Recent Developments in $\bar{B} \to X_s \gamma$
Motivation

- inclusive radiative $b \rightarrow s \gamma$ decay offers important precision tests of flavor sector in and beyond SM

- strong constraints on NP depend crucially on theoretical uncertainty of SM prediction

*Gambino & Misiak ’01
\[ \mathcal{B}(\bar{B} \to X_s \gamma)^{E_{\gamma}>1.6 \text{ GeV}} = \mathcal{B}(\bar{B} \to X_c e \bar{\nu}) \left[ \frac{\Gamma(b \to s \gamma)}{\Gamma(b \to c e \bar{\nu})} \right]_{\text{LO}} f \left( \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right) \]
\[
\times \left\{ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left( \frac{\Lambda^2}{m_b^2} \right) + \mathcal{O}\left( \frac{\Lambda^2}{m_c^2} \right) + \mathcal{O}\left( \frac{\alpha_s \Lambda}{m_b} \right) \right\}
\]
Effective Theory

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \ldots \]

\[ Q_{1,2} = (\bar{s} \Gamma_i c)(\bar{c} \Gamma'_i b) \]

\[ |C_{1,2}(m_b)| \approx 1 \]

\[ Q_{3-6} = (\bar{s} \Gamma_i b) \sum_q (\bar{q} \Gamma'_i q) \]

\[ |C_{3-6}(m_b)| < 0.07 \]

\[ Q_7 = \frac{e m_b}{16\pi^2} (\bar{s} L \sigma^{\mu\nu} b R) F_{\mu\nu} \]

\[ C_7(m_b) \approx -0.3 \]

\[ Q_8 = \frac{g m_b}{16\pi^2} (\bar{s} L \sigma^{\mu\nu} T^a b R) G^a_{\mu\nu} \]

\[ C_8(m_b) \approx -0.15 \]
Effective Theory

\[ L_{\text{eff}} = L_{\text{QCD}} \times \text{QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \ldots \]

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\[ Q_8 = \frac{g m_b}{16\pi^2} (\bar{s} L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a \]

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Error Budget at NLO

\[
B^{E_\gamma > 1.6 \text{ GeV}}_{\text{exp}} = \left( 3.55 \pm 0.24 \pm 0.09 \pm 0.03 \right) \times 10^{-4} \quad (*)
\]

\[
B^{E_\gamma > 1.6 \text{ GeV}}_{\text{NLO}} = (3.33 \pm 0.29) \times 10^{-4}, \quad m_c/m_b = 0.26
\]

- \(\alpha_s(M_Z)\) 25%
- scales 22%
- \(m_c, m_b\) 39%
- other 14%

\(*\) HFAG '06
Charm Quark Mass

- Charm quark mass dependence so pronounced since it first enters at NLO level
- Associated scheme ambiguity can only be resolved by NNLO calculation

\[
\frac{m_c}{m_b} = 0.22 \pm 0.04 \ (\text{MS})
\]

\[
\frac{m_c}{m_b} = 0.29 \pm 0.02 \ (\text{pole})
\]
Flavor of NNLO Calculation

- very involved task as $> 10^2$
  3-loop on-shell and $> 10^4$
  4-loop tadpole diagrams
  need to be computed
Flavor of NNLO Calculation

*Bobeth et al. ’00; Misiak & Steinhauser ’04
Gorbahn & UH ’04; Gorbahn et al. ’05;
Czakon et al. ’06
Bieri et al. ’03; Blokland et al. ’05;
Melnikov & Mitov ’05; Asatrian et al. ’05, ’06;
Misiak & Steinhauser ’06

.matching*
.running†
.matrix elements‡

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Melnikov & Mitov ’05; Asatrian et al. ’05, ’06;
Misiak & Steinhauser ’06
Results of 4-Loop Mixing

- numerical effect of $O(\alpha_s^2)$ mixing amounts to $-4.4\%$, $-2.9\%$ for $\mu_b=2.5, 5 \text{ GeV}$
- compared to accidentally small NLO mixing relatively large NNLO running

*$Czakon et al. ‘06*
Individual NNLO Contributions

\[ C_i = \sum_{n=0}^{2} \left( \frac{\alpha_s}{4\pi} \right)^n C_i^{(n)} \]

\[ \mathcal{B}_{\gamma\gamma > 1.6 \text{ GeV}}^{\text{NNLO}} \equiv \mathcal{B}_{\gamma\gamma > 1.6 \text{ GeV}}^{\text{NLO}} (1 + \delta_1 + \delta_2(r) + \delta_3(r)) \]

\[ r \equiv \frac{m_c(m_c)}{m_b^{1S}} \]

\[ \delta_1 \propto C_i^{(0)} C_j^{(2)}, C_i^{(1)} C_j^{(1)} \]

\[ \delta_2(r) \propto C_i^{(0)} C_j^{(0)} \]

\[ \delta_3(r) \propto C_i^{(0)} C_j^{(1)} \]

\[ \delta_2(r) = A n_f + B = -\frac{3}{2} \left( 11 - \frac{2}{3} n_f \right) A + \left( \frac{33}{2} A + B \right) \equiv \delta_2^{\beta_0}(r) + \delta_2^{\text{non-}\beta_0}(r) \]

• \( \delta_2^{\beta_0}(r) \) known for arbitrary \( r^* \) while

• \( \delta_2^{\text{non-}\beta_0}(r) \) only computed in limit \( r >> 1/2 \) so far\(^\dagger\)

*Bieri et al. ’03

\( ^\dagger \)Misiak & Steinhauser ’06
Interpolation in Charm Quark Mass

- Interpolate beyond large $\beta_0$ correction $\delta_2^{\text{non-}\beta_0}(r)$ by
  \[ a \frac{dB_{\text{NLO}}}{d \ln r} + b B_{\text{NLO}} + c \delta_2^{\beta_0}(r) + d \]

- Coefficients $a$, $b$, $c$, and $d$ are found from large $r$ behavior and requiring that correction vanishes at $r = 0$.

- Lower curve corresponds to $\delta_2^{\text{non-}\beta_0}(0) = 0$ upper one to $\delta_1 + \delta_2^{\text{non-}\beta_0}(0) + \delta_3(0) = 0$.

*Misiak & Steinhauser '06
Non-perturbative Effects

- Matrix elements of non-local operators promote corrections that scale like $\alpha_s \Lambda^3/m_b^3$ and $\alpha_s \Lambda^2/m_c^2$ in heavy quark limit to $\alpha_s \Lambda/m_b$

- Part involving $Q_7$ and $Q_8$ estimated in vacuum insertion approximation

\[
\frac{\Delta \Gamma_{VIA}^{78}}{\Gamma_{77}} = -\frac{2\pi \alpha_s}{9} \sum_{q=u,d} Q_q \frac{C_8}{C_7} \frac{f_B^2 m_B}{\lambda_B^2 m_b} \approx (-1.6 \pm 1.4)\%
\]

- Size of power corrections difficult to compute given present command of non-perturbative QCD on light cone

*Lee et al. ’06
First NNLO Estimate

\[ \mathcal{B}^{E_{\gamma}>1.6 \text{ GeV}}_{\text{NNLO}} = (3.15 \pm 0.23) \times 10^{-4} * \]

*Misiak et al. '06
Photon Energy Cut

• total rate cannot be measured

• at present experimental cut of $E_0 \geq 1.8$ GeV on photon energy

• with cut three relevant scales:
  $m_b \approx E_X$ (hard)
  $\sqrt{m_b \Delta} \approx M_X$ (jet)
  $\Delta = m_b - 2 E_0 \approx 1.5$ GeV (soft)

• operator product expansion in $\Lambda/\Delta$ instead of $\Lambda/m_b$

*BaBar ’06
†Neubert ’04
Second NNLO Estimate

\[ \mathcal{B}_{\text{NNLO}}^{E_{\gamma} > 1.6 \text{ GeV}} = \frac{F(1.6 \text{ GeV})}{F(1.0 \text{ GeV})} \mathcal{B}_{\text{NNLO}}^{E_{\gamma} > 1.0 \text{ GeV}} \]

\[ \frac{F(1.6 \text{ GeV})}{F(1.0 \text{ GeV})} = 0.93^{+0.03}_{-0.05\text{ per}}^{+0.02}_{-0.02\text{ had}}^{+0.02}_{-0.02\text{ par}} \]

- \( \Delta \approx 1.5 \text{ GeV}: \text{MSOPE NNLO}^* \)
- \( \Delta \approx 2.5 \text{ GeV}: \text{fixed order NNLO}^† \)

\[ \mathcal{B}_{\text{NNLO}}^{E_{\gamma} > 1.6 \text{ GeV}} = (2.98 \pm 0.26) \times 10^{-4} \]

- corrections at intermediate and soft scale lower BR value and increase theoretical uncertainty

*Becher & Neubert ’06
†Misiak et al. ’06
**$\bar{B} \to X_s\gamma$ in Universal Extra Dimensions**

- KK modes interfere destructively* with $b \to s\gamma$ SM amplitude

*Agashe et al. ’01; Buras et al. ’03
95% CL Bound on 1/R in UED

- lower limit 1/R > 600 GeV from $\bar{B} \to X_s \gamma$ independent of $m_h$

*Weiler & UH ’07
Conclusions and Outlook

- new NNLO results for $\bar{B} \to X_s \gamma$ reduce theoretical uncertainty to level of experimental one
- since SM predictions is now lower than WA more room in many models
- bounds on destructive NP become very strong

- NNLO charm quark contribution only known in approximation
- hard to estimate power corrections dominant source of error
- extrapolation in photon energy still based on old shape function model