Full of charm neutrino DIS
small-$x$ and non-conservation of weak currents

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weak currents are not conserved.

In what way they are not conserved? Gell-Mann, Levy and Nambu - PCAC:

\[ \partial_\mu A_\mu = m_\pi^2 f_\pi \phi \]

\[ A_\mu = \bar{u} \gamma_\mu \gamma_5 d \]

Adler converted PCAC into observables

\[ F_{ud}^{L}(x, Q^2 \to 0) = \frac{f_\pi^2}{\pi} \sigma_\pi(\nu) \]

\[ x = \frac{Q^2 + M^2}{2 m_N \nu} \]
to test PCAC

measure

\[ F_2(x, Q^2) = F_L(x, Q^2) + F_T(x, Q^2) \]

extrapolate \( F_2(x, Q^2) \) down to \( Q^2 = 0 \) and make use of

\[ F_T(x, Q^2 \to 0) \to 0 \]

then compare \( F_L(x, Q^2 = 0) \) with \( f_\pi^2 \sigma_\pi / \pi \)

Assumption: the excitation of charm is irrelevant
our findings

abundant production of charm and strangeness by longitudinally polarized EW bosons already at $x < 0.01$ and for $Q^2 \lesssim m_c^2$.

The longitudinal structure function

$$F_L = F_{L}^{ud} + F_{L}^{cs}$$

is dominated by its charmed-strange component, $F_{L}^{cs}$.

At $Q^2 = 0$ (Adler’s kinematics) $F_{L}^{cs}$ rises with $1/x$ much faster than $F_{L}^{ud}$
'small-$x$' means

- predominance of the $t$-channel gluon exchange
- weakening of the mass threshold effect

\[ x = \frac{Q^2 + M^2}{2m_N\nu} \ll 1 \]

\[ M^2 \sim (m_c + m_s)^2 \]
two fundamental reasons

for the $c\bar{s}$ excitation to prevail over $u\bar{d}$

1. non-conservation of the axial current

$$\partial_\mu A_\mu \propto m_c + m_s$$

2. non-conservation of the flavor changing vector current

$$\partial_\mu V_\mu \propto m_c - m_s$$

$$m_c \gg m_s$$

one more reason

(much less fundamental) is pQCD
DGLAP ordering of $c\bar{s}$ dipole sizes

$$m_c^{-2} < r^2 < m_s^{-2}$$

and the multiplication of log’s like

$$\alpha_s \log \left( \frac{m_c^2}{m_s^2} \right) \log \left( \frac{1}{x} \right)$$

to higher orders of perturbative QCD
color dipole factorization. BFKL

$F_L$ through the absorption cross section $\sigma_L(x, Q^2)$:

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \sigma_L(x, Q^2)$$

factorization:

$$\sigma_L(x, Q^2) = \int dzd^2r |\Psi_L(z, r)|^2 \sigma(x, r)$$

$|\Psi_L(z, r)|^2$ - light-cone density of $c\bar{s}$ states
$\sigma(x, r)$ - color dipole cross section
$r$ - $c\bar{s}$-dipole size
$z$ - Sudakov’s variable of $c$-quark
light-cone density of $c\bar{s}$ states

$$|\Psi_L(z, r)|^2 = |V_L(z, r)|^2 + |A_L(z, r)|^2$$

At $Q^2 \gg m_c^2$ the S-wave $c\bar{s}$ state dominates

$$|A_L(z, r)|^2 \propto Q^2 z^2 (1 - z)^2 K_0^2(\varepsilon r)$$

$$\varepsilon^2 = z(1 - z)Q^2 + (1 - z)m_c^2 + zm_s^2$$ - the attenuation parameter

At $Q^2 \ll m_c^2$ the P-wave component takes over

$$|A_L(z, r)|^2 \propto \frac{(m_c + m_s)^2}{Q^2} \varepsilon^2 K_1^2(\varepsilon r)$$

The P-wave $c\bar{s}$-state arises only due to the weak current non-conservation
P-wave, S-wave

\[ F_{L}^{cs} \]

\[ Q^2, \text{GeV}^2 \]

\[ x_{Bj} = 0.0001 \]

P-wave, S-wave
qualitative estimates: high $Q^2$

color dipole factorization again:

$$F_L(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \int dzd^2r |\Psi_L(z, r)|^2 \sigma(x, r)$$

for small dipoles (Nikolaev & Zakharov):

$$\sigma(x, r) \approx \frac{\pi^2 r^2}{N_c} \alpha_S(r^2) G(x, A/r^2)$$

for large dipoles:

$$|\Psi_L(z, r)|^2 \propto Q^2 z^2 (1 - z)^2 \exp[-2\varepsilon r]$$
integrating over $r$ yields a broad symmetric $z$-distribution

$$F_{L}^{cs} \sim Q^4 \int_{0}^{1} dz \frac{z^2(1-z)^2}{\varepsilon^4} \alpha_s(\varepsilon^2) G(x, A\varepsilon^2)$$

dominated by the “non-partonic” $z \sim 1/2$. That leads to

$$F_{L}^{cs} \sim \alpha_s(Q^2) G(x, Q^2)$$

$\mu DIS$ - Dokshitzer
$\nu DIS$ - Barone, Genovese, Nikolaev, Predazzi and Zakharov

At high $Q^2$ the S-wave component of $|\Psi_L|^2$ dominates $F_L$
z-distribution at $Q^2 \lesssim m_c^2$

becomes highly asymmetric, with the “parton model peak” at $z \to 1$ (the P-wave dominance),

$$\frac{dF_{L}^{cs}}{dz} \sim \frac{1}{1 + \delta - z},$$

where

$$\hat{\delta} = \frac{m_s^2}{m_c^2 + Q^2}.$$
relevant dipole sizes

integrate first over $z$

\[
F_{L}^{cs} \sim m_c^2 \int_{0}^{1} dz \int_{0}^{1/\varepsilon^2} \frac{dr^2}{r^2} \sigma(r)
\]

\[
\sim \frac{m_c^2}{m_c^2 + Q^2} \int_{1/(m_c^2 + Q^2)}^{1/m_s^2} \frac{dr^2}{r^4} \sigma(r)
\]

\[
\sim \frac{m_c^2}{m_c^2 + Q^2} \int_{1/(m_c^2 + Q^2)}^{1/m_s^2} \frac{dr^2}{r^2} \alpha_S(r^2) G(x, A/r^2)
\]

the DLLA ordering of sizes:

\[
m_s^{-2} < r^2 < (m_c^2 + Q^2)^{-1}
\]
Born approximation

The $c\bar{s}$ Fock state of the light-cone W-boson interacts with the target via the two-gluon exchange (not unreasonable phenomenologically at moderate $Q^2$ and $x \sim 0.01$)

$$F_{cs}^L \sim \frac{N_c C_F}{8} \frac{m_c^2}{m_c^2 + Q^2} \frac{1}{2!} L^2$$

where

$$L = \frac{\alpha_S}{\pi} \log \left( \frac{m_c^2 + Q^2}{m_s^2} \right)$$

the DLLA ordering of sizes:

$$m_s^{-2} < r^2 < (m_c^2 + Q^2)^{-1}$$
the rise of $F^{cs}_L$ to smaller $x$

is generated by interactions of the higher Fock states $c\bar{s} + \text{gluons}$. Leading term $c\bar{s} + \text{one gluon}$:

$$
\Delta F^{cs}_L \sim \frac{N_c C_F C_A}{8} \frac{m_c^2}{m_c^2 + Q^2} \log \left( \frac{x_0}{x} \right) \frac{1}{3!} L^3
$$

with $C_A \log(x_0/x)L$ as the DLLA expansion parameter. The slope parameter

$$
\Delta = \frac{1}{3} C_A L
$$

is rather large. Even at $Q^2 = 0$ and for moderate $x$, $\Delta \simeq 0.4$. $F^{cs}_L(x, Q^2)$ rises rapidly towards still smaller $x$. 
one can think of the DGLAP growth of the gluon density

\[ G \propto \exp(2\sqrt{\xi}) \]

with

\[ \xi = \frac{4C_A}{\beta_0} \log \left( \frac{\alpha_S(m_s^2)}{\alpha_S(m_c^2 + Q^2)} \right) \log \left( \frac{1}{x} \right) \]

which is tamed, however, by the P-wave factor

\[ \frac{m_c^2}{m_c^2 + Q^2} \]

in \[ F_{L}^{cs} \]
Adler’s theorem
allows only a slow rise of

\[ F_{L}^{ud}(x, 0) = \frac{f_{\pi}^{2}}{\pi} \sigma_{\pi}(\nu) \]

with \( \nu \propto x^{-1} \),

\[ F_{L}^{ud}(x, 0) \propto (1/x)^{\Delta_{soft}} \]

where

\[ \Delta_{soft} \approx 0.08 \]

is typical of the \( \pi N \) total cross section.

For \( \nu \) well above the charm-strangeness mass threshold
\( (x \lesssim 0.01) \) the diffractive excitation of charm dominates the
longitudinal structure function \( F_{L} \).
$F_L(x, Q^2), \ FT(x, Q^2)$

$F_2(x, Q^2)$

Conclusions

We developed the color dipole BFKL description of the CCNC phenomenon in the neutrino DIS at small Bjorken $x$,

quantified the effect in terms of the tree level light-cone wave functions,

found that the charmed-strange component of the longitudinal structure function prevails over its light quark component already at $x \sim 0.01$,

found that the excitation of charm and strangeness dominates the structure function $F_2(x, Q^2)$ at $Q^2 \lesssim m_c^2$ and small enough $x$.

the neutrino beam energy required is not too high, $E_\nu \gtrsim 100$ GeV, one can try to observe the phenomenon.