Recent developments in soft-collinear effective theory

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An effective theory for energetic processes

- Large momentum transfers $Q^2$, small invariant masses: $M_X^2 << Q^2$. Expansion in $M_X^2/Q^2$.
- Typical momentum regions / relevant scales:
  - In SCET, QCD fields are split into soft and collinear fields. Hard part is absorbed into Wilson coefficient.
An effective theory for energetic processes

• SCET provides Lagrangian framework to study interplay of soft and collinear emissions

• Scale separation
  • Physics associated with large scales is absorbed into Wilson coefficients.

• Factorization
  • In some cases soft interactions are absent at leading power.

• Sudakov resummation
  • RG-evolution in effective theory

• Power corrections
Many $B$-physics applications

- Exclusive $B$-decays
- Factorization theorems
- Inclusive $B$-decays: $B \rightarrow X_s \gamma$, $B \rightarrow X_u l \nu$
- Experimental cuts enforce small $M_X$
- Some recent results
  - Precision results for inclusive decays
    - Determination of $|V_{ub}|$ Bosch, Lange, Neubert, Paz
    - NNLO calculation of photon cut effects in $B \rightarrow X_s \gamma$ TB, Neubert ‘06
  - Fact. theorems for semi-inclusive decays Chay, Kim, Leibovich, Zupan ‘06

$E_\pi = \frac{M_B}{2} \gg M_\pi$
• Factorization theorems for $B$-decays contain a form factor piece which is usually left unfactorized. Involves
  • Non-factorizable “messenger” interactions between $B$ and light mesons.  
  • Divergent convolution integrals of hard scattering kernels with LCDAs (“wave functions”).
  • Manohar and Stewart, hep-ph/0605001, claim that this part can be factorized using “rapidity space factorization” with “zero-bin subtraction”
  • Note: form factor is also factorized in pQCD framework.
Complete factorization for exclusive B-decays to light mesons?

- Claims:
  - Confinement will eliminate non-factorizable interactions.
  - Divergencies in convolution integrals are UV and can be renormalized away.
- If true, this result would be quite spectacular, but many open questions
  - Renormalizability? RG evolution?
  - Gauge invariance? They use a gluon mass to eliminate nonfactorizable interactions.
  - Hadronic input?
Collider physics applications

• Natural habitat of large momentum transfers are high-energy collisions...
• Growing number of non-$B$ SCET papers in last year
• What should you expect from SCET in collider physics?
• Miracles!? Maybe not.
  • A prediction of the PDFs? A calculation of the fragmentation of quarks and gluons into hadrons?
    • No. Non-perturbative, with or without EFT ...
  • 8-point two-loop amplitudes?
    • No. A complicated perturbative problem, with or without EFT ...
SCET in collider physics

• Recent application
  • Resummations by RG evolution
    • Threshold resummation
      • DIS Manohar ‘03; Idilbi, Ji ‘05; TB, Neubert, Pecjak ‘06
      • DY and Higgs production Idilbi, Ji, Ma, Yuan ‘05 ‘06
      • in momentum space TB, Pecjak, Neubert ‘06
    • $p_T$-resummation for DY, Higgs Gao, Li, Liu ‘05; Idilbi, Ji, Yuan ‘06
  • Parton shower using SCET Bauer and Schwartz ‘06
  • Universality of non-perturbative effects in event shapes Lee and Sterman ‘06
  • Mass factorization TB and Melnikov ‘07
  • Bhabha scattering in the limit $m_e^2 \to 0$ at NNLO
Threshold resummation in momentum space

TB, M. Neubert ‘06
TB, M. Neubert, B. Pecjak ‘06
Resummation

- Fixed order perturbation theory problematic for problems with widely separated scales $Q_1 \gg Q_2$.
- Large logarithms $\alpha_s^n \log^n(Q_1/Q_2)$ and $\alpha_s^n \log^{2n}(Q_1/Q_2)$.
- Scale in coupling? $\alpha_s(Q_1)$ or $\alpha_s(Q_2)$?
- Many situations with scale hierarchies in collider processes. Simplest example: DIS for $x \to 1$

\[
Q^2 = -q^2
\]

\[
M_X = \frac{1 - x}{x} Q^2 \ll Q^2
\]
Any choice of the scale $\mu$ will lead to large perturbative logarithms. Hard function $H$ and jet-function $J$ are Wilson coefficients in SCET, fulfill RG equations.

Evaluate each part at its characteristic scale. Solve RG for individual parts, evolve to next scale:

$$H(\mu_h) \times U_1(\mu_h, \mu_i) \times J(\mu_i) \otimes U_2(\mu_i, \mu_f) \otimes \phi(\mu_f)$$

match $\rightarrow$ run $\rightarrow$ match $\rightarrow$ run
Result for resummed structure function

- Simple, analytic results for resummed structure function in momentum space.

\[
F_2^{\text{ns}}(x, Q^2) = H(Q^2, \mu_h) U(Q, \mu_h, \mu_i, \mu_f) \\
\times \widetilde{J} \left( \ln \frac{Q^2}{\mu_i^2} + \partial_\eta, \mu_i \right) \frac{e^{-\gamma E \eta}}{\Gamma(\eta)} \int_x^1 d\xi \frac{\phi_{q}^{\text{ns}}(\xi, \mu_f)}{(\xi - x)^{1-\eta}}
\]

- \( U, \eta \): (known) functions of anomalous dim’s.
- \( \widetilde{J} \): Laplace transform of \( J \)
- Resummed result obtained after plugging in fixed order results for coefficient \( C_V \), jet-function and anom. dimensions.
Comparison with traditional method

• Traditionally, resummation for hard processes is performed in moment space.
  • Landau pole ambiguities (in Sudakov exponent and Mellin inversion)
  • Mellin inversion only numerically

• By solving RG equations in SCET, one obtains resummed expressions directly in momentum space.
  • Clear scale separation. No Landau pole ambiguities.
  • Simple analytic expressions.
  • Trivial connection with fixed order expressions.
  • Freedom to choose matching scales

• Note: using the appropriate scale setting, we can rederive the standard resummation formulas from SCET.
slightly massive Bhabha scattering at NNLO

TB, K. Melnikov, to appear
Bhabha scattering

- Used to measure luminosity at $e^+ e^-$ colliders

$$\mathcal{L} = \frac{dN}{dtd\Omega} \bigg|_{\text{measured}} \frac{d\sigma}{d\Omega} \bigg|_{\text{theory}}$$

- **Large angle** scattering at low energy meson factories
  - Babar, Belle, BEPC-BES, CLEO-C, Daphne, VEPP-2M, ...

- **Small angle** scattering at high-energy machines
  - LEP, SLD, ILC, ...

- New physics search at large angles!

precise prediction crucial
QED theory status

- **State-of-the-art:** MC generators that implement NLO and resum logarithmically enhanced higher order corrections.

- **NNLO:**
  - Massless result calculated
    - Bern, Dixon, Ghinculov ‘01
  - Ongoing work on massive NNLO
    - Fermion loop contribution known
    - Bonciani et al. ‘04
  - Leading term for $m_e^2 \ll s, |t|$ known
    - inferred from massless result
    - sufficient for phenomenology
    - Penin ‘05
“Mass from no mass”

- Penin’s derivation of the $m_e^2 \ll s, |t|$ result is somewhat complicated
- uses photon mass as IR regulator
- depends on non-renormalization of leading Sudakov log’s
- Have much simpler method to restore logarithmic mass dependence of amplitudes
- Mass effects appear as wave function renormalization on external legs of massless amplitude $\tilde{M}(\{p_i\})$

$$\mathcal{M}(\{p_i\}, m) = Z_j(m)^{n/2} \tilde{M}(\{p_i\}) + \mathcal{O}(m^2/Q^2)$$

- this relation also works for QCD
- Note: relation is more complicated for diagrams with massive fermion loops.
Form factor in dimensional regularization

<table>
<thead>
<tr>
<th>Off-shell</th>
<th>On-shell massless</th>
<th>On-shell massive</th>
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<tbody>
<tr>
<td>$H = H(Q^2)$ same in all three cases! IR finite.</td>
<td>Jet and soft function scaleless! Soft and collinear divergencies for $d \to 4$</td>
<td>Jet function $J = J(m^2)$ Soft function scaleless! Soft divergencies for $d \to 4$</td>
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Jet and soft function scaleless!
Massive Bhabha

- Determine $Z_j (=J^2)$ by taking ratio of massive to massless form factor

$$Z_j = 1 + \left(\frac{\alpha}{\pi}\right) m^{-2\epsilon} \left[ \frac{1}{2\epsilon^2} + \frac{1}{4\epsilon} + \frac{\pi^2}{24} + 1 + \epsilon \left( 2 + \frac{\pi^2}{48} - \frac{\zeta(3)}{6}\right) + \epsilon^2 \left( 4 - \frac{\zeta(3)}{12} + \frac{\pi^4}{320} + \frac{\pi^2}{12}\right) \right]$$

$$+ \left(\frac{\alpha}{\pi}\right)^2 m^{-4\epsilon} \left[ \frac{1}{8\epsilon^4} + \frac{1}{8\epsilon^3} + \frac{1}{\epsilon^2} \left( \frac{17}{32} + \frac{\pi^2}{48}\right) + \frac{1}{\epsilon} \left( \frac{83}{64} - \frac{\pi^2}{24} + \frac{2\zeta(3)}{2}\right) \right]$$

$$+ \frac{561}{128} + \frac{61\pi^2}{192} - \frac{11}{24} \zeta(3) - \frac{\pi^2}{2} \ln(2) - \frac{77\pi^4}{2880} \right]$$

$Q^2$ independent ✓

- Multiply massless Bhabha amplitude with $Z_j^2$. Add soft radiation with $E_\gamma < \omega \ll m_e$

$$\frac{d\sigma}{d\Omega} = \exp \left( \frac{\alpha}{\pi} F_{\text{soft}} \right) \times Z_j^4 \times \left. \frac{d\sigma}{d\Omega} \right|_{\text{virtual},m_e=0}$$

see also Moch and Mitov ‘06
Result

• Input
  • 1-loop to $O(\varepsilon^2)$
  • 2-loop virtual  
  • (1-loop) x (1-loop)  
  • $F_{\text{soft}}$ to $O(\varepsilon)$

• Result:
  • Complete agreement with result of Penin!!
  • First independent check of his result.

Bern, Dixon, Ghinculov ‘01
Bern, Dixon, Ghinculov ‘01
inferred from Anastasiou et al. ‘00
our own evaluation
Summary

• Effective theory methods provide an efficient, powerful language to study factorization, resummation and power corrections to hard processes.

• Increased number of SCET application to collider physics problems during last year.

• No miracles, but a novel perspective and a number of promising results
  • new technique for resummation
  • proposal for improved parton showers
  • new factorization theorem for massive amplitudes

• Just the beginning, much room for future work!