Recent B-physics results from lattice QCD

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Moriond QCD
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Why compute B-decays in lattice QCD?

- B-factories and Tevatron have been pouring out data to pin down the CKM matrix elements -- lattice QCD calculations are needed to interpret many of their results.

- In order to accurately describe weak interactions involving quarks, must include effects of confining quarks into hadrons:

  - Absorb non-perturbative QCD effects into quantities such as decay constants, form factors, and bag-parameters.

  - Only way to calculate hadronic weak matrix elements with all systematic uncertainties under control is numerically using lattice QCD.
Lattice QCD and the CKM unitarity triangle

- In the Standard Model, the CKM matrix is unitary.
- Leads to relationships among matrix elements that can be expressed as the CKM unitarity triangle.
- New quark flavor-changing interactions & CP-violating phases would manifest themselves as apparent inconsistencies between experimental measurements that are predicted to be the same within the Standard Model framework.
- Schematically,
  \[ \text{expt.} = \text{CKM} \times \text{lattice} \times \text{known factors} \]
- \( \implies \) To test the Standard Model and observe new physics, need precise (few % or better) lattice QCD calculations.
Systematics in lattice calculations

- Lattice calculations typically quote the following sources of error:

  1. Monte carlo statistics & fitting
  2. Tuning lattice spacing, \( a \), and quark masses
  3. Matching lattice gauge theory to continuum QCD
     - (Sometimes split up into relativistic errors, discretization errors, perturbation theory, ...)
  4. Chiral extrapolation to physical up, down quark masses
  5. Extrapolation to continuum
     - (Often combined with chiral extrapolation)

- In order to verify understanding and control of systematic uncertainties in lattice calculations, **COMPARE RESULTS FOR KNOWN QUANTITIES WITH EXPERIMENT**

- Two such examples are the pion decay constant and the \( D \rightarrow K \Lambda \nu \) form factor . . .
The pion decay constant

- Tests:
  - Dynamical (sea) quark effects
  - Light quark formalism
  - Chiral and continuum extrapolation

- Because of limited computing resources, quark masses in lattice simulations are higher than those in the real world.

- Must extrapolate lattice results to physical values of up, down quark mass.

- Use expressions derived in chiral perturbation theory to extrapolate to the physical quark masses in a controlled way.

- Can also use symmetries of lattice action to incorporate discretization errors and extrapolate to the continuum.

- **Can compute $f_\pi$ to $\sim 2\%$ accuracy** and result agrees with experiment!
The $D \rightarrow K \ell \nu$ form factor

- Also tests:
  - Heavy-quark formalism
  - Lattice operator matching

- Generic lattice quark action will have discretization errors $\propto (am_Q)^n$

- Can use knowledge of the heavy quark or nonrelativistic quark limits of QCD to systematically eliminate HQ discretization errors order-by-order

- Requires tuning parameters of lattice action and matching lattice weak currents to continuum

  - Typically calculate matching coefficients in lattice perturbation theory

- Estimate errors using knowledge of short-distance coefficients and power-counting

- Successfully *predicted the shape and normalization of the $D \rightarrow K \ell \nu$ form factor!*

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Fermilab/MILC; Phys.Rev.Lett.94:011601,2005

![Graph showing $D \rightarrow K \ell \nu$ form factor](image)
Lattice calculations of B-meson quantities

- Currently two groups calculating heavy-light meson quantities with three dynamical quark flavors: Fermilab/MILC & HPQCD

- Both use the publicly available “2+1 flavor” MILC configurations [Phys.Rev.D70:114501,2004] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark ($\approx m_s$)
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$ (minimum $m_\pi \approx 240$-330 MeV)
  - Two or more lattice spacings with minimum $a \approx 0.09$ fm

- Groups use different heavy quark discretizations:
  - Fermilab/MILC uses Fermilab quarks
  - HPQCD uses nonrelativistic (NRQCD) heavy quarks

CAVEAT: This talk will be restricted to three-flavor unquenched lattice calculations
$B \rightarrow D^{*} \ell \nu$ decay and $|V_{cb}|$
\[ B \rightarrow D^* \ell \nu \text{ semileptonic decay} \]

- Experiments can only measure the product (form factor) \( \times |V_{cb}| \)

\[
\frac{d\Gamma(B \rightarrow D\ell\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 |\mathcal{F}_{B \rightarrow D}(w)|^2 \quad \text{w} = v' \cdot v
\]

- Lattice QCD calculations needed to determine normalization and extract the CKM matrix element \( |V_{cb}| \)

- Only need one \( q^2 \) point from lattice -- choose \( w=1 \) because easiest to calculate
|V_{cb}| normalizes the CKM unitarity triangle

- In order to make the base of the CKM triangle have unit length, the convention is to divide everything by |V_{cd} V_{cb}^{*}|.

\[ \Rightarrow |V_{cb}| \text{ enters all constraints on the apex of CKM unitarity triangle} \] (not the angles) except for those from ratios.

\[ \approx 2\% \text{ error in } |V_{cb}| \text{ already limits the constraint from neutral kaon mixing (the } \varepsilon_K \text{ band) will ultimately limit other constraints if it is not reduced} \ldots \]
Calculation of the $B \rightarrow D^* \ell \nu$ form factor and $|V_{cb}|$

\[ F(1) = 0.921(13)(20) \]

[Fermilab/MILC; Phys. Rev. D 79, 014506 (2009)]

- Mild quark mass dependence
- Largest uncertainties from statistics and discretization errors, and can be reduced in a straightforward manner:
  - MILC has recently generated 4x the configurations on the $a \approx 0.12$ fm lattices
  - Configurations with $a \approx 0.06$ fm, $a \approx 0.045$ fm still need to be analyzed
- Using the most recent experimental value of $F(1) \times |V_{cb}|$
  from the Heavy Flavor Averaging Group gives

\[ |V_{cb}| \times 10^3 = 38.2 \pm 0.6_{\text{exp.}} \pm 1.0_{\text{theo.}} \]
Comparison with other determinations

<table>
<thead>
<tr>
<th>Inclusive</th>
<th>Exclusive B → Dlν</th>
<th>Exclusive</th>
</tr>
</thead>
<tbody>
<tr>
<td>HFAG</td>
<td>41.67 ± 0.73 -0.73</td>
<td>~2%</td>
</tr>
<tr>
<td>FNAL-MILC '04 B → Dlν (preliminary)</td>
<td>39.5 +1.7 -1.7</td>
<td>~4%</td>
</tr>
<tr>
<td>FNAL-MILC '08</td>
<td>38.2 +1.2 -1.2</td>
<td>~3%</td>
</tr>
</tbody>
</table>

- Experiment updated since publication, with only slight change in |Vcb|
- Exclusive |Vcb| approximately $2\sigma$ lower than inclusive determinations
- Experiments not consistent for $B \rightarrow D^*\ell\nu$:
  - Confidence level of HFAG global fit is 0.01%
  - Calculation of $B \rightarrow D^*\ell\nu$ form factor at non-zero recoil could perhaps shed some light . . .
$B \rightarrow \pi \ell \nu$ decay and $|V_{ub}|$
B→πℓν semileptonic decay

- Experiments can only measure the product \( f_+(q^2) \times |V_{ub}| \)

\[
\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} \left[ (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 \right]^{3/2} |V_{ub}|^2 |f_+(q^2)|^2
\]

- Need lattice calculation of the B→πℓν form factor to determine |V_{ub}|

- Few percent determination of |V_{ub}| difficult because errors in experimental branching fraction smallest at low \( q^2 \), whereas errors in lattice form factor determination smallest at high \( q^2 \)
\[ |V_{ub}| \text{ constrains the apex } (\bar{\rho}, \bar{\eta}) \text{ of the unitarity triangle:} \]

\[
\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1 - \frac{\lambda^2}{2}} \sqrt{\rho^2 + \eta^2}
\]

\[ \lambda = |V_{us}| \text{ known to } \sim 1\% \]

\[ |V_{cb}| \text{ known to } \sim 2\% \]

\[ \text{Width of green error ring dominated by uncertainty in } |V_{ub}| \]

\[ \sin(2\beta) \text{ currently constrains the height to better than } 4\% \text{ and is still improving} \]

\[ \therefore \text{ A precise determination of } |V_{ub}| \text{ will allow a strong test of CKM unitarity} \]
Calculation of the $B \to \pi \ell \nu$ form factor $f_+(q^2)$

- Compute the form factor at 12 $q^2$ values from $\approx 18$ GeV$^2$ to $q^2_{\text{max}} = 26.5$ GeV$^2$
  - Shape and normalization consistent with other 2+1 flavor determinations
  - Errors smaller and more reliable due to use of second lattice spacing

Largest uncertainty from statistics and chiral extrapolation, and can be reduced with the following:

- MILC has recently generated 4× the configurations on the $a \approx 0.12$ fm lattices
- Configurations with larger spatial volumes exist and will allow lighter pion masses
Exclusive determination of $|V_{ub}|$

- Combine numerical lattice form factor data with experimentally-measured branching fraction from BABAR [Phys. Rev. Lett. 98, 091801 (2007)]

- Error in best-known experimental point is ~7%

- Fit lattice and BABAR data together to a **model-independent function based on analyticity, unitarity, and crossing-symmetry** leaving $|V_{ub}|$ as a free parameter [Arnesen et. al. Phys. Rev. Lett. 95, 071802 (2005) and refs. therein]

- Simultaneous fit optimizes extraction and minimizes error in $|V_{ub}|$
Recent B-physics results from lattice QCD

Comparison with other determinations

- New exclusive $|V_{ub}|$ approximately $1 - 2 - \sigma$ lower than inclusive determinations
- Consistent with preferred values from unitarity triangle analyses
Neutral B-meson mixing
B-mixing constraint on the unitarity triangle

- Underlying quark flavor-changing weak interaction is proportional to:
  - $|V_{td}^*V_{tb}|$ for $B_d$-mixing
  - $|V_{ts}^*V_{tb}|$ for $B_s$-mixing

- The ratio of $B_d$ to $B_s$ oscillation frequencies ($\Delta m_q$) constrains the apex of the CKM unitarity triangle:
  \[
  \frac{\Delta m_d}{\Delta m_s} = \left( \frac{f_{B_d} \sqrt{\hat{B}_{B_d}}}{f_{B_s} \sqrt{\hat{B}_{B_s}}} \right)^2 \frac{m_{B_d}}{m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} = \xi^2 \frac{m_{B_d}}{m_{B_s}} \left( \frac{\lambda}{1 - \lambda^2/2} \right)^2 \frac{(1 - \bar{\rho})^2 + \bar{\eta}^2}{\left(1 + \frac{\lambda^2}{\lambda^2/2 - \bar{\rho}}\right) + \lambda^4 \bar{\eta}^2}
  \]

- $\Delta m_q$ measured to better than 1%
- $\lambda = |V_{us}|$ known to $\sim 1$
- Dominant error currently from uncertainty in lattice QCD calculation of the ratio $\xi$
Calculation of B-meson mixing parameters

\[ \xi = 1.258(33) \]

\[ f_{B_d} \sqrt{\hat{B}_{B_d}} = 216(15) \text{ MeV} \]

\[ f_{B_s} \sqrt{\hat{B}_{B_s}} = 266(18) \text{ MeV} \]

- Almost no lattice spacing dependence in \( \xi \)
- Largest uncertainty in \( \xi \) (2%) from statistics and chiral extrapolation and can be reduced:
  - MILC has recently generated 4\( \times \) the configurations on the \( a \approx 0.12 \text{ fm} \) lattices
  - Configurations with larger spatial volumes exist and allow lighter pion masses
Comparison with other determinations

- Value of $\xi$ consistent with preliminary 2+1 flavor determination of Fermilab/MILC from Lattice 2008

- Leads to the following ratio of CKM matrix elements:

\[
\frac{|V_{td}|}{|V_{ts}|} = 0.214(1)_{\text{exp.}}(5)_{\text{theo.}}
\]

- Also consistent with (significantly less precise!) determination from $B \to \rho \gamma / B \to K^*\gamma$: $|V_{td}/V_{ts}| = 0.21(4)$
Summary and outlook

- Lattice QCD calculations of B-meson decays and mixing now allow **reliable determinations of CKM matrix elements**

- In the past year lattice QCD has produced:

  1. First 2+1 flavor calculation of the $B \to D^* \ell \nu$ form factor and $|V_{cb}|$ exclusive
  2. Best 2+1 flavor calculation of the $B \to \pi \ell \nu$ form factor and $|V_{ub}|$ exclusive
  3. First 2+1 flavor calculation of neutral B-meson mixing parameters and their ratio $\xi$

- Lattice QCD results will continue to improve with:
  - Higher statistics, finer lattice spacings
  - Improved heavy-quark actions
  - Improved form factor data at nonzero $q^2$

- Lattice QCD will soon allow **percent-level tests of the Standard Model** in the quark flavor sector and may eventually reveal new physics