XLIV Rencontres de Moriond

Forward jets and QCD coherence effects

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• Motivation — hard processes at forward rapidities at the LHC
• Theoretical issues on space-like parton showers and coherent gluon radiation
  • Applications in $ep$ and $p\bar{p}$ jet production
INTRODUCTION

High-$p_T$ production in the forward region at the LHC

- experimental coverage of large rapidities
- phase space opening up for large $\sqrt{s}$

- events with **multiple** hard scales: $q_1^2, \cdots, q_n^2$
- potentially large corrections to all orders in $\alpha_s, \sim \ln^k(q_i^2/q_j^2)$
• asymmetric parton kinematics
⇒ parton distributions probed near kinem. boundaries $x \to 0, 1 - x \to 0$

▶ coherence effects for multi-gluon radiation at small longitudinal momentum fractions $x$:
• not included AT ALL in standard shower Monte Carlo generators
• included ONLY partially in NLO multi-jet calculations
• present to all orders and enhanced by logs of $\sqrt{s}/E_T$
OUTLINE

I. QCD coherence effects in multi-parton emission at high energy

II. Issues on unintegrated parton distributions and matrix elements

III. Applications to jets: LHC prospects; examples from $p\bar{p}$ and $ep$ production
I. MULTI-PARTON EMISSION BY PARTON BRANCHING METHODS

Sudakov form factors

\[ d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \, \alpha_s \, P(z) \, \Delta(q^2, q_0^2) \]

- based on dominance of collinear evolution of jets
- Factorization of QCD cross sections in collinear limit
  \[ \rightarrow \text{probabilistic (Markov) picture} \]
- summation of logarithmically enhanced radiative contributions
  \[ (\alpha_s \, ln p_T/\Lambda)^n \]
- soft gluon radiation by coherent branching [e.g.: \textsc{Herwig}, new \textsc{Pythia}]
> soft gluons radiated over long times $\rightarrow$ quantum interferences

✓ factorization in soft limit

$$|M_{n+1}^{a_1 \ldots a_n}(p_1, p_n, q)\rangle = J^a |M_n^{a_1 \ldots a_n}(p_1, p_n)\rangle, \quad J^{a\mu} = \sum_i Q^a_i \frac{p_i^\mu}{p_i \cdot q}, \quad Q = \text{color charge}$$

interference terms ↓

$$d\sigma_{n+1} = d\sigma_n \frac{d^3 q}{(q^0)^3} \sum_{i,j} Q_i \cdot Q_j \ w_{ij}, \quad w_{ij} = \frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)}$$

→ not positive definite, non-Markov...?

→ spoils probabilistic picture? NO, owing to soft-gluon coherence ↩️
single-emission: separate singularities along emitters’ directions

\[
\frac{(q^0)^2 p_i \cdot p_j}{(p_i \cdot q)(p_j \cdot q)} = \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}}
\]

\[
= \frac{1}{2} \left( \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}} \right) + \frac{1}{2} \left( \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} - \frac{1}{\zeta_{jq}} + \frac{1}{\zeta_{iq}} \right)
\]

where \( \zeta_{nk} \equiv \frac{p_n \cdot p_k}{p_n^0 p_k^0} \simeq 1 - \cos \theta_{nk} \quad (m \to 0) \)

\[
\langle \frac{\zeta_{ij}}{\zeta_{iq}\zeta_{jq}} \rangle = \frac{1}{\zeta_{iq}} \Theta(\zeta_{ij} - \zeta_{iq}) + \frac{1}{\zeta_{jq}} \Theta(\zeta_{ij} - \zeta_{jq})
\]

large-angle emissions of soft gluons sum coherently outside angular-ordered cones
• multiple emission: \((q_1, q_2 \text{ with } q_2^0 \ll q_1^0)\)

\[
\mathbf{J}^{\mu a_1}_1 = Q_p^{a_1} \frac{p^\mu}{p \cdot q_1}, \quad \mathbf{J}^{\mu a_2}_2 = Q_p^{a_2} \frac{p^\mu}{p \cdot q_2} + Q_{q_1}^{a_2} \frac{q_1^\mu}{q_1 \cdot q_2}
\]

\[
\begin{align*}
\text{(a)} \\
&\quad \quad + \{1 \leftrightarrow 2\} + \\
\text{(b)} \\
\end{align*}
\]

\[
\mathcal{M}_{ki}^{a_1 a_2} = g_s^2 \langle a_1 k | \mathbf{J}_2 \cdot \epsilon_2 | a' i' \rangle \langle i' | \mathbf{J}_1 \cdot \epsilon_1 | i \rangle \\
= g_s^2 \frac{p \cdot \epsilon_1}{p \cdot q_1} \left( \frac{p \cdot \epsilon_2}{p \cdot q_2} t^{a_2} t^{a_1} + \frac{q_1 \cdot \epsilon_2}{q_1 \cdot q_2} [t^{a_1}, t^{a_2}] \right)_{ki}
\]

• small angle: bremsstrahlung cones
• large angle \((\theta_{pq_2} \gg \theta_{pq_1})\): sees total charge \(Q_p + Q_{q_1}\)
COHERENCE IN HIGH-ENERGY, SMALL-X PARTON SHOWERS

- Arguments used above rely on soft vector emission current from external legs → leading IR singularities
- Appropriate in single-scale hard processes

LHC forward hard processes are multi-scale: $s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$

- Initial-state branchings not collinearly-ordered potentially non-negligible
- Emissions from internal legs become leading for $x \ll 1$ ⇒ associated coherence effects
▷ internal-emission current also factorizable at high-energy
\[ |M_{n+1}⟩ = J^a |M_n⟩ \]
\[ J = J_{\text{ext.}} + δJ \]

▷ BUT:
- \( J \) depends on total transverse momentum transmitted
  ⇒ matrix elements and pdf at fixed \( k_\perp \) (“unintegrated”)
- virtual corrections not all in \( Δ \) form factor
  ⇒ modified branching probability \( P(z, k_\perp) \)

◊ radiative enhancements \( \alpha_s^k \ln^m s/p_T^2 \)
◊ superleading logs \( m > k \) cancel in fully inclusive quantities
  (e.g: corrections \( O(\alpha_s^k) \) to space-like splitting functions)
◊ not in exclusive final-state correlations
II. TOWARD PRECISE CHARACTERIZATIONS OF UNINTEGRATED PDF’s

Example 1: High energy factorization:

◊ single gluon polarization dominates \( s \gg M^2 \gg \Lambda_{QCD}^2 \)

\( \leftarrow \) gauge invariance rescued (despite gluon off-shell)

[Lipatov; Ciafaloni; Catani, H]

• corrections down by \( 1/\ln s \) rather than \( 1/Q \)

\( \leftarrow \) NLO to BFKL (+ its variants)

• can go to arbitrarily high \( k_\perp \)

\( \Rightarrow \) well-defined summation of higher-order logarithmic corrections

\( \Rightarrow \) suitable for simulations of jet physics at the LHC
BUT: Gauge-invariant characterization over whole phase space is more difficult!

Example 2: General operator matrix elements:

\[
\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle , \quad y = (0, y^-, y^\perp)
\]

\[
V_y(n) = \mathcal{P} \exp \left( i g_s \int_0^\infty d\tau \ n \cdot A(y + \tau n) \right) \text{ eikonal Wilson line in direction } n
\]

- works at tree level \cite{Mulders,Belitsky et al., 2003}
- subtler at level of radiative corrections \cite{Collins & Zu; Hautmann; Cherednikov et al.}

\[ \leftarrow x \rightarrow 1 \Rightarrow \text{explicit regularization method} \text{ (unlike inclusive case)} \]

- non-abelian Coulomb phase → effects from spectators possibly non-decoupling?

\cite{Mulders, Bomhof; Collins, Qiu; Brodsky et al]
CUT-OFF REGULARIZATION

▷ cut-off in Monte-Carlo generators using u-pdf's

LDCMC Lönblad & Sjödahl, 2005; Gustafson, Lönblad & Miu, 2002 (LDC)
CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)
SMALLX Marchesini & Webber, 1992 (CCFM)

▷ cut-off from gauge link in non-lightlike direction $n$:

\[ \eta = \left( \frac{p \cdot n}{n} \right)^2 / n^2 \]

Collins, Rogers & Stasto, arXiv:0708.2833
Ji, Ma & Yuan, 2005, 2006
earlier work from 80’s and 90’s

finite $\eta \Rightarrow$ singularity is cut off at \[ 1 - x \gtrsim \sqrt{k_\perp / 4\eta} \]

* Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]
III. APPLICATIONS TO JET PHYSICS

FORWARD JET CROSS SECTION:

\[ \sigma = \sum_a \int \phi_{a/A} \otimes M_H \otimes \phi_{g^*/B} \]

- \( M_H \) from perturbative off-shell amplitudes
  \((\leftrightarrow q \text{ and } g \text{ channels})\) [Fadin-Lipatov,
  Catani-Ciafaloni-H]

- \( k_{\perp} \)-dependent shower from branching equation + data fits
K_\perp\text{-DEPENDENT PARTON BRANCHING}

- implement all-order summation of \((\alpha_s \ln s/p_T^2) \oplus \text{IR} \ x \to 1\) behavior

branching eq.: \(\mathcal{A}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \times \Delta(\mu, zq) \mathcal{P}(z, q, k_T) \mathcal{A}(\frac{x}{z}, k_T + (1 - z)q, q)\)

\[\text{Sudakov}\quad \text{unintegr. splitting}\]

\[\text{(left) Coherent radiation in the space-like parton shower for } x \ll 1; \quad \text{(right) the unintegrated splitting function } \mathcal{P}, \text{ including small-}x \text{ virtual corrections.}\]

\[\alpha/x > \alpha_1 > \alpha \quad (\text{small } - x \text{ coherence region})\]
COHERENT MATRIX ELEMENTS FOR HARD EVENT

Ex.: \( q_T = \) final-state transverse energy (out of two hardest jets)
\[ \alpha = \frac{k_T^2}{4q_T^2}, \quad \beta^{-1} = x_1 x_2 S / (4q_T^2) \]

[Deak, Jung, Kutak & H, in progress]

- measures transverse momentum distribution of third jet
- dynamical cut-off at large \( k_T \) set by coherence effects
WHAT DO WE LEARN FROM $p \bar{p}$ AND EP JET FINAL STATES

Ex.: azimuthal $\Delta \phi$ correlation between two hardest jets

- Tevatron $\Delta \phi$ dominated by leading-order processes
  - good description by HERWIG as well as by NLO
  - used for MC parameter tuning in PYTHIA

- HERA $\Delta \phi$ not well described by standard MC
  $\hookrightarrow$ see next

- accessible at the LHC relatively early
  $\hookrightarrow$ how do MC describe multiple radiation?
DI-JET EP CORRELATIONS: COMPARISON WITH NLO RESULTS

(Left) Azimuth dependence and (right) Bjorken-\(x\) dependence of di-jet distributions

\[ Q^2 > 10 \text{ GeV}^2 \quad , \quad 10^{-4} < x < 10^{-2} \]

[Z. Nagy]

◊ large variation from order-\(\alpha_s^2\) to order-\(\alpha_s^3\) prediction as \(\Delta\phi\) and \(x\) decrease

⇒ sizeable theory uncertainty at NLO (underestimated by “\(\mu\) error band”)
ANGULAR JET CORRELATIONS FROM $K_{\perp}$-SHOWER ($C_{\text{ASCADE}}$) AND COLLINEAR-SHOWER ($\text{HERWIG}$)

(left) di-jet cross section; (right) three-jet cross section


- largest differences at small $\Delta \phi$
- good description of measurement by $k_{\perp}$-shower
- collinear shower insufficient to describe shapes
Normalize to the back-to-back cross section:

\[
\frac{1}{\sigma d^2 \sigma dx d|\Delta \phi|}
\]

\[
1.7 \times 10^{-4} < x < 3 \times 10^{-4}
\]

\[
3 \times 10^{-4} < x < 5 \times 10^{-4}
\]

\[
5 \times 10^{-4} < x < 1 \times 10^{-3}
\]

\[
\text{updf} \oplus \text{ME}_{\text{collin.}} : \mathcal{M} \rightarrow \mathcal{M}_{\text{collin.}}(k_T) = \mathcal{M}(0_{\perp}) \Theta(\mu - k_T)
\]

\[
\text{no resolved branching} : \mathcal{A} \rightarrow \mathcal{A}_{\text{no-res.}}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0) \Delta(\mu, Q_0)
\]

\[\triangleright \text{high-}k_{\perp}, \text{ coherent effect essential for correlation at small } \Delta \phi\]

(cfr., e.g., MC by Höche, Krauss & Teubner, arXiv:0705.4577: u-pdf but no ME correction)
Tevatron $b$-jets correlations

[B. Webber, CERN Lectures, 2008]

These observables are very involved ($b$-jets at hadron level) and cannot be computed with analytical techniques; the underlying event in Pythia is fitted to data; default Herwig model (used in MC@NLO) does not fit data well (lack of MPI).

- **HERWIG** description not satisfactory
- $k_\perp$ distribution of underlying event?
PROSPECTS FOR FURTHER FINAL STATES

◊ **MC and radiative corrections to gluon fusion processes:**

- production of $b$, $c$ — what size NLO uncertainties at LHC energies?
  
  [see MC@NLO; Nason et al.]
  
  ▶ sizeable corrections from $g\rightarrow b\bar{b}$ coupling to spacelike jet
  
  ▶ coherence effects to $b\bar{b} + 2\text{ jets}$ for $m_b \ll p_T^{(b\bar{b})} \ll p_T^{(\text{jet})}$

- multi-scale effects in $b\bar{b} + W/Z$ production

- $k_\perp$-shower vs. MC@NLO for top-antitop pair production
  
  ($\leftrightarrow$ see $p_T$ spectrum)

- final states with Higgs
  
  $\rightarrow$ possibly $10 \div 20\%$ effects in $p_T$ spectrum from $x \ll 1$ terms?

  [Kulesza, Sterman & Vogelsang, 2004]

  see also: Marzani, Ball, Del Duca et al., 2008; H, 2002
CONCLUSIONS

• Correlations of high-$p_T$ probes across large rapidity intervals will be explored with forward detectors at the LHC to unprecedented level

• Branching methods based on u-pdfs and $k_{\perp}$-MEs useful to
  ▶ simulate high-energy parton showers
  ▶ investigate possibly new effects from QCD physics

• Systematic theoretical studies of u-pdf’s ongoing
  ▶ relevant to turn these Monte-Carlo’s into general-purpose tools
EXTRA SLIDES
**HERWIG** K-factor of 2 (from two-jet region)

(left) di-jet cross section; (right) three-jet cross section


- different shapes from the two MC
- small $\Delta \phi$ not well described by HERWIG
- good description of 3-jet by $k_\perp$-shower but not by HERWIG
AZIMUTHAL DISTRIBUTION OF THE THIRD JET

Cross section in the azimuthal angle between the hardest and the third jet for small (left) and large (right) azimuthal separations between the leading jets


- small $\Delta \phi \Rightarrow$ non-negligible initial $k_\perp \Rightarrow$ larger corrections to collinear ordering
  - curves become closer at large $\Delta \phi$
Extensive collider data studies emphasize the phenomenological relevance of coherence effects. Example: $p\bar{p}$ dijets.

[CDF Preliminary]

$M_{jj} = 82$ GeV; $M_{jj} = 105$ GeV; $M_{jj} = 140$ GeV

$M_{jj} = 183$ GeV; $M_{jj} = 229$ GeV; $M_{jj} = 293$ GeV

$M_{jj} = 378$ GeV; $M_{jj} = 488$ GeV; $M_{jj} = 628$ GeV

$Q_{eff} = 256 \pm 13$ MeV

[B. Webber, CERN seminar, 2008]
AZIMUTHAL DISTRIBUTION IN EP 3-JET CROSS SECTION

[ZEUS, 2007]

- grey dashed band: NLO result [NLOJET++]
- NLO results more stable for more inclusive distributions
$p_T$ distribution of top-antitop pairs
from $k_{\perp}$-shower and from MC@NLO at LHC energies

[Deak, Jung, Schwennsen, prelim.]

- small-$x$ effects not large in this case
- probe shower in region of finite $x$ and large virtualities on the order of $m_{\text{top}}$
Suppose a gluon is absorbed or emitted by eikonal line:

\[ n = (0, 1, 0_\perp) \]

\[ (0, 0, 0_\perp) \quad p \quad (0, y_\perp, y_\perp) \quad p \quad + \quad q \quad p \quad + \quad \ldots \]

\[ f^{(1)} = P_R(x, k_\perp) - \delta(1 - x) \delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp) \]

where \[ P_R = \frac{\alpha_s \, C_F}{\pi^2} \left[ \frac{1}{1 - x} \frac{1}{k^2_\perp + \rho^2} + \{\text{regular at } x \to 1\} \right] \]

\[ \rho = \text{IR regulator} \]

endpoint singularity \( (q^+ \to 0, \forall k_\perp) \)

Physical observables:

\[ \mathcal{O} = \int dx \, dk_\perp \, f^{(1)}(x, k_\perp) \varphi(x, k_\perp) \]

\[ = \int dx \, dk_\perp \left[ \varphi(x, k_\perp) - \varphi(1, 0_\perp) \right] P_R(x, k_\perp) \]

inclusive case: \( \varphi \) independent of \( k_\perp \Rightarrow 1/(1 - x)_+ \) from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)
UPDF’s WITH SUBTRACTIVE REGULARIZATION

- Endpoint divergences $x \to 1$ from incomplete KLN cancellation


- gauge link still evaluated at $n$ lightlike, but multiplied by “subtraction factors”

$$\tilde{f}^{(\text{subtr})}(y^-, y_\perp) = \frac{\langle P|\bar{\psi}(y)V_y^\dagger(n)\gamma^+V_0(n)\psi(0)|P\rangle}{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle / \langle 0|V_{\bar{y}}(u)V_{\bar{y}}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}$$

\(\bar{y} = (0, y^-, 0_\perp); \ u = \text{auxiliary non-lightlike eikonal} (u^+, u^-, 0_\perp)\)

H, arXiv:0708.1319

\(\diamond \) $u$ serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$
One loop expansion:

\[ f^{(\text{subtr})}_{(1)}(x, k_\perp) = P_R(x, k_\perp) - \delta(1-x) \delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp) \quad (\text{from numerator}) \]

\[ - W_R(x, k_\perp, \zeta) + \delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) \quad (\text{from vev's}) \]

with \( P_R = \alpha_s C_F / \pi^2 \left\{ 1/[ (1-x) \ (k^2_\perp + m^2 (1-x)^2) ] + \ldots \right\} = \text{real emission prob.} \)

\( W_R = \alpha_s C_F / \pi^2 \left\{ 1/[ (1-x) \ (k^2_\perp + 4\zeta(1-x)^2) ] + \ldots \right\} = \text{counterterm} \)

- \( \zeta\)-dependence cancels upon integration in \( k_\perp \) \[ \zeta = (p^+ / 2) u^- / u^+ \]

\[ \Rightarrow \mathcal{O} = \int dx \ dk_\perp \ f^{(\text{subtr})}_{(1)}(x, k_\perp) \varphi(x, k_\perp) \]

\[ = \int dx \ dk_\perp \left\{ P_R [\varphi(x, 0_\perp) - \varphi(1, 0_\perp)] + (P_R - W_R) [\varphi(x, k_\perp) - \varphi(x, 0_\perp)] \right\} \]

- first term: usual \( 1/(1-x)_+ \) distribution
- second term: singularity in \( P_R \) cancelled by \( W_R \)
FURTHER ISSUES AT HIGHER ORDER

- soft gluon exchange with spectator partons
  ⇒ factorization breaking in higher loops?

◊ likely suppressed for small-x, small-Δϕ

◊ could affect physical picture near back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Collins, arXiv:0708.4410
Vogelsang and Yuan, arXiv:0708.4398
Bomhof and Mulders, arXiv:0709.1390

Forshaw, Kyrieleis & Seymour, 2006