The mass of $\eta_b$

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RENCONTRES DE MORIOND
QCD and High Energy Interactions

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<table>
<thead>
<tr>
<th>Charmonium</th>
<th>Bottomonium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872), X(3940)$</td>
<td>$Y(3940), Y(4260)$</td>
</tr>
<tr>
<td>$Y(3940), Y(4260)$</td>
<td>$Z(3930)$</td>
</tr>
</tbody>
</table>

nothing
Desciery of $\eta_b$ - June 2008


Single photon spectrum at $s = M(\Upsilon(3S))$

Peak at $E_\gamma = 921.2^{+2.1}_{-2.8}\text{stat} \pm 2.4\text{syst}\text{ MeV} \Leftrightarrow \Upsilon(3S) \rightarrow \gamma\eta_b$
Discovering of $\eta_b$ - June 2008


Inclusive spectrum
Desciery of $\eta_b$ - June 2008


![Inclusive spectrum graph]

$\Upsilon(3S) \rightarrow \gamma \chi_b J(2P)$

Inclusive spectrum
Descivery of $\eta_b$ - June 2008


Non-peak background subtracted

$\Upsilon(3S) \rightarrow \gamma \chi_{bJ}(2P)$

$e^+e^- \rightarrow \gamma \Upsilon(1S)$

$\Upsilon(3S) \rightarrow \gamma \eta_b$
Descivery of $\eta_b$ - June 2008


$\Upsilon(3S) \rightarrow \gamma \eta_b$

All background subtracted
Hyperfine Splitting $E_{\text{hfs}} = M(\Upsilon_{1S}) - M(\eta_b)$

$E_{\text{hfs}}^{\text{exp}} = 71.4 \pm 2.7 \text{ (syst)} \pm 2.3 \text{ (stat)} \text{ MeV}$

$E_{\text{hfs}} \sim \alpha_s^4 \Rightarrow$ very sensitive to the strong coupling constant at the scale $\alpha_s m_b \sim 2 \text{ GeV}$

$a'\text{priori}$ can be computed from the first principles of QCD

Can be used for a very accurate $\sim 1\%$ determination of $\alpha_s(M_Z)$
Theory of Hyperfine Splitting

✔ Perturbative QCD

✔ Lattice QCD

✘ Potential models
Perturbative QCD

NRQCD, pNRQCD $\Rightarrow$ systematic expansion in $\alpha_s$

(Caswell, Lepage; Pineda, Soto, ...)

Nonperturbative contribution $O(v^2) \Rightarrow$ NNLO

(Voloshin; Leutwiller)

NLO $O(\alpha_s) \Rightarrow$

\[
E_{hfs}^{NLO} = \frac{C_F^4 \alpha_s^4 m_b}{3} \left\{ 1 + \frac{\alpha_s}{\pi} \left[ \frac{7}{4} C_A \ln (C_F \alpha_s) - \frac{C_F}{2} + \frac{2\pi^2 - 26}{9} n_f T_F + \frac{543 - 44\pi^2}{72 n} C_A \right] \right\}
\]

\[
= E_{hfs}^{LO} \left\{ 1 + \frac{\alpha_s}{\pi} \left[ \frac{21}{4} \ln \left( \frac{4 \alpha_s}{3} \right) + \frac{1061}{72} - \frac{23}{18} \pi^2 \right] \right\}
\]

\[
\approx E_{hfs}^{LO} \left\{ 1 + \alpha_s \left[ 1.67 \ln (1.33 \alpha_s) + 0.68 \right] \right\}
\]

(Penin, Steinhauser)

NRG resummation $\Rightarrow$ NLL $O(\alpha_s^m \ln^{m-1} \alpha_s)$

(Kniehl, Penin, Pineda, Smirnov, Steinhauser)
HFS in charmonium

HFS in bottomonium

\[ E_{\text{hfs}} = M(\Upsilon_{1S}) - M(\eta_b) \]

- **QCD NLL approximation:**
  \[ E_{\text{hfs}}^{\text{th}} = 39 \pm 11 \, (\text{th}) +9^{+9}_{-8} (\delta \alpha_s) \, \text{MeV} \]

- **Experiment:**
  \[ E_{\text{hfs}}^{\text{exp}} = 71.4 \pm 2.7 \, (\text{syst}) +2.3^{+2.3}_{-3.1} (\text{stat}) \, \text{MeV} \]
HFS Puzzle

Why perturbative QCD works for charmonium but fails for bottomonium HFS?

- Nonperturbative correction scales as $1/m_q^4$

Perturbative correction scales as $\alpha_s(\alpha_s m_q)$

- Cancellation of huge perturbative and nonperturbative corrections for charmonium, no cancellation for bottomonium?

- Hard to believe since leading $\mathcal{O}(v^2)$ nonperturbative contribution is positive $\Rightarrow$ bottomonium HFS would be even smaller!
Lattice QCD

- HPQCD and UKQCD collaborations (NRQCD)
  \[ E_{\text{hfs}}^{\text{lat}} = 61 \pm 14 \text{ MeV} \]

- TWQCD collaboration (QCD)
  \[ E_{\text{hfs}}^{\text{lat}} = 70 \pm 10 \text{ MeV} \]

- Experiment:
  \[ E_{\text{hfs}}^{\text{exp}} = 71.4 \pm 2.7 \text{ (syst)} \pm 2.3 \pm 3.1 \text{ (stat)} \text{ MeV} \]
Lattice QCD

- No continuum limit $\Leftrightarrow$ lattice spacing $a$ is determined by fitting the experimental spectrum
  
  - *HPQCD and UKQCD* $1/a \approx 2.3$ GeV
  - *TWQCD* $1/a \approx 7.7$ GeV

- For $a > 1/m_b$ the one-loop logarithmic contribution of the hard relativistic modes is
  
  $$ -\frac{21}{4} \frac{\alpha_s}{\pi} \ln(a m_b) E_{hfs} \approx -20 \text{ MeV} \quad \text{for} \quad \frac{1}{a} \approx 2.3 \text{MeV} $$

  - *brings lattice NRQCD into agreement with perturbative QCD*

- *TWQCD result* $m_b = 4.65(5)$ GeV is suspicious, world average is $m_b = 4.20(7)$ GeV
There is an apparent discrepancy among QCD predictions for bottomonium HFS
Summary

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- Further progress $\Rightarrow$ matching lattice and perturbative QCD
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Summary

- There is an apparent discrepancy among QCD predictions for bottomonium HFS
- Further progress $\Rightarrow$ matching lattice and perturbative QCD
- If it does not work $\Rightarrow$ check the experiment
- The prize $\Rightarrow$ record precision of $\alpha_s(M_Z)$