Phenomenology of the rare decay $\bar{B} \to X_s \ell^+ \ell^-$

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General Features of $\bar{B} \to X_s \ell^+ \ell^-$

- Rare decay, FCNC process, probes SM directly at one-loop level
- Sensitive to new physics beyond the SM
- A complementary SM test to $\bar{B} \to X_s \gamma$
- Precision in both experiment and theory needed and achievable
General features of $B \to X_s \ell^+ \ell^-$

- Differential decay width: $(q^2 : \text{lepton inv. mass}; \hat{s} \equiv q^2/m_b^2)$

\[
\frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu) (1 - \hat{s})^2}{768\pi^5} \\
\times \left\{ (4 + \frac{8}{\hat{s}}) |\tilde{C}_7^{\text{eff}}|^2 + (1 + 2\hat{s})(|\tilde{C}_9^{\text{eff}}|^2 + |\tilde{C}_{10}^{\text{eff}}|^2) + 12 \text{Re}(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\ast \text{eff}}) + \frac{d\Gamma^{\text{brems}}}{d\hat{s}} \right\}
\]

- Compare to:
  \[\Gamma(\bar{B} \to X_s \gamma) \propto |\tilde{C}_7^{\text{eff}}|^2\]

- SM size and signs of amplitudes
  - $\tilde{C}_7^{\text{eff}} \simeq -0.30$
  - $\tilde{C}_9^{\text{eff}} \simeq +4.05$
  - $\tilde{C}_{10}^{\text{eff}} \simeq -4.26$

[Akeroyd et. al.]
Forward backward asymmetry

Forward backward asymmetry:

\[ \mathcal{A}_{FB}(q^2) = \frac{dBR_{\ell\ell}/dq^2(\cos \theta_{l} > 0) - dBR_{\ell\ell}/dq^2(\cos \theta_{l} < 0)}{dBR_{\ell\ell}/dq^2(\cos \theta_{l} > 0) + dBR_{\ell\ell}/dq^2(\cos \theta_{l} < 0)} \]

\[ \mathcal{A}_{FB}(\hat{s}) = \frac{G_F^2 m_b^5 |V_{ts}^* V_{tb}|^2 \alpha_{em}^2(\mu) (1 - \hat{s})^2}{768 \pi^5} \times \left\{ -6 \Re(\tilde{C}_{7,F B}^{\text{eff}} \tilde{C}_{10,F B}^{\ast \text{eff}}) - 3\hat{s} \Re(\tilde{C}_{9,F B}^{\text{eff}} \tilde{C}_{10,F B}^{\ast \text{eff}}) + A_{FB}^{\text{brems}} \right\} \]

Zero of FBA represents SM precision observable (theor. uncertainty \( \sim 5\% \))

A measurement of \( dBR_{\ell\ell}/d\hat{s} \) and \( \mathcal{A}_{FB}(\hat{s}) \) can provide information on the sign of \( \tilde{C}_7^{\text{eff}} \), which again will allow to constrain parameter space of new physics models.

[Ali,Greub,Hiller,Lunghi, 1: \( C_{10}=-C_{10,SM} \), 2: \( C_7=-C_7,SM \), 3: \( C(7,10)=-C(7,10,SM) \)]
Effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} \times \mathcal{L}_{\text{QED}} (u, \ldots, b, e, \mu, \tau) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \cdot \left[ \sum_{i=1}^{10} C_i P_i + \sum_{i=3}^{6} C_i Q P_i Q + C_b P_b \right] \]

\[ \text{for QED corrections} \]

\[ P_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L), \]
\[ P_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q), \]
\[ P_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L), \]
\[ P_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\sigma q), \]
\[ P_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma_\mu q), \]
\[ P_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\sigma T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\sigma T^a q), \]
\[ P_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \]
\[ P_8 = \frac{g}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a, \]
\[ P_9 = (\bar{s}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma_\mu l), \]
\[ P_{10} = (\bar{s}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma_\mu \gamma_5 l), \]
Effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}}(u, \ldots, b, e, \mu, \tau) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \cdot \left[ \sum_{i=1}^{10} C_i P_i + \sum_{i=3}^{6} C_i Q P_{iQ} + C_b P_b \right] \]

for QED corrections

\[ P_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), \quad P_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q), \]

\[ P_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L), \quad P_5 = (\bar{s}_L \gamma_\mu \gamma^\nu \gamma^\sigma b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma q), \]

\[ P_3 = (\bar{s}_L \gamma_\mu b_L) \sum_q (\bar{q} \gamma^\mu q), \quad P_6 = (\bar{s}_L \gamma_\mu \gamma^\nu \gamma^\sigma T^a b_L) \sum_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma T^a q), \]

\[ P_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \quad P_9 = (\bar{s}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma^\mu l), \]

\[ P_8 = \frac{q}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}, \quad P_{10} = (\bar{s}_L \gamma_\mu b_L) \sum_l (\bar{l} \gamma^\mu \gamma_5 l), \]

\[ P_{3Q} = (\bar{s}_L \gamma_\mu b_L) \sum_q Q_q (\bar{q} \gamma^\mu q), \]

\[ P_{4Q} = (\bar{s}_L \gamma_\mu T^a b_L) \sum_q Q_q (\bar{q} \gamma^\mu T^a q), \]

\[ P_{5Q} = (\bar{s}_L \gamma_\mu \gamma^\nu \gamma^\sigma b_L) \sum_q Q_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma q), \]

\[ P_{6Q} = (\bar{s}_L \gamma_\mu \gamma^\nu \gamma^\sigma T^a b_L) \sum_q Q_q (\bar{q} \gamma^\mu \gamma^\nu \gamma^\sigma T^a q), \]

\[ P_b = \frac{1}{12} \left[ (\bar{s}_L \gamma_\mu \gamma^\nu \gamma^\sigma b_L)(\bar{b} \gamma^\mu \gamma^\nu \gamma^\sigma b) - 4(\bar{s}_L \gamma_\mu b_L)(\bar{b} \gamma^\mu b) \right]. \]
Perturbative and non-pert. Corrections

QCD corrections to quark level decay rate are known to NNLO

[Refs: Misiak, Buras, Münz, Bobeth, Urban, Asatrian, Asatryan, Greub, Walker]
[Bobeth, Gambino, Gorbahn, Haisch, Bieri, Ghinculov, Hurth, Isidori, Yao]

Diagrams involved:

Order $\mathcal{O}(\alpha_s^2)$ ME of $P_9$ for high-$q^2$ region now also known analytically

[Refs: Blokland, Czarnecki, Melnikov, Slusarczyk]

$1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections

[Bauer, Burrell, Buchalla, Isidori, Rey]

Impact in the low-$q^2$ region is at the few percent level

Large impact in the high-$q^2$ region due to breakdown at the endpoint

Factorizable $c\bar{c}$ contributions implemented via the KS approach

[Refs: Krüger, Sehgal]
QED Corrections

NLO QED corrections

- are expected to be larger than N^3LO QCD corrections.
- reduce ±4% scale uncertainty due to
  \[ \alpha_e(m_b) \approx \frac{1}{133} \quad \text{vs.} \quad \alpha_e(m_Z) \approx \frac{1}{128}. \]
- This ±4% uncertainty is as large as NNLO QCD precision.

Calculation of NLO QED corrections is threefold

- Matching and running calculation;
  checked to agree with [Bobeth, Gambino, Gorbahn, Haisch]

- Finite corrections, matrix elements of the \( P_i \)

- Phenomenologically most important: IR divergent ME's

\[
\int dPS_3 \left| \begin{array}{c}
\bar{b} \quad b \\
\gamma \quad \gamma
\end{array} \right| P_9 \quad + \quad 6 \times \left| \begin{array}{c}
\bar{b} \quad b \\
\gamma \quad \gamma
\end{array} \right| P_9 \quad + \quad \text{UV c.t.} \quad + \quad \int dPS_4 \left| \begin{array}{c}
\bar{b} \quad b \\
\gamma \quad \gamma
\end{array} \right| P_9
\]
NLO QED Matrix Elements

Contrary to the integrated decay width, the differential decay width is not an infrared
safe object with respect to the emission of collinear photons from lepton lines.

Collinear divergence manifests itself through large EM logarithm

$$\frac{\alpha_e}{4\pi} \left[ \log \left( \frac{m_b^2}{m_l^2} \right) \cdot h(\hat{s}) + k(\hat{s}) \right]$$

with

$$\int_0^1 d\hat{s} \ h(\hat{s}) = 0.$$

Plot: Log-enhanced contribution to $|\langle P_9 \rangle|^2$

Relative effect of this log is much larger for high $q^2$ than for low $q^2$.

Include also log-enhanced corrections to $|\langle P_7 \rangle|^2$, $|\langle P_{10} \rangle|^2$, $Re[\langle P_i \rangle \langle P_j \rangle^*]$.

Presence of $\log \left( \frac{m_b^2}{m_l^2} \right)$ depends on exptl. setup (finite detector resolution)

Separation of muons and collinear photons is no problem

$e^-$: inside an angle $\theta_c$ collinear $\gamma$'s are included in 4-momentum

$$q^2 = (p_+ + p_\gamma)^2 \quad m^2_\ell \leq (p_\ell + p_\gamma)^2 \leq \Lambda^2 \simeq 2E_\ell^2 (1 - \cos \theta_c) \quad \Lambda \sim \mathcal{O}(m_\mu)$$

[Berryhill, Ishikawa]
**Results: BR in low-$q^2$-region**

- Including all NLO-QED corrections:  
  \[
  BR(\bar{B} \to X_s \mu\mu) = (1.59 \pm 0.08_{\text{scale}} \pm 0.06_{m_t} \pm 0.015_{m_b} \\
  \pm 0.024_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\text{BR}_{sl}}) \cdot 10^{-6} = (1.59 \pm 0.11) \cdot 10^{-6}
  \]
  \[
  BR(\bar{B} \to X_{s\ell\ell}) = (1.64 \pm 0.08_{\text{scale}} \pm 0.06_{m_t} \pm 0.015_{m_b} \\
  \pm 0.025_{C,m_c} \pm 0.02_{\alpha_s(M_Z)} \pm 0.015_{CKM} \pm 0.026_{\text{BR}_{sl}}) \cdot 10^{-6} = (1.64 \pm 0.11) \cdot 10^{-6}
  \]

- Experimental values:
  - \[
  BR(\bar{B} \to X_{s\ell\ell}) = (1.493 \pm 0.504_{\text{stat.}}^{+0.411}_{-0.321_{\text{sys.}}}) \cdot 10^{-6} \quad \text{[Belle, 152 M evts.]} 
  \]
  - \[
  BR(\bar{B} \to X_{s\ell\ell}) = (1.8 \pm 0.7_{\text{stat.}} \pm 0.5_{\text{sys.}}) \cdot 10^{-6} \quad \text{[BaBar, 89 M events]} 
  \]
  - weighted average: \((1.60 \pm 0.51) \cdot 10^{-6}\)

- With reversed sign of $\tilde{C}_7^{\text{eff}}$:
  - \[
  BR(\bar{B} \to X_s \mu\mu) = 3.11 \cdot 10^{-6} 
  \]
  - \[
  BR(\bar{B} \to X_s eee) = 3.19 \cdot 10^{-6} 
  \]
  - SM-sign of $\tilde{C}_7^{\text{eff}}$ is favored
  - Models with positive $\tilde{C}_7^{\text{eff}}$ require sizable contributions to $\tilde{C}_9^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$

[Refs: Lunghi, Misiak, Wyler, TH; Belle, 152 M evts.; BaBar, 89 M events; Gambino, Haisch, Misiak]
**BR in high-\(q^2\)-region**

Branching ratio integrated over \(q^2 > 14.4\) GeV\(^2\)  

\[
\mathcal{B}_{\mu\mu}^{\text{high}} = 2.40 \times 10^{-7} \left( 1 + \left[ \begin{array}{c} +0.02 \end{array} \right]_0 + \left[ \begin{array}{c} +0.06 \end{array} \right]_b \pm 0.02 m_t + \left[ \begin{array}{c} +0.006 \end{array} \right] C, \pm 0.05 m_b \\
+ \left[ \begin{array}{c} +0.002 \end{array} \right] \alpha_s \pm 0.002_{\text{CKM}} \pm 0.02_{\text{BR_{sl}}} \pm 0.05 \lambda_2 \pm 0.19 \rho_1 \pm 0.14 f_s \pm 0.02 f_u \right)
\]

\[
= 2.40 \times 10^{-7} \left( 1^{+0.29}_{-0.26} \right)
\]

\[
\mathcal{B}_{ee}^{\text{high}} = 2.09 \times 10^{-7} \left( 1 + \left[ \begin{array}{c} +0.04 \end{array} \right]_0 + \left[ \begin{array}{c} +0.08 \end{array} \right]_b \pm 0.02 m_t + \left[ \begin{array}{c} +0.005 \end{array} \right] C, \pm 0.05 m_b \\
+ \left[ \begin{array}{c} +0.002 \end{array} \right] \alpha_s \pm 0.002_{\text{CKM}} \pm 0.02_{\text{BR_{sl}}} \pm 0.05 \lambda_2 \pm 0.22 \rho_1 \pm 0.16 f_s \pm 0.02 f_u \right)
\]

\[
= 2.09 \times 10^{-7} \left( 1^{+0.32}_{-0.30} \right)
\]

Largest uncertainties from poorly known \(O(1/m_b^3)\) power corrections

Impact of EM logs is \(-8\% \) (\(\mu\)) and \(-20\% \) (\(e\)), that of KS res. \(-10\% \) (\(\mu\)) and \(-12\% \) (\(e\))

Experimental values:

- \(BR(\bar{B} \to X_s ll) = (4.18 \pm 1.17_{\text{stat.}}^{+0.61}_{-0.68_{\text{sys.}}}) \cdot 10^{-7} \)  
  \[\text{[Belle, 152 M evts.]}\]

- \(BR(\bar{B} \to X_s ll) = (5 \pm 2.5_{\text{stat.}}^{+0.8}_{-0.7_{\text{sys.}}}) \cdot 10^{-7} \)  
  \[\text{[BaBar, 89 M events]}\]
The ratio $\mathcal{R}(s_0)$

Introduction of the ratio

$$\mathcal{R}(s_0) = \frac{\int_{s_0}^{1} d\hat{s} \, d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{s_0}^{1} d\hat{s} \, d\Gamma(\bar{B}^0 \to X_u \ell \nu)/d\hat{s}}$$

Normalize to semileptonic $\bar{B}^0 \to X_u \ell \nu$ rate with the same cut

Impact of non-perturbative $1/m_b^2$ and $1/m_b^3$ power corrections drastically reduced

For lower integration limit $s_0 = 14.4$ GeV$^2$ one obtains

$$\mathcal{R}(s_0)_{\mu\mu} = 2.29 \times 10^{-3} \left( 1 \pm 0.04_{\text{scale}} \pm 0.02 m_t \pm 0.01 C, m_c \pm 0.006 m_b \pm 0.005 \alpha_s \\
\quad \pm 0.09_{\text{CKM}} \pm 0.003 \lambda_2 \pm 0.05 \rho_1 \pm 0.03 \left( f_0^u + f_s \right) \pm 0.05 \left( f_0^u - f_s \right) \right) \approx 2.29 \times 10^{-3} (1 \pm 0.13)$$

$$\mathcal{R}(s_0)_{ee} = 1.94 \times 10^{-3} \left( 1 \pm 0.06_{\text{scale}} \pm 0.02 m_t \pm 0.02 C, m_c \pm 0.004 m_b \pm 0.006 \alpha_s \\
\quad \pm 0.09_{\text{CKM}} \pm 0.01 \lambda_2 \pm 0.09 \rho_1 \pm 0.05 \left( f_0^u + f_s \right) \pm 0.05 \left( f_0^u - f_s \right) \right) \approx 1.94 \times 10^{-3} (1 \pm 0.16)$$

Uncertainties from poorly known $O(1/m_b^3)$ power corrections are under control

Largest source of error is $V_{ub}$

Impact of EM logs is $-9\% (\mu)$ and $-23\% (e)$, that of KS reson. $-11\% (\mu)$ and $-12\% (e)$
Zero of FBA and integrated FBA

\[(q_0^2)_{\mu\mu} = \left[ 3.50 \pm 0.10_{\text{scale}} \pm 0.002 m_t \pm 0.04 m_c, C \pm 0.05 m_b \pm 0.03 \alpha_s(M_Z) \pm 0.001 \lambda_1 \pm 0.01 \lambda_2 \right] \text{GeV}^2\]

\[= (3.50 \pm 0.12) \text{GeV}^2\]

\[(q_0^2)_{ee} = \left[ 3.38 \pm 0.09_{\text{scale}} \pm 0.002 m_t \pm 0.04 m_c, C \pm 0.04 m_b \pm 0.03 \alpha_s(M_Z) \pm 0.002 \lambda_1 \pm 0.01 \lambda_2 \right] \text{GeV}^2\]

\[= (3.38 \pm 0.11) \text{GeV}^2\]

Integrated FBA: Pronounced sensitivity to NP

Bin 1 \((q^2 \in [1, 3.5] \text{ GeV}^2)\)

\((\bar{A}_{\mu\mu})_{\text{bin1}} = [-9.09 \pm 0.91] \%\)

\((\bar{A}_{ee})_{\text{bin1}} = [-8.14 \pm 0.87] \%\)

Bin 2 \((q^2 \in [3.5, 6] \text{ GeV}^2)\)

\((\bar{A}_{\mu\mu})_{\text{bin2}} = [+7.80 \pm 0.76] \%\)

\((\bar{A}_{ee})_{\text{bin2}} = [+8.27 \pm 0.69] \%\)

low-\(s\) \((q^2 \in [1, 6] \text{ GeV}^2)\)

\((\bar{A}_{\mu\mu})_{\text{low}} = [-1.50 \pm 0.90] \%\)

\((\bar{A}_{ee})_{\text{low}} = [-0.86 \pm 0.85] \%\)
Recent proposal: 3rd independent combination of Wilson Coefficients: \((z = \cos \theta)\)

\[
\frac{d^2 \Gamma}{dq^2 \ dz} = \frac{3}{8} \left[ (1 + z^2) \, H_T(q^2) + 2 \, z \, H_A(q^2) + 2 \left( 1 - z^2 \right) \, H_L(q^2) \right]
\]

Note: \(
\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) , \quad \frac{dA_{FB}}{dq^2} = \frac{3}{4} H_A(q^2)
\)

Current data extrapolated to 1ab\(^{-1}\).

Size of \(C_7 (< 0)\) taken from \(\bar{B} \rightarrow X_s \gamma\). Constraints in \(C_9\)-\(C_{10}\) plane:
Summary and Outlook

- $\bar{B} \to X_s \ell^+ \ell^-$ serves as a precision test for the SM and is a sensitive probe for new physics.

- Higher order perturbative corrections are indispensable for obtaining precision data to test the SM.

- Together with $\bar{B} \to X_s \gamma$, all relevant WC’s can be predicted within the SM and constraints on NP can be set.

- FCNC precision observables serve as an important ingredient for synergy and complementarity between flavour and collider physics.
Backup slides
**Numerical inputs**

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<th>Value</th>
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<td>$\alpha_s(M_z)$</td>
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<td>$\alpha_e(M_z)$</td>
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<td>V_{ts}V_{tb}/V_{ub}</td>
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<td>$f_u^\pm$</td>
<td>$(0 \pm 0.4)$ GeV$^3$</td>
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Organizing the Expansion

Amplitude: \[ \kappa = \frac{\alpha_e(\mu_b)}{\alpha_s(\mu_b)} \]
\[ A = \kappa \left[ A_{LO} + \tilde{\alpha}_s A_{NLO} + \tilde{\alpha}_s^2 A_{NNLO} + O(\tilde{\alpha}_s^3) \right] \]
\[ + \kappa^2 \left[ A_{em}^{LO} + \tilde{\alpha}_s A_{em}^{NLO} + \tilde{\alpha}_s^2 A_{em}^{NNLO} + O(\tilde{\alpha}_s^3) \right] + O(\kappa^3) \]

Decay width:
\[ |A|^2 = \kappa^2 \left[ A_{LO}^2 + 2 A_{LO} A_{NLO} + A_{NLO}^2 \right] \quad \Leftarrow \text{QCD, NLO} \]
\[ \text{QCD, NNLO} \Rightarrow + \kappa^2 \left[ 2 \tilde{\alpha}_s^2 A_{LO} A_{NNLO} + 2 \tilde{\alpha}_s^3 A_{NLO} A_{NNLO} + \ldots \right] \]
\[ \text{QED} \Rightarrow + \kappa^3 \left[ 2 A_{LO} A_{em}^{LO} + 2 \tilde{\alpha}_s \left( A_{NLO} A_{em}^{NLO} + A_{LO} A_{em}^{NLO} \right) \right. \]
\[ \left. + 2 \tilde{\alpha}_s^2 \left( A_{NLO} A_{em}^{NLO} + A_{NNLO} A_{LO}^2 + A_{LO} A_{em}^{NNLO} \right) \right. \]
\[ \left. + 2 \tilde{\alpha}_s^3 \left( A_{NLO} A_{em}^{NNLO} + A_{NNLO} A_{em}^{NNLO} \right) + \ldots \right] \]

Accidentally: \[ A_{LO} \sim \tilde{\alpha}_s A_{NLO} \text{ and } A_{em}^{LO} \sim \tilde{\alpha}_s A_{em}^{NLO} \]
\[ \Rightarrow \text{quite high terms in the expansion remain numerically important} \]
Definition of observables

\[ \frac{d\mathcal{B}(\bar{B} \to X_s\ell^+\ell^-)}{d\hat{s}} = BR_{b \to c e \nu}^{\exp.} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{C} \frac{d\Gamma(\bar{B} \to X_s\ell^+\ell^-)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})} \]

Normalize the differential decay width and bare FBA to the semilep. \( \bar{B} \to X_u e \bar{\nu} \) rate

- Removes pre-factor \( m_{b,\text{pole}}^5 \) and avoids phase space factors involving \( m_{c,\text{pole}} \)

\[ C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \to X_c e \bar{\nu})}{\Gamma(\bar{B} \to X_u e \bar{\nu})} = 0.58 \pm 0.01 \]

- We assume 100% correlation between the errors on \( C \) and \( m_c \)

\[ \frac{dA_{FB}(\bar{B} \to X_s\ell^+\ell^-)}{d\hat{s}} = BR_{b \to c e \nu}^{\exp.} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{C} \frac{d\Gamma^{\text{FB}}(\bar{B} \to X_s\ell^+\ell^-)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})} \]

\[ \frac{d\Gamma^{\text{FB}}(\bar{B} \to X_s\ell^+\ell^-)}{d\hat{s}} = \int_{-1}^{1} dz \text{sgn}(z) \frac{d^2\Gamma(\bar{B} \to X_s\ell^+\ell^-)}{d\hat{s} \, dz} \quad (z = \cos \theta_{\ell}) \]
Definition of observables

Total Forward backward asymmetry: \[ \frac{dA_{FB}(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} \] / \[ \frac{dB(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}} \]

- Each of the brackets is normalized to \( \Gamma_u \) and gets fully expanded in the couplings, but no overall expansion is done:
- No expansion of badly converging \( d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s} \)
- Closest to experiment
- Small scheme dependence on \( b \) quark mass due to removal of \( m_{b,\text{pole}}^5 \)-factor
- Stable, well-converging perturbative expansion
- Each for the square brackets alone is made renormalon-free and is observable

Introduction of the ratio \( R(s_0) = \frac{\int_{\hat{s}_0}^{1} d\hat{s} \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}}}{\int_{\hat{s}_0}^{1} d\hat{s} \frac{d\Gamma(\bar{B}^0 \to X_u \ell \nu)}{d\hat{s}}} \) [Ligeti, Tackmann]

- Normalize to semileptonic \( \bar{B}^0 \to X_u \ell \nu \) rate with the same cut
- Impact of non-perturbative \( 1/m_b^2 \) and \( 1/m_b^3 \) power corrections drastically reduced
More on sign of $\tilde{C}^{\text{eff}}_7$

Model-independent constraints on additive new physics contributions to $\tilde{C}^{\text{eff}}_{9,10}$ at 90% C.L.

- Extract bounds on $|\tilde{C}^{\text{eff}}_7|$ from $B(\bar{B} \to X_s\gamma) = (3.52 \pm 0.70) \cdot 10^{-4}$
- Use bounds on low $q^2$-region from $B(\bar{B} \to X_sll) = (1.60 \pm 0.90) \cdot 10^{-6}$
  to extract allowed region for $\tilde{C}^{\text{eff}}_{9,10}$
- Regions outside the rings are excluded
- Dot at the origin indicates the SM case for $\tilde{C}^{\text{eff}}_{9,10}$.

SM-like sign of $\tilde{C}^{\text{eff}}_7$

[Gambino,Haisch,Misiak]
More on sign of $\tilde{C}_{7}$

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[Extract bounds on $|\tilde{C}_{7}|$ from $B(\bar{B} \rightarrow X_s\gamma) = (3.52 \pm 0.70) \cdot 10^{-4}$]

[Gambino, Haisch, Misiak]
More on sign of $\tilde{C}_{7}^{\text{eff}}$

- Dashed cross:
  Maximal MFV MSSM contributions to $\tilde{C}_{9,10}^{\text{eff}}$
  \[ \text{[Ali,Lunghi,Greub,Hiller]} \]

- Dashed cross too small to reach border of allowed region

- Extensions of SM with reversed sign of $\tilde{C}_{7}^{\text{eff}}$ but only small corrections to $\tilde{C}_{9,10}^{\text{eff}}$ are disfavored

- Models with positive $\tilde{C}_{7}^{\text{eff}}$ require sizable contributions to $\tilde{C}_{9}^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$

Enlarged surroundings of the origin.
opposite sign of $\tilde{C}_{7}^{\text{eff}}$

[\text{Gambino,Haisch,Misiak}]
Numerical values for $\tilde{\alpha}_s(\mu_b)$ and $\kappa(\mu_b)$ with $\mu_b = 5$ GeV

- $\tilde{\alpha}_s(\mu_b) = 0.0170$
- $\kappa(\mu_b) = 0.0354$
- $\tilde{\alpha}_{em}(\mu_b) \ln(m^2_b/m^2_e) = 0.011$