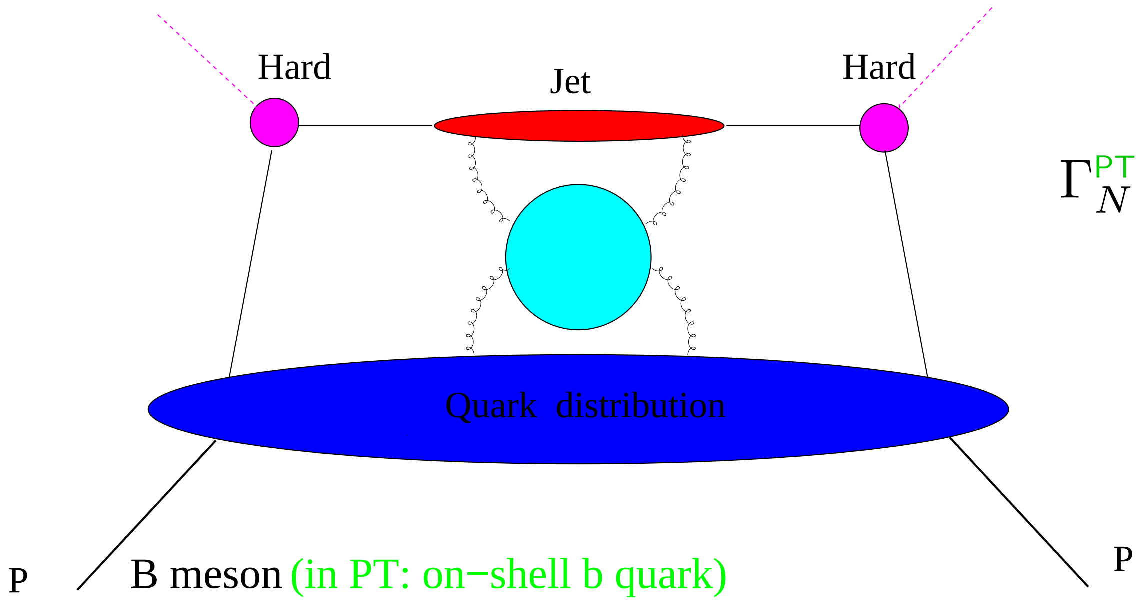
$k \longrightarrow$

$n \downarrow$

-1.56	0.67	0	0	0	0	0
1.24	0.90	1.39	0	0	0	0
61.17	28.32	8.28	3.38	0	0	0
1096.06	515.20	166.25	34.89	9.04	0	0
20399.23	10078.43	3231.40	793.25	131.33	25.95	0
444615.21	221481.03	73268.94	17791.58	3514.66	482.12	78.49
11342675.74	5665794.49	1883129.50	468180.33	91361.30	15080.79	1768.50
334032127.30	166960507.50	55609620.17	13867704.58	2760946.21	449959.01	63745.75

- $C_{n,k}$ **increase** for lower powers of $\ln N$, building up $\sum_{k=1}^{n+1} C_{n,k}\ln^k N \sim \mathbf{n!} f_n(N)$
- Truncation at fixed logarithmic accuracy is **not** a good approximation.
- **Renormalon divergence** sets in already at low orders — requires a prescription!

Large–x factorization — going beyond PT



Perturbation theory (on-shell quark):

$$\Gamma_N^{\rm PT} \equiv \int dE_\gamma \left(\frac{2E_\gamma}{m}\right)^{N-1} \frac{1}{\Gamma_{\rm tot}^{\rm PT}}\frac{d\Gamma^{\rm PT}}{dE_\gamma}$$

$$\simeq H(m)J(m^2/N;\mu)S_{\rm PT}(m/N;\mu)$$

$$= H(m)\,{\rm Sud}(N,m)$$

Non-perturbatively (meson):

$$\Gamma_N \equiv \int dE_\gamma \left(\frac{2E_\gamma}{M}\right)^{N-1} \frac{1}{\Gamma_{\rm tot}}\frac{d\Gamma}{dE_\gamma}$$

$$\simeq H(m)J(m^2/N;\mu)S(M/N;\mu)$$

Quark distribution in the meson in terms of that in an on-shell heavy quark:

$$S(m/N;\mu) = S_{\rm PT}(m/N;\mu)\ {\rm e}^{-(N-1)\bar{\Lambda}/M}\,\mathcal{F}((N-1)\Lambda/M)$$

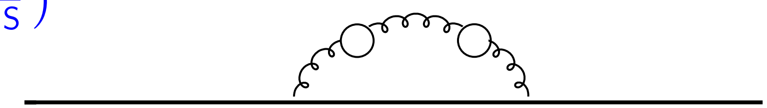
where $\bar{\Lambda} \equiv M - m$ captures the dependence on the mass; $\mathcal{F}$ has no linear term

Renormalon ambiguity in the pole mass

Infrared renormalons: a perturbative probe of large–distance effects

The propagator: $\frac{i}{\not p - m_{\overline{\rm MS}} - \Sigma(p, m_{\overline{\rm MS}})}$

Computed in the large–N_f limit



Off shell $\Sigma(p, m_{\overline{\rm MS}})$ has no renormalons
But applying the on–shell condition (inverse propagator vanishes at $p^2 = m^2$):

$$\frac{m}{m_{\overline{\rm MS}}} = 1 + \frac{C_F}{\beta_0}\int_0^\infty du \left(\frac{\Lambda^2}{m_{\overline{\rm MS}}^2}\right)^u \left[3e^{\frac{5}{3}u}\,\frac{(1-u)\Gamma(1+u)\Gamma(-2u)}{\Gamma(3-u)} + \frac{3}{4u} - R_{\Sigma_1}(u)\right].$$

Beyond PT the pole mass is ambiguous...

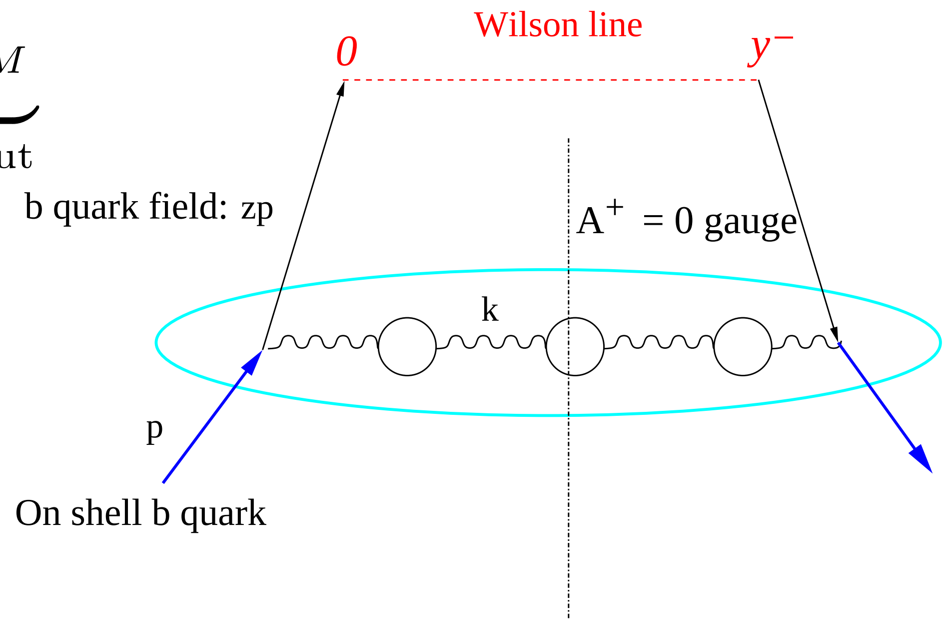
Beneke & Braun; Bigi, Shifman, Uraltsev & Vainshtein (94)

and so is $\bar\Lambda = M - m$.

Cancellation of the leading renormalon ambiguity

The non-perturbative quark distribution function is **renormalon free**:

$$S(m/N;\mu) \simeq \underbrace{S_{\rm PT}(m/N;\mu)\ {\rm e}^{-(N-1)\bar{\Lambda}/M}}_{\text{leading}-\text{renormalon cancels out}}$$



Dressed Gluon Exponentiation (DGE):

$$S_{\rm PT}(m/N;\mu) = \exp\left\{\int_0^\infty du\, T(u)\,\left(\frac{\Lambda^2}{m^2}\right)^u \times \frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) + \left(\frac{m^2}{\mu^2}\right)^u B_{\mathcal{A}}(u)\ln N\right]\right\}$$

$B_{\mathcal{S}}(u) = \frac{C_F}{\beta_0}{\rm e}^{\frac{5}{3}u}\,(1-u)\,+\mathcal{O}(1/\beta_0^2)$; $\mathcal{A}$ is the cusp anomalous dimension.

The renormalon at $u=\frac{1}{2}$ **cancels** between $S_{\rm PT}(m/N;\mu)$ and ${\rm e}^{-(N-1)\bar{\Lambda}/M}$!

Renormalons at work — going beyond the large–β_0 limit

The cancellation mechanism is general and it can be **used**:

- Calculation of the E_γ spectrum using the Principal Value prescription: Upon neglecting additional power corrections, $\mathcal{F} = 1$,

$$\Gamma_N = \Gamma_N^{\text{PT,PV}} \, \mathrm{e}^{-(N-1)\bar{\Lambda}_{\text{PV}}/M} \; = H(m) \; \mathrm{Sud}(N,m)|_{\text{PV}} \; \mathrm{e}^{-(N-1)\bar{\Lambda}_{\text{PV}}/M}$$

where $\bar{\Lambda}_{\text{PV}} \equiv M - m_{\text{PV}}$;

$\mathrm{Sud}(N,m)|_{\text{PV}}$ is **real–valued**: $\mathrm{Sud}(m,N)|_{\text{PV}} = \left[\mathrm{Sud}(m,N^*)|_{\text{PV}} \right]^*$.

$$\frac{d\Gamma(E_\gamma)}{dE_\gamma} = \frac{M}{2} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \Gamma_N \left(\frac{2E_\gamma}{M} \right)^{-N} = \frac{m_{\text{PV}}}{2} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \, \Gamma_N^{\text{PT,PV}} \left(\frac{2E_\gamma}{m_{\text{PV}}} \right)^{-N}$$

This spectrum is **free** of the leading renormalon ambiguity!

- The residue of the $u = \frac{1}{2}$ renormalon in the pole mass can be used to **improve the determination of** $B_{\mathcal{S}}(u)$.

Sudakov resummation beyond logarithmic accuracy

$$\mathrm{Sud}(m,N)\big|_{\mathrm{PV}} = \exp\left\{\mathrm{PV}\int_0^\infty du\,T(u)\left(\frac{\Lambda^2}{m^2}\right)^u \times\frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(N^{u}-1\right)\right]\right\}.$$

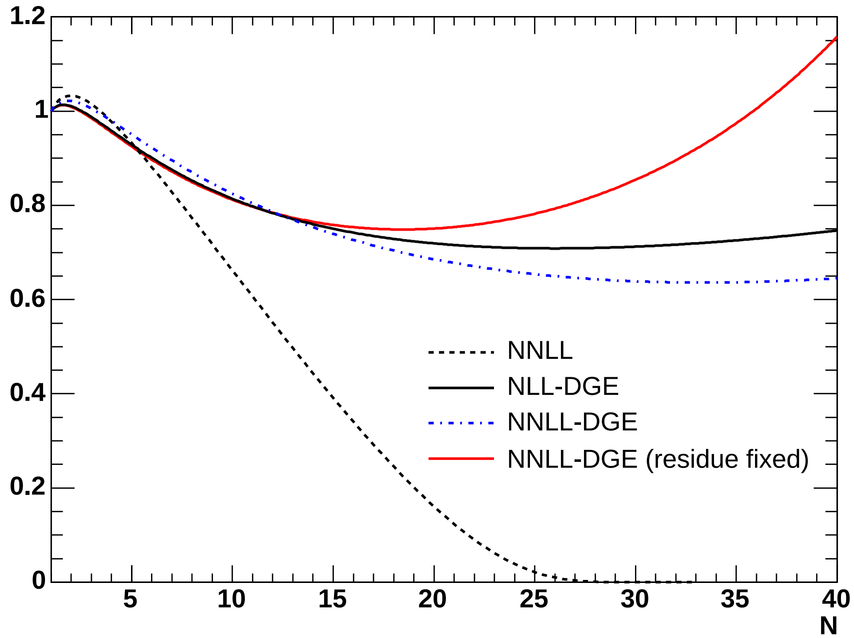
What do we know about $B_{\mathcal{S}}(u)$?

- All orders in the large–β_0 limit: $B_{\mathcal{S}}(u) = \dfrac{C_F}{\beta_0}\mathrm{e}^{\frac{5}{3}u}\left(1-u\right) + \mathcal{O}(1/\beta_0^2)$
- NNLO in the full theory: $B_{\mathcal{S}}(u) = 1 + s_1\dfrac{u}{1!} + s_2\dfrac{u^2}{2!} + \cdots$
- Renormalon cancellation in $\mathrm{Sud}(m,N)\,\mathrm{e}^{-(N-1)\bar{\Lambda}/M}$ implies: $B_{\mathcal{S}}(u=1/2)$ is equal in magnitude and opposite in sign to the residue of the $u=1/2$ renormalon in $m/m_{\overline{\mathrm{MS}}}$, which can be determined from the known NNLO expansion in $\overline{\mathrm{MS}}$ within a few percent.

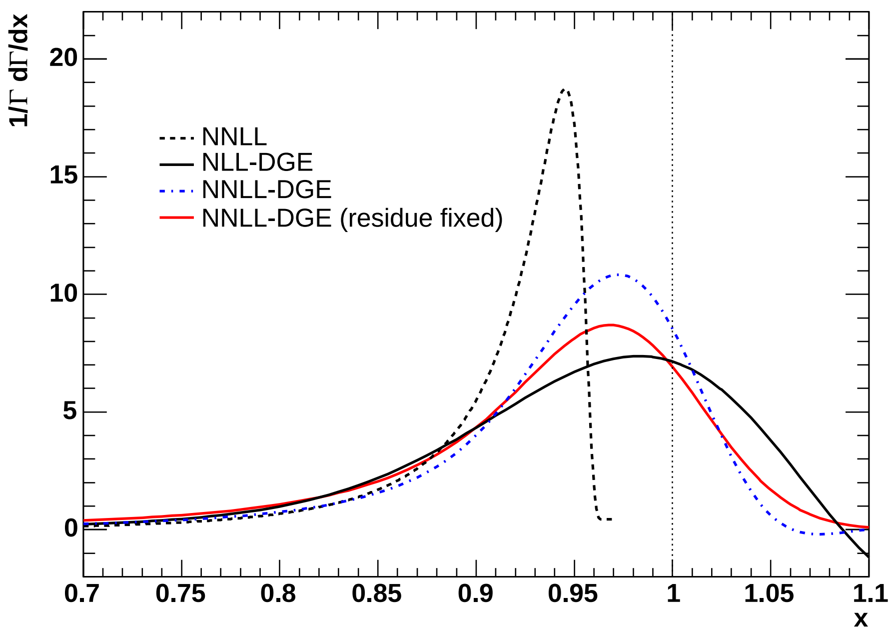
Dressed Gluon Exponentiation: Results

$$\text{Sud}(m,N)|_{\text{PV}} = \exp\left\{\text{PV}\int_0^\infty du\, T(u)\left(\frac{\Lambda^2}{m^2}\right)^u \right.$$
$$\left.\times\frac{1}{u}\left[B_{\mathcal{S}}(u)\Gamma(-2u)\left(N^{2u}-1\right) - B_{\mathcal{J}}(u)\Gamma(-u)\left(N^u-1\right)\right]\right\}.$$

$\text{Sud}(N,m)|_{\text{PV}}$ with various approx. for $B_{\mathcal{S}}(u)$

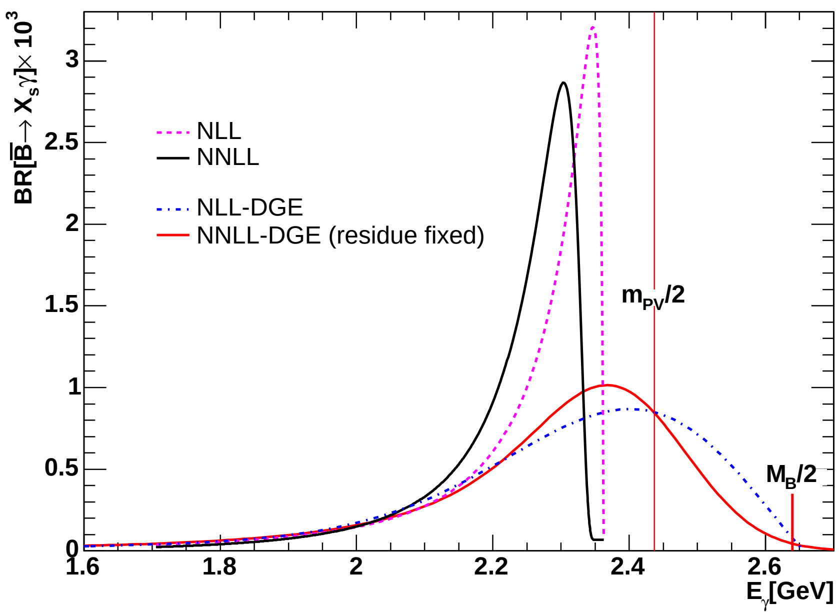


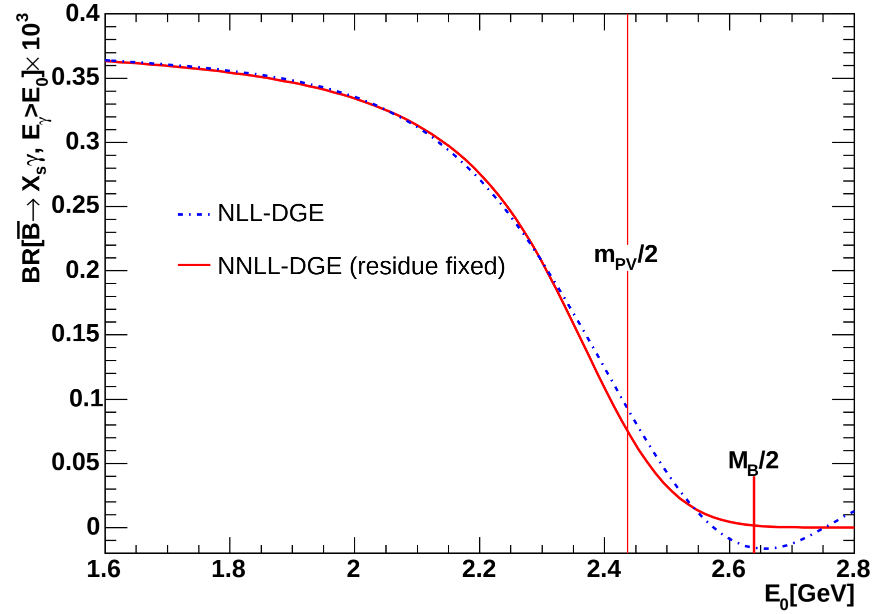
Corresponding spectra



Conclusions

- Resummed perturbation theory **predicts** the photon energy spectrum in $\bar{B} \longrightarrow X_s\gamma$.



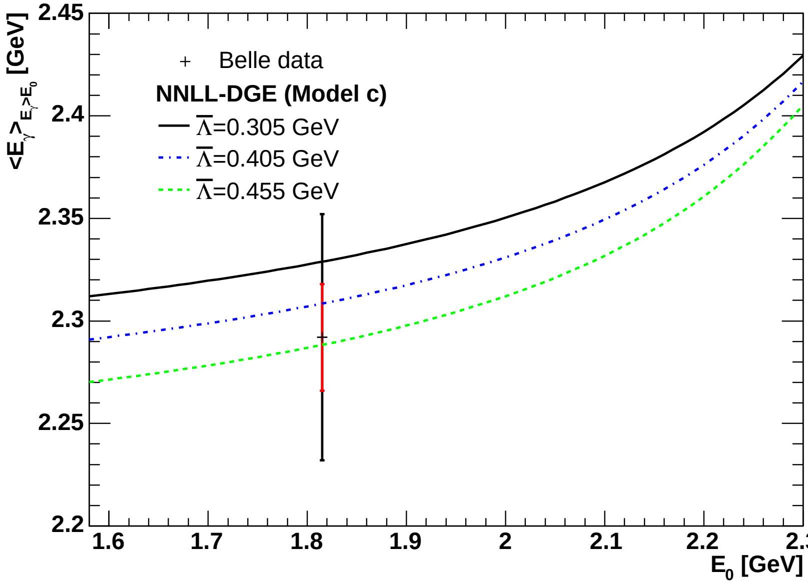


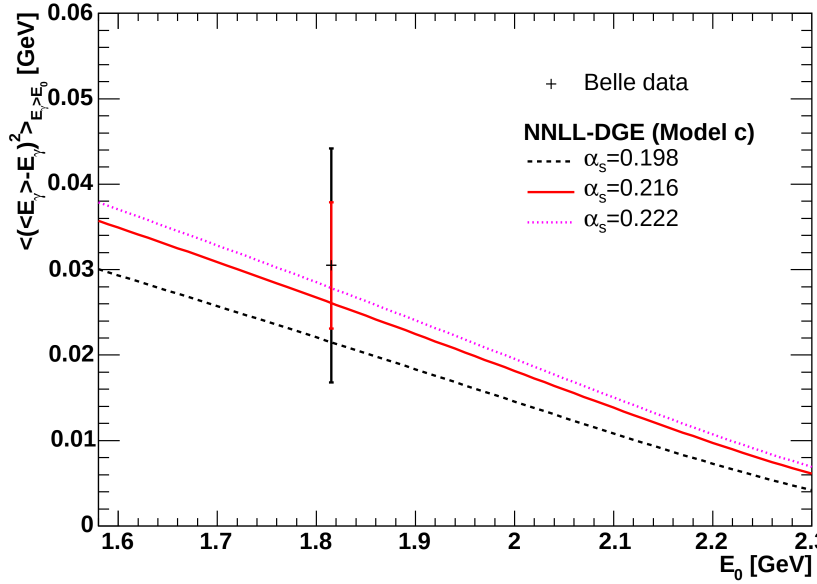
- Renormalon cancellation $(u = 1/2)$ is respected by using the **same prescription** for both the Sudakov exponent and the pole mass.
- Contrary to Sudakov resummation with fixed logarithmic accuracy, DGE is **stable**. This is owing to resumming running–coupling effects with a definite prescription.
- The DGE spectrum smoothly extends beyond the perturbative endpoint and tends to zero for $E_\gamma = (m + \mathcal{O}(\Lambda))/2$, close to the physical endpoint $E_\gamma = M/2$.

Cut moments: comparison to Belle data with $E_\gamma > 1.815$ GeV

$$\left\langle E_\gamma \right\rangle_{E_\gamma>E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \, E_\gamma$$

$$\left\langle \left(\left\langle E_\gamma \right\rangle_{E_\gamma>E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma>E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \, \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\left\langle E_\gamma \right\rangle_{E_\gamma>E_0} - E_\gamma \right)^n .$$





- Future comparison with experiment (varying energy cut; higher moments) will facilitate quantifying power corrections.
- Good prospects for determination of V_{ub} from **inclusive semileptonic decays**.