The $f_{Ds}$ Puzzle

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based on

*Accumulating Evidence for Nonstandard Leptonic Decays of $D_s$ Mesons*


with Bogdan Dobrescu

2008 Rencontres de Moriond: QCD and High-energy Interactions
Context
\( D_s \rightarrow l \nu \)

- The leptonic decay \( D_s \rightarrow l \nu \) has been advertised as a good test of lattice QCD.
- Counting experiment at CLEO, B factories.
- A simple matrix element \( \langle 0 | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle \)
- No light valence quarks.
- New physics thought to be very unlikely.
The Decay

• In Standard Model the branching fraction is

\[ B(D_s \to \ell \nu) = \frac{m_{D_s} \tau_{D_s}}{8\pi} f_{D_s}^2 |G_F V_{cs} m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 \]

where the decay constant \( f_{D_s} \) is defined by

\[ \langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle = i f_{D_s} p_\mu \]

• Experiments assume SM & quote \( f_{D_s} \).
Something funny happened ...
Something funny happened ...

a $3.8\sigma$ discrepancy, or $2.7\sigma \oplus 2.9\sigma$. 
A Puzzle

- Excluding BaBar [Rosner, Stone], it is $3.5\sigma$; including the old experiments, it is $4.1\sigma$.
- What is the origin of the discrepancy?
  - experiments or radiative corrections
  - lattice QCD
  - non-Standard phenomena
• The $\mu\nu$ final state is helicity suppressed,
\[
\frac{m^2_\mu}{m^2_{D_s}} = 2.8 \times 10^{-3}
\]

• The $\tau\nu$ final state is phase-space suppressed
\[
\left(1 - \frac{m^2_\tau}{m^2_{D_s}}\right)^2 = 3.4 \times 10^{-2}
\]
Experiments

• Measurements by BaBar, CLEO, Belle do not depend on models* for interpretation of the central value or the error bar.

• CLEO and Belle have absolute $B(D_s \to l\nu)$.

• Hard to see a misunderstood systematic.

• Could all fluctuate high?

• * except the Standard Model!
• Experiments take $|V_{cs}|$ from 3-generation unitarity, either with PDG’s global CKM fit or setting $|V_{cs}| = |V_{ud}|$. No difference.

• Even $n$-generation CKM requires $|V_{cs}| < 1$; would need $|V_{cs}| > 1.1$ to explain effect.
Radiative Corrections

- Fermi constant from muon decay, so its radiative corrections implicit in $\mu\nu$ and $\tau\nu$.

- Standard treatment [Marciano & Sirlin] has a cutoff, set (for $f_\pi$) to $m_\rho$. Only 1–2%.

- More interesting is $D_s \rightarrow D_s^*\gamma \rightarrow \mu\nu\gamma$, which is not helicity suppressed. Applying CLEO’s cut 1% for $\mu\nu$ [Burdman, Goldman, Wyler].

- Only 9.3 MeV kinetic energy in $D_s \rightarrow \tau\nu$. 
Lattice QCD
2+1 Sea Quarks

• There are two calculations of $f_{D_s}$ with 2+1 flavors of sea quarks:

  $f_{D_s} = 249 \pm 3 \pm 16 \text{ MeV, \; hep-lat/0506030}$
  $f_{D_s} = 241 \pm -\pm 03 \text{ MeV, \; 0706.1726 [hep-lat]}$

• Compared with experimental averages:

  $f_{D_s} = 277 \pm 09 \text{ MeV, \; } \ell\nu$
  $f_{D_s} = 273 \pm 11 \text{ MeV, \; } \mu\nu$
  $f_{D_s} = 285 \pm 15 \text{ MeV, \; } \tau\nu$
Elements of HPQCD

• Staggered valence quarks
  • HISQ (highly improved staggered quark) action;
  • lattice artifacts $O(\alpha_s \Lambda m_c a^2)$, $O(\Lambda m_c^3 a^4)$;
  • absolutely normalized via PCAC;
  • less “taste breaking” (see below);
  • tiny statistical errors: 0.5% on $f_{D_s}$.
• 2+1 rooted staggered sea quarks [MILC]:
  • Lüscher-Weisz gluon + asqtad action;
  • discretization errors $O(\alpha_s a^2), O(a^4)$;
  • rooting + discretization cause small violations of unitarity, controllable by chiral perturbation theory.

• Combined fit to $a^2, m_{\text{sea}}, m_{\text{val}}$ dependence: not fully documented, but irrelevant for $f_{Ds}$. 
Staggered Fermions
[Susskind; Karsten & Smit; Sharatchandra, Thun & Weisz]

• come in 4 species, or *tastes*; the staggered Dirac operator can be written

$$(\sl{D} + m)_{\text{stag}} = \begin{pmatrix} \sl{D} + m & \sl{D} + m \\ \sl{D} + m & \sl{D} + m \end{pmatrix}$$

• All theoretical and numerical results suggest that $a\Delta$ vanishes in continuum limit.
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\[
(D + m)_{stag} = \begin{pmatrix}
\not{D} + m & a\Delta \\
a\Delta & \not{D} + m \\
a\Delta & \not{D} + m
\end{pmatrix}
\]

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\[
(D + m)_{stag} = \begin{pmatrix}
D + m & a\Delta \\
D + m & D + m \\
a\Delta & D + m \\
\end{pmatrix}
\]

- All theoretical and numerical results suggest that \( a\Delta \) vanishes in continuum limit.
Rooting

- For sea quarks, reduce the number of tastes, by assuming

\[
\left[ \det_4 (\slashed{D}_{stag} + m) \right]^{1/4} = \det_1 (\slashed{D}_{cont} + m)
\]

[Hamber, Marinari, Parisi, Rebbi].

- Uncontroversial for 20 years, until we saw that it reproduces experiment.
The taste-breaking defect $a\Delta$ must vanish in the continuum limit:

- supported by papers of Shamir (et al.), and experience with scaling in QCD.

Need chiral perturbation theory to control (small) unitarity violations:

- supported by papers of Bernard (et al.), and experience fitting numerical data.

hep-lat/0509026, hep-lat/0610094, 0711.0699 [hep-lat]
High-precision determination of the $\pi$, $K$, $D$ and $D_s$ decay constants from lattice QCD


[arXiv:0706.1726 [hep-lat]]
• The key to HPQCD’s result for $f_{Ds}$ is the extrapolation to the continuum limit.

• RS$\chi$PT needed only for benign $m_K^2 \ln m_K^2$.

• I will show their plots, followed by my own back-of-the-envelope analysis.
$m_K$ and $m_\pi$ set $m_s, m_q$

charmonium sets $m_c$
$m_K$ and $m_\pi$ set $m_s, m_q$

charmonium sets $m_c$

Assuming flat in $m_{\text{sea}}$. 
As the lattice gets finer, the discrepancy grows:

\[ f_{Ds} \text{ (MeV)} \]

\[ a^2 \text{ (fm}^2) \]

HPQCD

\[ 241 \pm 3 \text{ MeV} \]

my “amateur” fits: linear in \( a^2 \): 239; quad in \( a^2 \): 242; linear in \( a^4 \): 245.

\[ 277 \pm 9 \text{ MeV} \]

errors from reading off plot
The slopes on these to plots are consistent with $\alpha_s \Lambda m_c a^2 \oplus \Lambda m_c^3 a^4$. 
HPQCD Summary

- The trend in lattice spacing drives a value around 240 MeV.
- Systematic errors are always devilish.
- Doubling theirs still leaves a discrepancy of a $3.5\sigma$, or two of $2.5\sigma$ ($\mu\nu$) & $2.7\sigma$ ($\tau\nu$).
- So I believe their result, i.e., values around 240-250 MeV, will prove to be robust.
New Physics
Necessary Condition

• To mediate $D_s \rightarrow l\nu$ we need

$$L_{\text{eff}} = \frac{C_A^\ell}{M^2} (\bar{s}\gamma_\mu \gamma_5 c) (\bar{\nu}_L \gamma^\mu \ell_L) + \frac{C_P^\ell}{M^2} (\bar{s}\gamma_5 c) (\bar{\nu}_L \ell_R) + \text{H.c.}$$

• In rate, replace

$$G_F V_{cs}^* m_\ell \rightarrow G_F V_{cs}^* m_\ell + \frac{1}{\sqrt{2}M^2} \left( C_A^\ell m_\ell + \frac{C_P^\ell m_D^2}{m_c + m_s} \right)$$

because

$$\langle 0|\bar{s}\gamma_5 c|D_s \rangle = -i f_{D_s} m_D^2 (m_c + m_s)^{-1}$$
• Because $V_{cs}$ has a small imaginary part (in PDG parametrization), one of $C_A, C_P$ must be real and positive, to explain the effect.

• To reduce each effect to $1\sigma$,

\[
\frac{M}{(\text{Re} C_{A}^{\ell})^{1/2}} \lesssim \begin{cases} 
710 \text{ GeV} & \text{for } \ell = \tau \\
850 \text{ GeV} & \text{for } \ell = \mu 
\end{cases}, \\
\frac{M}{(\text{Re} C_{P}^{\ell})^{1/2}} \lesssim \begin{cases} 
920 \text{ GeV} & \text{for } \ell = \tau \\
4500 \text{ GeV} & \text{for } \ell = \mu 
\end{cases}.
\]
New Particles

- The effective interactions can be induced by heavy particles of charge $+1, +\frac{2}{3}, -\frac{1}{3}$.

\[
\begin{align*}
\begin{array}{c}
\text{new } W' \\
\text{charged Higgs}
\end{array}
\end{align*}
\]
• Contributes only to $C_A$.

• New gauge symmetry, but couplings to left-handed leptons constrained by other data.

• With $W$-$W'$ or $f$-$f'$ mixing, it may be possible to find a model, but...

• ... seems unlikely, barring contrived, finely tuned scenarios.
Charged Higgs

• Multi-Higgs models include Yukawa terms

\[ y_c \bar{c}_R s_L H^+ + y_s \bar{c}_L s_R H^+ + y_\ell \bar{\nu}^\ell_L \ell_R H^+ + \text{H.c.,} \]

leading to

\[ C_P^\ell = \frac{1}{2} (y_c^* - y_s^*) y_\ell, \quad M = M_{H^\pm} \]
\[ \propto (m_c - m_s \tan^2 \beta) m_\ell \quad \text{in Model II} \]

• Note that \( C_P \) can have either sign.
• But consider a two-Higgs-doublet model
  • one for $c, u, l$, with VEV 2 GeV or so;
  • other for $d, s, b, t$, with VEV 245 GeV.
• Has CKM suppression, & no tree FCNCs.
• Need to look at one-loop FCNCs.
• Naturally has same-sized increase for $\mu$ & $\tau$. 
Leptoquarks

- Eight $J \leq 1$ kinds of leptoquark can couple to SM; only six can mediate leptonic decays.

- Most have $C_P \neq 0$, some have $C_A \neq 0$

- Many disfavored by LFV $\tau \rightarrow \mu s\bar{s}$:
  - $J = 0$, $(3, 2, +7/6)$ and $(3, 3, -1/3)$;
  - $J = 1$, $(3, 1, +2/3)$ and $(3, 3, +2/3)$.
• But $J = 0, (3, 1, -1/3)$ seems promising:

$$\kappa_\ell (\bar{c}_L \ell^c_L - \bar{s}_L \nu^c_L) \bar{d} + \kappa'_\ell \bar{c}_R \ell^c_R \bar{d} + \text{H.c.}$$

(an interaction in $R$-violating SUSY), with

$$C_A^\ell = \frac{1}{4} |\kappa_\ell|^2$$

$$C_P^\ell = \frac{1}{4} \kappa_\ell \kappa'_\ell^*$$

• If $|\kappa'_\ell / \kappa_\ell| \ll m_\ell m_c / m^2_{D_s}$, then automatically the interference is constructive and creates same-sized deviation for $\mu\nu$ and $\tau\nu$. 
Summary
• Experiments are statistics limited and we hope they will improve: Belle, BES, Super-B.

• Radiative corrections should, perhaps, be collected into a single place.

• Lattice calculations must be done by other groups, with other sea quarks.

• Prejudice against new physics should be questioned.
LHC

- The generic bounds on mass/coupling suggest that any non-Standard explanation of the effect is observable at the LHC.
- Charged Higgs: similar to usual search.
- Leptoquarks: $gg \rightarrow \tilde{d}\tilde{d} \rightarrow \ell_1^+ \ell_2^- j_c j_c$. 