Introduction: Top physics
I.: Virtual corrections
II: Real corrections
III: Results

Phenomenological relevance of $t\bar{t} + \text{jet}$

- Interesting signal process
  - Top quark physics plays an important role at the LHC.
  - Large fraction of inclusive $t\bar{t}$ events are due to $t\bar{t} + \text{jet}$.
  - Search for anomalous couplings.
  - Measurement of the forward-backward charge asymmetry at the Tevatron.

- Important background process for Higgs searches at the LHC:
  - Vector-boson-fusion: $pp(WW \rightarrow H) \rightarrow H + 2 \text{ jets}$.
  - Associated Higgs production: $pp \rightarrow t\bar{t}H$.

- Important background process for SUSY searches at the LHC.
The forward-backward charge asymmetry

Forward-backward charge asymmetry in $q\bar{q} \rightarrow t\bar{t}(+\text{jets})$

**Origin:** Interference of $C$-odd with $C$-even parts.

$q\bar{q} \rightarrow t\bar{t}$: asymmetry appears first at NLO (Kühn, Rodrigo '98).

$A_{FB} @ \text{NLO}$ not feasible in the near future (requires $\sigma_{NNLO}$).

$q\bar{q} \rightarrow t\bar{t} + \text{jet}$: asymmetry is LO effect (Halzen, Hoyer, Kim, '87).

$A_{FB} @ \text{NLO}$ can be deduced from $\sigma_{NLO}$.

LO-study for the Tevatron Bowen, Ellis, Rainwater, '05
“Technical” importance of $t\bar{t} + \text{jet}$

Benchmark process for one-loop calculations at the LHC.

**Significant complexity** due to:

- All partons coloured.
- Additional mass scale $m_t$.
- Complicated infra-red structure.
- Many sub-processes, many diagrams, ...

**Test ground for the development of new methods** for one-loop calculations.
The master formula for the calculation of observables

\[
\langle O \rangle = \frac{1}{2K(s)} \frac{1}{(2J_1 + 1)} \frac{1}{(2J_2 + 1)} \sum_n \int d\phi_{n-2} O(p_1, \ldots, p_n) \sum \left| A_n \right|^2
\]

Perturbative expansion of the amplitude (LO, NLO):

\[
\left| A_n \right|^2 = \left| A_n^{(0)} \right|^2 + \left( \left| A_n^{(0)} \right|^2 \left| A_n^{(1)} \right|^2 + \left| A_n^{(1)} \right|^2 \left| A_n^{(0)} \right|^2 \right),
\]

\[
\left| A_{n+1} \right|^2 = \left| A_{n+1}^{(0)} \right|^2 \left| A_{n+1}^{(0)} \right|^2.
\]
The virtual corrections

Main problem:
Numerically fast and stable evaluation of tensor integrals:

\[ \int d^D k \frac{k^\mu k^\nu k^\rho \ldots}{(k^2 - m_0^2) ((k + p_1)^2 - m_1^2) \ldots} \]

Many one-loop diagrams:
\sim 350 \text{ for } gg \rightarrow t\bar{t}g, \\
\sim 100 \text{ for } q\bar{q} \rightarrow t\bar{t}g.

High degree of automatisation needed!

Most complicated diagrams:

Pentagons
The virtual corrections

Two independent calculations of the virtual corrections:

Calculation 1:
- Diagram generation with FeynArts Küblbeck, Böhm, Denner '90, Hahn '01
- Symbolic part: Mathematica, numerics: Fortran
- Reduction of pentagons Denner, Dittmaier '02
- Analytical extraction of soft/collinear singularities Beenakker et al. '02, Dittmaier '03
- Treatment of critical phase space regions Denner, Dittmaier '05

Calculation 2:
- Diagram generation with Qgraf Nogueira '93
- Symbolic part: Form Vermaseren '00, numerics: C++
- Reduction of pentagons Giele, Glover '04
- Treatment of critical phase space regions Giele, Glover, Zanderighi '04
In addition to ultraviolet divergences, *loop integrals* can have *infrared divergences*.

For each IR divergence there is a *corresponding divergence with the opposite sign* in the real emission amplitude, when particles becomes *soft* or *collinear* (e.g. unresolved).

The *Kinoshita-Lee-Nauenberg* theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.
The dipole formalism

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

\[ \sigma^{NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V \]

\[ = \int_{n+1} (d\sigma^R - d\sigma^A) + \int_n \left( d\sigma^V + \frac{1}{1} d\sigma^A \right) \]

The approximation \( d\sigma^A \) has to fulfill the following requirements:

- \( d\sigma^A \) must be a proper approximation of \( d\sigma^R \) such as to have the same pointwise singular behaviour in \( D \) dimensions as \( d\sigma^R \) itself. Thus, \( d\sigma^A \) acts as a local counterterm for \( d\sigma^R \) and one can safely perform the limit \( \varepsilon \rightarrow 0 \).

- Analytic integrability in \( D \) dimensions over the one-parton subspace leading to soft and collinear divergences.

Catani and Seymour '96, Phaf and S.W. '01, Catani, Dittmaier, Seymour and Trócsányi '02.
Two independent calculations of the real corrections minus subtractions:

Calculation 1:
- Helicity amplitudes calculated numerically with recursion relations Berends, Giele '88
- Automated subtraction terms S.W. '05

Calculation 2:
- Analytical expressions for helicity amplitudes and/or Madgraph Stelzer, Long '94
- Semi-automated subtraction terms
Numerical results on $t\bar{t} + \text{jet}$ production

Dependence of the cross section on renormalisation and factorisation scale:

**Leading order is proportional to $\alpha_s^3$!**

**Tevatron:**

\[
p\bar{p} \rightarrow t\bar{t} + \text{jet} + X
\]

\[\sqrt{s} = 1.96 \text{ TeV} \quad p_{T,\text{jet}} > 20 \text{ GeV}\]

Jet definition: $k_\perp$-algorithm with $R = 1$ applied to particles other than $t$ or $\bar{t}$.

**LHC:**

\[
p p \rightarrow t\bar{t} + \text{jet} + X
\]

\[\sqrt{s} = 14 \text{ TeV} \quad p_{T,\text{jet}} > 20 \text{ GeV}\]
Results on the forward-backward asymmetry

\[ \sigma^\pm = \sigma(y_t > 0) \pm \sigma(y_t < 0), \]

\[ A_{FB,LO}^t = \frac{\sigma^-}{\sigma^+}, \]

\[ A_{FB,NLO}^t = \frac{\sigma^-}{\sigma^+} \left( 1 + \frac{\delta\sigma^-}{\sigma^-} - \frac{\delta\sigma^+}{\sigma^+} \right). \]

\((\mu = \mu_{ren} = \mu_{fact})\)

- \(A_{FB,LO}^t = O(\alpha_s^0)\), i.e. no dependence on \(\mu_{ren}\)
  mild dependence on \(\mu_{fact} \ll \) theoretical uncertainty!

- \(A_{FB,NLO}^t\) depends on \(\mu_{fact}\) and \(\mu_{ren}\)
  asymmetry almost washed out by scale dependence.
Distribution of the additional jet at the Tevatron

$$\left( \frac{d\sigma}{dp_{T,jet}} \right) [\text{fb/GeV}]$$

$$p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$$

$$\sqrt{s} = 1.96 \text{ TeV}$$

$$K = \frac{\text{NLO}}{\text{LO}}$$
Distribution of the additional jet at the LHC

\[ \frac{d\sigma}{dp_{T,jet}} \left[ \text{fb} \right] \]

\[ \text{pp} \rightarrow t\bar{t} + \text{jet} + X \]

\[ \sqrt{s} = 14 \text{ TeV} \]

\[ K = \text{NLO/LO} \]

\[ \frac{d\sigma}{dy_{jet}} \left[ \text{fb} \right] \]

\[ \text{pp} \rightarrow t\bar{t} + \text{jet} + X \]

\[ \sqrt{s} = 14 \text{ TeV} \]

\[ K = \text{NLO/LO} \]
Distribution of the top quark at the Tevatron

\[ \left( \frac{d\sigma}{dp_{T,t}} \right) \left[ \frac{fb}{GeV} \right] \]

\[ p\bar{p} \rightarrow t\bar{t} + \text{jet} + X \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]

\[ K = \text{NLO/LO} \]

\[ \left( \frac{d\sigma}{dy_t} \right) [fb] \]

\[ p\bar{p} \rightarrow t\bar{t} + \text{jet} + X \]

\[ \sqrt{s} = 1.96 \text{ TeV} \]

\[ K = \text{NLO/LO} \]
Summary

Top-pair production + jet:

- Study of top-quark properties at LHC and Tevatron.
- Important background process for Higgs and Susy searches.

NLO QCD corrections:

- NLO corrections stabilise LO cross section
- Forward-backward asymmetry receives large NLO corrections
- Differential distributions