Parton showers with quantum interference

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Monte Carlo event generators: what are they?

- An “event” is a list of particles (pions, protons, ...) with their momenta.
- The MCs generate events.
- The probability to generate an event is proportional to the (approximate!) cross section for such an event.
- Alternatively, cross section could be a weight given by the program times the probability to generate the event.
How MCs work

- The first ingredient is the cross section for some “hard” process of interest.
- The second ingredient is parton showering.
- Finally one needs a model for hadronization.
A reformulation of the parton shower

- QCD is a rather complicated quantum theory.
- In typical parton shower algorithms, the main approximation is small angle or soft splitting.
- But color, spin, and quantum interference from soft gluon emissions not fully accounted for.
Approximations

- Interference between different graphs sometimes treated in “angular ordering” approximation.
- Color is treated in limit $1/N_c \rightarrow 0$.
- Average over parton spins.
Our aim

- Base parton shower on approximation of small angle or soft splitting.
- Eliminate the other approximations.
- Beware: there is no code and there are negative weights.
The matrix element

• The basic object is the quantum matrix element

\[ M(\{p, f\}_m^{c_a, c_b, c_1, \ldots, c_m}_{s_a, s_b, s_1, \ldots, s_m}) \]

• This is a function of the momenta and flavors and carries color and spin indices. Consider it as a vector in color and spin space

\[ \lvert M(\{p, f\}_m) \rangle \]
The cross section

The cross section with a measurement function $F$ is

$$\sigma[F] = \sum \frac{1}{m!} \int [d\{p, f\}_m] \frac{f_{a/A}(\eta_a, \mu^2_F) f_{b/B}(\eta_b, \mu^2_F)}{4n_c(a)n_c(b) 2\eta_a\eta_b p_A \cdot p_B}$$

$$\times \langle M(\{p, f\}_m) | F(\{p, f\}_m) | M(\{p, f\}_m) \rangle$$
The density matrix

$$\sigma[F] = \sum_m \frac{1}{m!} \int [d\{p, f\}_m] \Tr\{\rho(\{p, f\}_m) F(\{p, f\}_m)\}$$

where

$$\rho(\{p, f\}_m)$$

$$= \langle M(\{p, f\}_m) \rangle \frac{f_{a/A}(\eta_a, \mu_F^2) f_{b/B}(\eta_b, \mu_F^2)}{4n_c(a)n_c(b) 2\eta_a\eta_b p_A \cdot p_B} \langle M(\{p, f\}_m) \rangle$$
Density matrix in statistical notation

\[ \rho(\{p, f\}_m) = \sum_{s,c} \sum_{s',c'} \langle \{s, c\}_m | \rho(\{p, f, s', c', s, c\}_m) \langle \{s', c'\}_m | \{p, f, s', c', s, c\}_m \rangle \]

\[ \rho(\{p, f, s', c', s, c\}_m) = \langle \{p, f, s', c', s, c\}_m | \rho \rangle \]

- For QCD partons have momenta and flavors.
- There are two sets of spin indices and two sets of color indices.
- \( |\rho\rangle \) is the “state.”
- \( |\{p, f, s', c', s, c\}_m\rangle \) are basis states.
Measurement function

- Define the measurement function as a bra vector on the statistical states,

\[
(F|\{p, f, s', c', s, c\}_m) = \langle \{s', c'\}_m | F(\{p, f\}_m) | \{s, c\}_m \rangle
\]

- Then

\[
\sigma[F] = (F|\rho)
\]

- Vector corresponding to the unit measurement is \((1|\),

\[
(1|\{p, f, s', c', s, c\}_m) = \langle \{s'\}_m | \{s\}_m \rangle \langle \{c'\}_m | \{c\}_m \rangle
\]
Color basis states

• We use a set of “string” basis states for color.

• With this basis, splitting is simple.
Color approximations

- Implement a large $N_c$ approximation if desired.

An interference diagram, to be decomposed in basis states.

The leading contribution

A subleading contribution.
Spin

- Use ordinary helicity states

\[ U(p, s) \quad \varepsilon(p, s; \hat{Q}) \]

- For gluons, define polarization vectors with

\[ \hat{Q} \cdot \varepsilon(p, s; \hat{Q}) = 0 \]

\[ \hat{Q} = \sum_{i=1}^{m+1} \hat{p}_i \]
Splitting amplitudes

- Take splitting amplitude directly from the Feynman rules,

\[ v_l = - \varepsilon_\mu (\hat{p}_{m+1}, \hat{s}_{m+1}; \hat{Q})^* \]

\[ \times \frac{\overline{U}(p_l, s_l)\gamma^\mu (\hat{p}_l - \hat{p}_{m+1} + m(f_l)) g \gamma^\mu U(\hat{p}_l, \hat{s}_l)}{2p_l \cdot n_l \left[ (\hat{p}_l - \hat{p}_{m+1})^2 - m^2(f_l) \right]} \]

- Here \( n_l \) is the lightlike combination of \( p_l \) and \( \hat{Q} \).
Soft splitting amplitude

- For soft gluon emission, one can use a simple eikonal approximation,

\[ v_{l}^{\text{soft}} = g \delta_{s_{l},s_{l}} \frac{\varepsilon(\hat{p}_{m+1}, \hat{s}_{m+1}; \hat{Q})^{*} \cdot \hat{p}_{l}}{\hat{p}_{m+1} \cdot \hat{p}_{l}} \]
Soft gluon emission

Splitting includes interference graphs.

The soft gluon approximation is used for the splitting function.

Since we use the interference graphs, we do not need the angular ordering approximation.
Shower evolution

• Showers develop in "shower time."
• Hardest interactions first.
• Eg. $t = \log \left( \frac{Q_0^2}{Q^2} \right)$. 

Real time picture

Shower time picture

$t$
Evolution

\[ |\rho(t)\rangle = U(t, t')|\rho(t')\rangle \]

\[ \frac{d}{dt} U(t, t') = [\mathcal{H}_I(t) - V(t)] U(t, t') \]

- Evolution doesn't change the cross section,

\[ (1|U(t', t)|\rho) = (1|\rho) \]

- This gives probability conservation relation,

\[ 0 = (1|[\mathcal{H}_I(t) - V(t)] \]

- This gives relation determines \( V \).
• With our choices, the solution of our evolution equation is

\[ U(t, t') = \mathcal{N}(t, t') \]

\[ + \int_{t'}^{t} d\tau \, U(t, \tau) \left[ H_I(\tau) - V_S(\tau) \right] \mathcal{N}(\tau, t') \]

\[ \mathcal{N}(t, t') = \exp \left\{ - \int_{t'}^{t} d\tau \, V_E(\tau) \right\} \]

• Get a Sudakov factor diagonal in spin and color plus perturbative corrections.
Picture for evolution

Evolution equation

Iterative solution
Status

• I have presented the general scheme, based on the quantum density matrix in spin and color.

• If we take the leading color limit and average over spin, we get something structurally similar to standard Monte Carlo parton showers.

• We are working on putting the spin back.

• Then we want to put the color back.

• We are working to build code for all of this.