Conformal Symmetry and the Standard Model

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Hierarchy Problem

- **Fact:** Standard Model (≡ SM) of elementary particle physics is conformally invariant except for tree level mass term $-m^2\Phi^\dagger\Phi \rightarrow$ masses for vector bosons (via *Brout-Englert-Higgs* mechanism), quarks and leptons.

- Quantum corrections $m^2 \sim \Lambda^2 \Rightarrow$ why $m_H \ll M_P$? (with UV cutoff $\Lambda =$ scale of ‘new physics’)
  - explanation of hierarchy?
  - stabilization of hierarchy?

- Most popular proposal: SM $\rightarrow$ MSSM or NMSSM: use supersymmetry to control quantum corrections via cancellation of quadratic divergences $\Rightarrow$
  $$m^2 \sim \Lambda_{SUSY}^2 \ln(\Lambda^2/\Lambda_{SUSY}^2)$$
Coleman-Weinberg Mechanism (1973)

- Idea: spontaneous breaking of conformal symmetry by quantum corrections \( \Rightarrow \) small mass scales arise via conformal anomaly and effective potential

\[
\lambda \phi^4 \rightarrow V_{\text{eff}}(\phi) = \lambda \phi^4 + \frac{\lambda^2 \phi^4}{64\pi^2} \left( \frac{\lambda \phi^2}{v^2} \right) - \frac{1}{2}
\]

- In its original form this proposal does not work:
  - Higgs mass too small (\( \sim \mathcal{O}(10\text{ GeV}) \)), or
  - Scalar self-couplings too large \( \rightarrow \) Landau poles


- Also, must accommodate:
  - \( m_H > 115\text{ GeV} \) and \( m_{\text{top}} = 178\text{ GeV} \)
  - \( m_\nu < 1\text{ eV} \) \( \rightarrow \) large intermediate scale?
Our Proposal [\(\text{\hspace{1cm}}\text{\texttt{\textsc{hep-th/0612165}}\text{\hspace{1cm}}}\)]

- Classical conformal symmetry (i.e. no tree level mass terms) in SM
- plus right-chiral neutrinos
- plus enlarged scalar sector: \(\Phi\) and \(\varphi\)
- All mass scales from effective (CW) potential
- all coupling constants small and positive up to \(M_{Pl}\)
- No large intermediate scales \(\Rightarrow\)
  - no grand unification (GUTs)
  - no new scales required to explain \(m_\nu < 1\) eV
- No low energy SUSY
The Model

- Start from conformally invariant (and therefore renormalizable) Lagrangian $\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}'$ with:

$$ \mathcal{L}' = \left( L^i \Phi Y^E_{i} E^j + Q^i \epsilon \Phi * Y^D_{ij} D^j + Q^i \epsilon \Phi * Y^U_{ij} U^j + \right.$$ 

$$+ \bar{L}^i \epsilon \Phi * Y^\nu_{ij} \nu^j_R + \varphi \nu^i_R C Y^M_{ij} \nu^j_R + \text{h.c.}\right) - \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \varphi^2 (\Phi^\dagger \Phi) - \frac{\lambda_3}{4} \varphi^4 $$

[See also: Shaposhnikov, Tkachev, PLB639(2006)104; Nishino, Rajpoot, hep-th/0403039]

- Besides usual $SU(2)$ doublet $\Phi$: new scalar field $\varphi$ (here taken to be real, but could be complex and in non-trivial representation of family symmetry)

- No mass terms, all coupling constants dimensionless

- $Y^U_{ij}$, $Y^E_{ij}$, $Y^M_{ij}$ real and diagonal

- $Y^D_{ij}$, $Y^\nu_{ij}$ complex → parametrize family mixing
Effective Potential

\[ V_{\text{eff}}(H, \varphi) = \frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2 \varphi^2}{2} + \frac{\lambda_3 \varphi^4}{4} \]

\[ + \frac{3}{256\pi^2}(\lambda_1 H^2 + \lambda_2 \varphi^2)^2 \ln \left[ \frac{\lambda_1 H^2 + \lambda_2 \varphi^2}{v^2} \right] \]

\[ + \frac{1}{64\pi^2} F_+^2 \ln \left[ \frac{F_+}{v^2} \right] + \frac{1}{64\pi^2} F_-^2 \ln \left[ \frac{F_-}{v^2} \right] \]

\[ - \frac{6}{32\pi^2} g^4_i (H^2)^2 \ln \left[ \frac{H^2}{v^2} \right] - \frac{1}{32\pi^2} g_M^4 \varphi^4 \ln \left[ \frac{\varphi^2}{v^2} \right] \]

\[ g_M^4 \equiv \text{Tr} Y_M^4, \ v \text{ dimensionful scale (conformal anomaly)} \]

Not included: contributions from SU(2)_w \times U(1)_Y gauge fields (because respective gauge couplings are small), nor from SU(3)_c (because it is a two-loop effect)

Search for minima: must perform numerical analysis!
Numerics

Choice of parameters is *strongly constrained* by experimental data and RGE analysis → ‘trial and error’ method leads to following choice of parameters

\[ \lambda_1 = 3.4, \quad \lambda_2 = 2.6, \quad \lambda_3 = 3.3, \quad g_t = 1, \quad g_M^2 = 0.4 \]

Minimum lies at

\[ \langle H \rangle = 0.415 \cdot 10^{-5} \nu, \quad \langle \varphi \rangle = 2.506 \cdot 10^{-5} \nu \]

Normalize this by setting \( \langle H \rangle = 174 \text{ GeV} \quad \Rightarrow \)

\[ H' = H \cos \beta + \varphi \sin \beta, \quad \varphi' = -H \sin \beta + \varphi \cos \beta \]

\[ m_{H'} = 217 \text{ GeV}, \quad m_{\varphi'} = 439 \text{ GeV}; \quad \sin \beta = 0.119 \]

‘Higgs mixing’: only the components along \( H \) of the mass eigenstates couple to the usual SM particles.

NB: Not (yet?) a definitive prediction.
Renormalization Group Equations

Effective couplings (≡ 4th derivatives at minimum):

\[
\lambda_1^{\text{eff}} = 1.463, \quad \lambda_2^{\text{eff}} = 0.348, \quad \lambda_3^{\text{eff}} = 0.626
\]

With

\[
y_1 = \frac{\lambda_1^{\text{eff}}}{4\pi^2}, \quad y_2 = \frac{\lambda_2^{\text{eff}}}{4\pi^2}, \quad y_3 = \frac{\lambda_3^{\text{eff}}}{4\pi^2}, \quad x = \frac{g_i^2}{4\pi^2}, \quad u = \frac{g_M^2}{4\pi^2}, \quad z = \frac{\alpha_s}{\pi}
\]

we get

\[
\begin{align*}
\mu \frac{dy_1}{d\mu} &= \frac{3}{2} y_1^2 + \frac{1}{8} y_2^2 - 6x^2, \\
\mu \frac{dy_2}{d\mu} &= \frac{3}{8} y_2(2y_1 + y_3 + \frac{4}{3} y_2), \\
\mu \frac{dy_3}{d\mu} &= \frac{9}{8} y_3^2 + \frac{1}{2} y_2^2 - u^2, \quad \mu \frac{du}{d\mu} = \frac{3}{4} u^2, \\
\mu \frac{dx}{d\mu} &= \frac{9}{4} x^2 - 4xz, \quad \mu \frac{dz}{d\mu} = -\frac{7}{2} z^2.
\end{align*}
\]
Evolution of Coupling Constants

\[ \frac{\lambda_1}{4\pi^2} \]

\[ \frac{\lambda_2}{4\pi^2} \]
Evolution of Coupling Constants

Important: top Yukawa stays bounded up to $M_{Pl}$ due to large $\alpha_s$!
Evolution of Coupling Constants

\[ \frac{YM^2}{4\pi^2} \]

\[ \frac{\alpha_s}{\pi} \]
**Phenomenology**

- New scalar is a 'heavier brother' of the SM Higgs (the same BRs except for the mass difference and lower cross sections)

- **if** $m_\phi > 2m_Z$ a clear signal in LHC

- no other new particles at LHC except standard Higgs and a new scalar (scalars?)
Discussion and outlook

● conformal symmetry is a different (much simpler than SUSY...) explanation of hierarchy of scales

● SM with massive neutrinos and the new scalar can be viable up to the Planck scale so LHC may see just the SM Higgs and (several?) new scalar particles...

● LHC will tell...