SYNCHROTRON-LIKE GLUON EMISSION IN THE QUARK-GLUON PLASMA

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A quasiclassical theory of the synchrotron-like gluon emission is discussed. We show that the synchrotron emission may be important in the jet quenching if the plasma instabilities generate a chromomagnetic field $H \sim m_D^2/g$. Our gluon spectrum disagrees with that obtained by Shuryak and Zahed within Schwinger's proper time method.

1. In this talk, I discuss a quasiclassical approach to the synchrotron-like gluon emission in the quark-gluon plasma (QGP) developed in my recent paper [1]. The synchrotron energy loss may potentially be important in the jet quenching since the plasma instabilities can generate chromomagnetic/electric fields in the QGP created in AA-collisions [2,3,4,5,6]. The non-Abelian synchrotron radiation was previously addressed in the soft gluon limit by Shuryak and Zahed [7] within Schwinger's proper time method. Our quasiclassical approach is very simple and applicable in the quantum regime. The gluon spectrum derived in our formalism disagrees with that obtained in [7]. We give arguments that the spectrum of [7] is wrong.

2. One can show that in the quasiclassical regime (when the parton energies are much bigger than their masses), similarly to the photon radiation in QED, the coherence length of the gluon emission, L_c, is small compared to the minimal parton Larmor radius R_L. It allows one to calculate the radiation rate per unit length by considering a slab of chromomagnetic field of the thickness $L \gg L_c$, which is, however, small compared to the parton Larmor radii. In this case the transverse momenta of the final partons are small compared to their longitudinal momenta (we choose the z-axis along the initial parton momentum, which is perpendicular to the slab with transverse chromomagnetic field, $\mathbf{H}_a$). One may consider the magnetic field with the only nonzero color components in the Cartan subalgebra, i.e., for $a = 3$ and $a = 8$ for the $SU(3)$ color group. The interaction of the gluons with the external field is diagonalized by introducing the fields with definite color isospin, Q_A, and color hypercharge, Q_B, (we denote the color charge by the two-dimensional vector $Q = (Q_A, Q_B)$). There are two neutral gluons $A = G_3$ and $B = G_8$, and six charged gluons X, Y, Z, $\bar{X}$, $\bar{Y}$, $\bar{Z}$, where in terms of the usual gluon vector potential, G, (the Lorentz indices are omitted) $X = (G_1 + iG_2)/\sqrt{2}$ ($Q = (-1, 0)$), $Y = (G_4 + iG_5)/\sqrt{2}$ ($Q = (-1/2, -\sqrt{3}/2)$), $Z = (G_6 + iG_7)/\sqrt{2}$ ($Q = (1/2, -\sqrt{3}/2)$).

The S-matrix element of the $q \to gq'$ transition can be written as (we omit the color factors and indices)

$$\langle gq'|\hat{S}|q\rangle = -ig \int dy \bar{\psi}_{q'}(y)\gamma^{\mu} G^{*}_{\mu}(y)\psi_q(y)\,, \qquad (1)$$

where $\psi_{q,q'}$ are the wave functions of the initial quark and final quark, G is the wave function of the emitted gluon. We write each quark wave function in the form $\psi_i(y) = \exp[-iE_i(t -$

$z)]\hat{u}_\lambda \phi_i(z,\boldsymbol{\rho})/\sqrt{2E_i}$ (hereafter the bold vectors denote the transverse vectors), where λ is the quark helicity, $\hat{u}_\lambda$ is the Dirac spinor operator. The z-dependence of the transverse wave functions ϕ_i is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial\phi_i(z,\boldsymbol{\rho})}{\partial z}=\Big\{\frac{(\mathbf{p}-gQ_n\mathbf{G}_n)^2+m_q^2}{2E_i}+gQ_n(G_n^0-G_n^3)\Big\}\phi_i(z,\boldsymbol{\rho})\,, \tag{2}$$

where now G denotes the external vector potential (the superscripts are the Lorentz indexes and $n=A,B$), Q_n is the quark color charge. The wave function of the emitted gluon can be represented in a similar way. We will assume that in the QGP for the parton masses one can use the corresponding quasiparticle masses.

We take the external vector potential in the form $G_n^3=[\mathbf{H}_n\times\boldsymbol{\rho}]^3$, $\mathbf{G}_n=0$, $G_n^0=0$ (we assume that chromoelectric field is absent, however, it can be included as well). The term $-gQ_nG_n^3$ in (2) can be viewed as a potential energy $U_i=-\mathbf{F}_i\cdot\boldsymbol{\rho}$, where $\mathbf{F}_i$ is the corresponding Lorentz force. The solution of (2) can be taken in the form

$$\phi_i(z,\boldsymbol{\rho})=\exp\left\{i\mathbf{p}_i(z)\boldsymbol{\rho}-\frac{i}{2E_i}\int_0^z dz'[\mathbf{p}_i^2(z')+m_q^2]\right\}. \tag{3}$$

Here $\mathbf{p}_i(z)$ is the solution to the equation $d\mathbf{p}_i/dz=\mathbf{F}_i(z)$. From (1), (3) one can obtain

$$\langle gq'|\hat{S}|q\rangle=-ig(2\pi)^3\delta(E_g+E_{q'}-E_q)\int_{-\infty}^{\infty}dzV(z,\{\lambda\})\delta(\mathbf{p}_g(z)+\mathbf{p}_{q'}(z)-\mathbf{p}_q(z))$$
$$\times\exp\left\{-i\int_0^z dz'\left[\frac{\mathbf{p}_q^2(z')+m_q^2}{2E_q}-\frac{\mathbf{p}_g^2(z')+m_g^2}{2E_g}-\frac{\mathbf{p}_{q'}^2(z')+m_q^2}{2E_{q'}}\right]\right\}, \tag{4}$$

where V is the spin vertex factor, $\{\lambda\}$ is the set of the parton helicities. The argument of the transverse momentum δ-function does not depend on z (since $\mathbf{F}_q=\mathbf{F}_g+\mathbf{F}_{q'}$), and can be replaced by $\mathbf{p}_g(\infty)+\mathbf{p}_{q'}(\infty)-\mathbf{p}_q(\infty)$. From (4) one can obtain for the gluon spectrum

$$\frac{dP}{dx}=\frac{1}{(2\pi)^2}\int d\mathbf{p}_g(\infty)\int dz_1dz_2g(z_1,z_2)$$
$$\times\exp\left\{i\int_{z_1}^{z_2}dz\left[\frac{\mathbf{p}_q^2(z)+m_q^2}{2E_q}-\frac{\mathbf{p}_g^2(z)+m_g^2}{2E_g}-\frac{\mathbf{p}_{q'}^2(z)+m_q^2}{2E_{q'}}\right]\right\}, \tag{5}$$

$$g(z_1,z_2)=\frac{C\alpha_s}{8E_q^2x(1-x)}\sum_{\{\lambda\}}V^*(z_2,\{\lambda\})V(z_1,\{\lambda\})=g_1\mathbf{q}(z_2)\mathbf{q}(z_1)/\mu^2+g_2\,, \tag{6}$$

where x is the longitudinal gluon fractional momentum, $\mu=E_qx(1-x)$, $\mathbf{q}(z)=\mathbf{p}_g(z)(1-x)-\mathbf{p}_{q'}(z)x$, $g_1=C\alpha_s(1-x+x^2/2)/x$ and $g_2=C\alpha_s m_q^2x^3/2\mu^2$ (the two terms in (6) correspond to the non-flip and spin-flip processes), $C=|\lambda^a_{fi}\chi^*_a/2|^2$, where i,f are the color indexes of the initial and final quarks, χ_a is the color wave function of the emitted gluon.

For a uniform external field we have $\mathbf{q}(z_2)\mathbf{q}(z_1)=\bar{\mathbf{q}}^2-\mathbf{f}^2\tau^2/4$, where $\bar{\mathbf{q}}=\mathbf{q}(\bar{z})$, $\bar{z}=(z_1+z_2)/2)$, $\tau=z_2-z_1$, and $\mathbf{f}=d\mathbf{q}/dz=\mathbf{F}_g(1-x)-\mathbf{F}_{q'}x$. After replacing in (5) the integration over $\mathbf{p}_g(\infty)$ by the integration over $\bar{\mathbf{q}}$ we obtain for the radiation rate per unit length

$$\frac{dP}{dLdx}=\frac{1}{(2\pi)^2}\int d\bar{\mathbf{q}}\int_{-\infty}^{\infty}d\tau\left[\frac{g_1}{\mu^2}\left(\bar{\mathbf{q}}^2-\frac{\mathbf{f}^2\tau^2}{4}\right)+g_2\right]\exp\left\{-i\left[\frac{(\epsilon^2+\bar{\mathbf{q}}^2)\tau}{2\mu}+\frac{\mathbf{f}^2\tau^3}{24\mu}\right]\right\} \tag{7}$$

with $\epsilon^2=m_q^2x^2+m_g^2(1-x)$. With the help of τ integration by parts one can remove $\bar{\mathbf{q}}^2$ from the left square brackets in (7), and after integrating over $\bar{\mathbf{q}}$ one obtains

$$\frac{dP}{dLdx}=\frac{i\mu}{2\pi}\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\left[\frac{g_1}{\mu^2}\left(\epsilon^2+\frac{\mathbf{f}^2\tau^2}{2}\right)-g_2\right]\exp\left\{-i\left[\frac{\epsilon^2\tau}{2\mu}+\frac{\mathbf{f}^2\tau^3}{24\mu}\right]\right\}. \tag{8}$$

Here it is assumed that τ has a small negative imaginary part. One can easily show that in (8) the integral around the lower semicircle near the pole at $\tau = 0$ plays the role of the $\mathbf{f} = 0$ subtraction term. The formula (8) can be written in terms of the Airy function

$$\frac{dP}{dLdx} = \frac{a}{\kappa}\mathrm{Ai}'(\kappa) + b\int_{\kappa}^{\infty} dy\mathrm{Ai}(y)\,, \tag{9}$$

where $a = -2\epsilon^2 g_1/\mu$, $b = \mu g_2 - \epsilon^2 g_1/\mu$, $\kappa = \epsilon^2/(\mu^2\mathbf{f}^2)^{1/3}$.

3. For neutral gluons at $m_g = 0$ our spectrum (8) agrees with prediction of the quasiclassical operator approach[8] for the photon spectrum. For charged gluons (8) disagrees with the spectrum obtained by Shuryak and Zahed[7] in the soft gluon limit within Schwinger's proper time method. In[7] (Eq. (20) of[7]) the argument of the exponential contains (we use our notation) $\mathbf{F}_{q'}^2 x_g^2 + \mathbf{F}_g^2$ instead of our $\mathbf{f}^2$. Also, in the pre-exponential factor instead of $\mathbf{f}^2$ there appears $\mathbf{F}_{q'}^2 x_g^2$. Due to the absence of the interference term the spectrum of[7] is insensitive to the relation between the signs of the color charges of the final partons. It is strange enough, since the difference in the bending of the final parton trajectories (which is responsible for the synchrotron radiation) is sensitive to the relation between the color charges of the final partons. Also, Eq. (20) of[7] in the massless limit gives a vanishing spectrum for the $q_1 \to g_Z q_3$ transition for magnetic field in the color state A (since in this case $\mathbf{F}_{q'} = 0$). This process except for the spin effects is analogous to the synchrotron radiation in QED, and there is no physical reason why it should vanish. One more objection to the result of[7] is that due to a non-symmetric form of the pre-exponential factor in the case of $g \to gg$ process it should give the spectrum with incorrect permutation properties. Thus, one sees that the formula obtained in[7] clearly leads to absurd predictions, and cannot be correct.

4. In Fig. 1 we present the averaged over the color states gluon spectrum for the chromomagnetic field in the color state A for different initial parton energies. The computations are performed for $\alpha_s = 0.3$, and $gH_A/m_D^2 = 0.05$, 0.25 and 1, where m_D is the Debye mass (we assume that as for an isotropic weakly coupled plasma $m_D^2 = 2m_g^2$). For the quasiparticle masses we take $m_q \approx 0.3$ and $m_g \approx 0.4$ GeV[9]. For the magnetic field in the color state B the spectrum is very close to that shown in Fig. 1. The decrease of the spectra at $x \to 0$ (and $x \to 1$ for $g \to gg$ process) which is well seen for the smallest value of the field is due to the Ter-Mikaelian mass effect. This suppression decreases with increase of the chromomagnetic field.

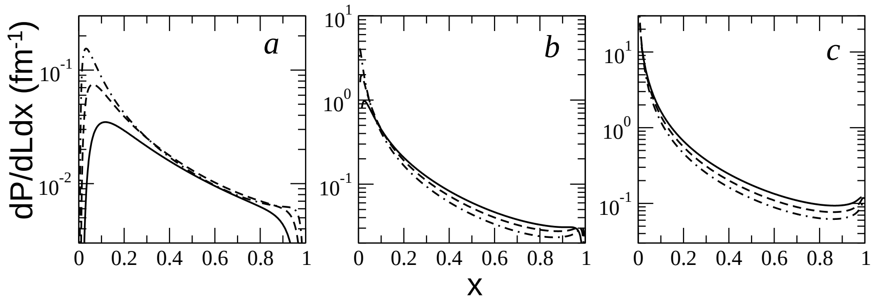

Figure 1: The spectrum for the $q \to gq$ process in the chromomagnetic field in the color state A for $\alpha_s = 0.3$, $gH_A/m_D^2 = 0.05$ (a), 0.25 (b) and 1 (c), for the initial quark energies $E_q = 20$ GeV (solid line), $E_q = 40$ GeV (dashed line), $E_q = 80$ GeV (dash-dotted line).

To estimate the synchrotron energy loss in the QGP produced in AA-collisions we take $gH \approx$

m_D^2. Approximately such a chromomagnetic field is necessary in the scenario with turbulent magnetic fields [11] for agreement with the small viscosity of the QGP observed at RHIC. In this case the ratio of the magnetic energy to the thermal parton energy is ~ 0.3, and a higher fraction of the magnetic energy looks unrealistic. For $\alpha_s = 0.3$ and $L \sim 2-4$ fm we obtained $\Delta E/E \sim 0.1-0.2$ for quarks and $\Delta E/E \sim 0.2-0.4$ for gluons at $E \sim 10-20$ GeV (for $\alpha_s = 0.5$ the results are about two times bigger). However, these estimates neglect any finite-size effects which may be important if $L_c \gtrsim L$ [10]. The dominating contribution to the energy loss comes from the soft gluon emission where $L_c \sim 1-2$ fm. In this situation the finite-size effects may suppress the energy loss by a factor ~ 0.5. The finite coherence length of the turbulent magnetic field, L_m, can suppress the radiation as well. If for the unstable magnetic field modes the wave vector $k^2 \lesssim \xi m_D^2$ [11] ($\xi \sim 1$ is the anisotropy parameter), this suppression should not be very strong since we have $L_m/L_c \sim 1$. As a plausible estimate one can take the turbulent suppression factor ~ 0.5. Even with these suppression factors the synchrotron loss turns out to be comparable with the collisional energy loss [12,13], which in turn is about 20-30% of the radiative energy loss. Thus our analysis demonstrates that the synchrotron radiation can be important in jet quenching and deserves further more accurate investigations. In particular, it would be interesting to treat the synchrotron radiation and emission due to multiple rescatterings on even footing. This can be done within the light-cone path integral formalism [14].

5. In summary, we have developed a quasiclassical theory of the synchrotron-like gluon radiation. Our calculations show that the parton energy loss due to the synchrotron radiation may be important in the jet quenching if the QGP instabilities generate magnetic field $H \sim m_D^2/g$. Our gluon spectrum disagrees with that obtained by Shuryak and Zahed [7]. We give simple physical arguments that the spectrum derived in [7] is incorrect.

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