Global analysis for determining fragmentation functions and their uncertainties

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XLIIInd Rencontres de Moriond
QCD and High Energy Hadronic Interactions

La Thuile, Italy, March 22, 2007
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Introduction
Fragmentation Function

Fragmentation: hadron production from a quark, antiquark, or gluon

Fragmentation function is defined by

\[ F_h(z, Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+ e^- \to hX)}{dz} \]

\( \sigma_{\text{tot}} = \) total hadronic cross section

Variable \( z \)
- Hadron energy / Beam energy
- Hadron energy / Primary quark energy

A fragmentation process occurs from quarks, antiquarks, and gluons, so that \( F^h \) is expressed by their individual contributions:

\[ F^h(z, Q^2) = \sum_i \int_z^1 dy C_i \left( \frac{z}{y}, Q^2 \right) D^h_i(y, Q^2) \]

Calculated in perturbative QCD

\( C_i(z, Q^2) = \) coefficient function

\( D^h_i(z, Q^2) = \) fragmentation function of hadron \( h \) from a parton \( i \)

Non-perturbative
(determined from experiments)

\[ z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2 \]
Momentum (energy) sum rule

\[ D_{i}^{h}(z,Q^{2}) = \text{probability to find the hadron } h \text{ from a parton } i \]
with the energy fraction \( z \)

Energy conservation: \( \sum_{h} \int_{0}^{1} dz \, z \, D_{i}^{h}(z,Q^{2}) = 1 \)

\( h = \pi^{+}, \pi^{0}, \pi^{-}, K^{+}, K^{0}, \bar{K}^{0}, K^{-}, p, \bar{p}, n, \bar{n}, \ldots \)

**Favored and disfavored fragmentation functions**

Simple quark model: \( \pi^{+}(ud), \, K^{+}(us), \, p(uud), \ldots \)

Favored fragmentation: \( D_{u}^{\pi^{+}}, \, D_{d}^{\pi^{+}}, \ldots \)

(from a quark which exists in a naive quark model)

Disfavored fragmentation: \( D_{d}^{\pi^{+}}, \, D_{u}^{\pi^{+}}, \, D_{s}^{\pi^{+}}, \ldots \)

(from a quark which does not exist in a naive quark model)
Status of determining fragmentation functions

<table>
<thead>
<tr>
<th></th>
<th>Nulceonic PDFs</th>
<th>Polarized PDFs</th>
<th>Nuclear PDFs</th>
<th>FFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determination</td>
<td>****</td>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>x</td>
</tr>
<tr>
<td>Comments</td>
<td>Accurate determination from small x to large x</td>
<td>Gluon &amp; antiquark polarization?</td>
<td>Gluon? Antiquark at medium x?</td>
<td>Flavor separation?</td>
</tr>
</tbody>
</table>

Uncertainty ranges of determined fragmentation functions were not estimated, although there are such studies in nucleonic and nuclear PDFs.

The large differences indicate that the determined FFs have much ambiguities.
Situation of fragmentation functions

There are two widely used fragmentation functions by Kretzer and KKP. An updated version of KKP is AKK.

(Kretzer) S. Kretzer, PRD 62 (2000) 054001
(KKP) B. A. Kniehl, G. Kramer, B. Pötter, NPB 582 (2000) 514
(AKK) S. Albino, B.A. Kniehl, G. Kramer, NPB 725 (2005) 181

The functions of Kretzer and KKP (AKK) are very different.
Purposes of investigating fragmentation functions

Semi-inclusive reactions have been used for investigating

- **origin of proton spin**
  \[ e^- + p \to e^- + h + X \text{ (e.g. HERMES), } \quad \bar{p} + \bar{p} \to h + X \text{ (RHIC-Spin)} \]

Quark, antiquark, and gluon contributions to proton spin
  (flavor separation, gluon polarization)

- **properties of quark-hadron matters** \[ A + A' \to h + X \text{ (RHIC, LHC)} \]

Nuclear modification
  (recombination, energy loss, …)

\[
\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2) \otimes \hat{\sigma}(ab \to cX) \otimes D_c^\pi(z, Q^2)
\]
Determination of Fragmentation Functions

Determination of fragmentation function and their uncertainties

M. Hirai, SK, T.-H. Nagai, K. Sudoh


A code for calculating the FFs is available at

http://research.kek.jp/people/kumanos/ffs.html
New aspects in our analysis

- Determination of fragmentation functions (FFs) and their uncertainties in LO and NLO.
- Discuss NLO improvement in comparison with LO by considering the uncertainties. (Namely, roles of NLO terms in the determination of FFs)
- Comparison with other parametrizations
- Avoid assumptions on parameters as much as we can, Avoid contradiction to the momentum sum rule
- SLD (2004) data are included.
## Comparison with other analyses

<table>
<thead>
<tr>
<th></th>
<th>HKNS (Ours)</th>
<th>Kretzer</th>
<th>KKP (AKK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function form</td>
<td>$N_i^{\pi^+} z^{\alpha_i^{\pi^+}} (1 - z)^{\beta_i^{\pi^+}}$</td>
<td>$N_i^{\pi^+} z^{\alpha_i^{\pi^+}} (1 - z)^{\beta_i^{\pi^+}}$</td>
<td>$N_i^{\pi^+} z^{\alpha_i^{\pi^+}} (1 - z)^{\beta_i^{\pi^+}}$</td>
</tr>
<tr>
<td># of parameters</td>
<td>14</td>
<td>11</td>
<td>15 (18)</td>
</tr>
<tr>
<td>Mass threshold</td>
<td>$m_Q^2$</td>
<td>$m_Q^2$</td>
<td>$4m_Q^2$</td>
</tr>
<tr>
<td></td>
<td>$(m_{c,b} = 1.43, 4.3 \text{ GeV})$</td>
<td>$(m_{c,b} = 1.4, 4.5 \text{ GeV})$</td>
<td>$(2m_{c,b} = 2.98, 9.46 \text{ GeV})$</td>
</tr>
<tr>
<td>Initial scale $Q_0^2$ (NLO)</td>
<td>1.0 GeV$^2$</td>
<td>0.4 GeV$^2$</td>
<td>2.0 GeV$^2$</td>
</tr>
<tr>
<td>Major ansatz</td>
<td>One constraint: A gluon parameter is fixed.</td>
<td>Four constraints: $D_u^{\pi^+} = (1 - z) D_u^{\pi^+}$</td>
<td>$M_h^i \equiv \int_{0.05}^{1} z D_h^i(z,Q^2)dz$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_g = \frac{M_u + M_{\bar{u}}}{2}$</td>
<td>(issue of momentum sum)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No $\pi^+$, $\pi^-$ separation</td>
<td></td>
</tr>
</tbody>
</table>
Initial functions for pion

Note: constituent-quark composition $\pi^+ = u\bar{d}$, $\pi^- = \bar{u}d$

$$D^\pi_u (z, Q_0^2) = N_u^\pi z^{\alpha_u^\pi} (1 - z)^{\beta_u^\pi} = D^\pi_d (z, Q_0^2)$$

$$D^\pi_{\bar{u}} (z, Q_0^2) = N_{\bar{u}}^\pi z^{\alpha_{\bar{u}}^\pi} (1 - z)^{\beta_{\bar{u}}^\pi} = D^\pi_d (z, Q_0^2) = D^\pi_s (z, Q_0^2) = D^\pi_{\bar{s}} (z, Q_0^2)$$

$$D^\pi_c (z, m_c^2) = N_c^\pi z^{\alpha_c^\pi} (1 - z)^{\beta_c^\pi} = D^\pi_{\bar{c}} (z, m_c^2)$$

$$D^\pi_b (z, m_b^2) = N_b^\pi z^{\alpha_b^\pi} (1 - z)^{\beta_b^\pi} = D^\pi_{\bar{b}} (z, m_b^2)$$

$$D^\pi_g (z, Q_0^2) = N_g^\pi z^{\alpha_g^\pi} (1 - z)^{\beta_g^\pi}$$

$$\begin{equation}
\begin{aligned}
D^\pi_u &= D^\pi_{\bar{u}} \\
D^\pi_d &= D^\pi_s \\
D^\pi_{\bar{c}} &= D^\pi_{\bar{b}} \\
D^\pi_c &= D^\pi_b \\
D^\pi_{\bar{c}} &= D^\pi_{\bar{b}} \\
D^\pi_{\bar{d}} &= D^\pi_{\bar{u}}
\end{aligned}
\end{equation}$$

$$D_q^\pi = D_{\bar{q}}^\pi$$

Constraint: 2nd moment should be finite and less than 1

$$n_f = \begin{cases} 
3, & \mu_0^2 < Q^2 < m_c^2 \\
4, & m_c^2 < Q^2 < m_b^2 \\
5, & m_b^2 < Q^2 < m_t^2 \\
6, & m_t^2 < Q^2 
\end{cases}$$

$$N = \frac{M}{B(\alpha + 2, \beta + 1)}, \quad M \equiv \int_0^1 zD(z)dz \quad (\text{2nd moment}), \quad B(\alpha + 2, \beta + 1) = \text{beta function}$$

$$0 < M_i^h < 1 \quad \text{because of the sum rule} \quad \sum_h M_i^h = 1$$
## Experimental data for pion

**Total number of data: 264**

<table>
<thead>
<tr>
<th></th>
<th>$\sqrt{s}$ (GeV)</th>
<th># of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>TASSO</td>
<td>12, 14, 22, 30, 34, 44</td>
<td>29</td>
</tr>
<tr>
<td>TCP</td>
<td>29</td>
<td>18</td>
</tr>
<tr>
<td>HRS</td>
<td>29</td>
<td>2</td>
</tr>
<tr>
<td>TOPAZ</td>
<td>58</td>
<td>4</td>
</tr>
<tr>
<td>SLD</td>
<td>91.2</td>
<td>29</td>
</tr>
<tr>
<td>SLD [light quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>SLD [c quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>SLD [b quark]</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>ALEPH</td>
<td>91.2</td>
<td>22</td>
</tr>
<tr>
<td>OPAL</td>
<td>91.2</td>
<td>22</td>
</tr>
<tr>
<td>DELPHI</td>
<td>91.2</td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [light quark]</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>DELPHI [b quark]</td>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>

![Graph showing data points and distribution](image)
Analysis

Initial scale: \( Q_0^2 = 1 \text{ GeV}^2 \)

Scale parameter: \( \Lambda_{QCD}^{n_f=4} = 0.220 \text{ (LO), 0.323 (NLO)} \)
\[ \alpha_s \text{ varies with } n_f \]

Heavy-quark masses: \( m_c = 1.43 \text{ GeV}, \, m_b = 4.3 \text{ GeV} \)

Results for the pion \( \chi^2/\text{d.o.f.} = 1.81 \text{ (LO), 1.73 (NLO)} \)

Uncertainty estimation: Hessian method

\[
\Delta \chi^2 \equiv \chi^2(\hat{a} + \delta a) - \chi^2(\hat{a}) = \sum_{i,j} H_{ij} \delta a_i \delta a_j, \quad H_{ij} = \frac{\partial^2 \chi^2(\hat{a})}{\partial a_i \partial a_j}
\]

\[
\left[ \delta D(z) \right]^2 = \Delta \chi^2 \sum_{i,j} \frac{\partial D(z,\hat{a})}{\partial a_i} H^{-1}_{ij} \frac{\partial D(z,\hat{a})}{\partial a_j}
\]
Comparison with pion data

\[ F_{\pi^{\pm}}(z, Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow \pi^{\pm}X)}{dz} \]

Our fit is successful to reproduce the pion data.

The DELPHI data deviate from our fit at large z.

Rational difference between data and theory

\[ \frac{F_{\pi^{\pm}}(z, Q^2)_{\text{data}} - F_{\pi^{\pm}}(z, Q^2)_{\text{theory}}}{F_{\pi^{\pm}}(z, Q^2)_{\text{theory}}} \]
Comparison with pion data: \( \frac{(\text{data-theory})}{\text{theory}} \)
Determined fragmentation functions for pion

- Gluon and light-quark fragmentation functions have large uncertainties.

- Uncertainty bands become smaller in NLO in comparison with LO.
  → The data are sensitive to NLO effects.

- The NLO improvement is clear especially in gluon and disfavored functions.

- Heavy-quark functions are relatively well determined.
Comparison with kaon data
Determined functions for kaon

- Gluon and light-quark fragmentation functions have large uncertainties.

- Uncertainty bands become smaller in NLO in comparison with LO.

- Heavy-quark functions are relatively well determined.
Comparison with other parametrizations in pion

(KKP) Kniehl, Kramer, Pötter
(AKK) Albino, Kniehl, Kramer
(HKNS) Hirai, Kumano, Nagai, Sudoh

• Gluon and light-quark fragmentation functions have large uncertainties, but they are within the uncertainty bands.

→ The functions of KKP, Kretzer, AKK, and HKNS are consistent with each other.

All the parametrizations agree in charm and bottom functions.
Comparison with other parametrizations in kaon and proton

**kaon**

- Gluon: $Q^2 = 2$ GeV$^2$
- $u$ quark: $Q^2 = 2$ GeV$^2$
- $c$ quark: $Q^2 = 10$ GeV$^2$

**Proton**

- Gluon: $Q^2 = 2$ GeV$^2$
- $d$ quark: $Q^2 = 2$ GeV$^2$
- $b$ quark: $Q^2 = 10$ GeV$^2$
- $s$ quark: $Q^2 = 2$ GeV$^2$
- $c$ quark: $Q^2 = 10$ GeV$^2$

Legend:
- KKP
- HKNS
- AKK
- Kretzer
Comments on “low-energy” experiments, Belle & BaBar

Gluon fragmentation function is very important for hadron production at small $p_T$ at RHIC (heavy ion, spin) and LHC, (see the next transparency) and it is “not determined” as shown in this analysis.

→ Need to determine it accurately.
→ Gluon function is a NLO effect with the coefficient function and in $Q^2$ evolution.

We have precise data such as the SLD ones at $Q=Mz$, so that accurate small-$Q^2$ data are needed for probing the $Q^2$ evolution, namely the gluon fragmentation functions. (Belle, BaBar ?)
Pion production at RHIC: \( p + p \rightarrow \pi^0 + X \)

S. S. Adler et al. (PHENIX), PRL 91 (2003) 241803

- Consistent with NLO QCD calculation up to \( 10^{-8} \)
- Data agree with NLO pQCD + KKP
- Large differences between Kretzer and KKP calculations at small \( p_T \)

\[ \sqrt{s} = 200 \text{ GeV} \]

\[
\begin{array}{c}
p \\
\rightarrow \\
\pi \\
\rightarrow \\
p
\end{array}
\]

Blue band indicates the scale uncertainty by taking \( Q = 2p_T \) and \( p_T/2 \).
Summary

Determination of the optimum fragmentation functions for π, K, p in LO and NLO by a global analysis of e^++e^−→ h+X data.

- This is the first time that uncertainties of the fragmentation functions are estimated.
- Gluon and disfavored light-quark functions have large uncertainties.
  → The uncertainties could be important for discussing physics in
    \( \bar{p} + \bar{p} \rightarrow \pi^0 + X, \ A + A' \rightarrow h + X \) (RHIC, LHC), HERMES, JLab, ...
  → Need accurate data at low energies (Belle and BaBar).
- For the pion and kaon, the uncertainties are reduced in NLO in comparison with LO.
  For the proton, such improvement is not obvious.
- Heavy-quark functions are well determined.
- Code for calculating the fragmentation functions is available at http://research.kek.jp/people/kumanos/ffs.html.