Generalized D-dimensional unitarity and applications

Kirill Melnikov
Johns Hopkins University
Based on collaboration with K. Ellis, W. Giele, Z.Kunszt, G. Zanderighi
QCD Moriond, March 14-21, 2009
Introduction

- My talk is about next-to-leading order computations
- NLO QCD/EW is a step beyond ALPGEN, Madgraph, Comphep and similar programs
- A NLO QCD correction to a process X includes one-loop corrections to X and X+1 parton process
- NLO is the first approximation to QCD where some degree of control over theoretical predictions is possible
Introduction

• NLO QCD predictions are still unavailable for many processes of interest

• There is a sharp cut-off:
  • processes with less than three particles in the final state are ``easy''
  • processes with more than three particles in the final state are ``prohibitively difficult''.

• However, NLO QCD computations are of higher relevance for large-multiplicity processes where dependencies on renormalization and factorization scales are most severe
Introduction

- A typical NLO computation consists of two parts
  - one-loop virtual correction
  - real emission correction
- Focus on one-loop virtual
- Any one-loop amplitude can be expressed through certain (known) scalar integrals using Lorentz invariance
  \[ A = \sum_j c_j I_j \]
- The problem of NLO computations is to develop an efficient numerical method to compute coefficients $c_j$ in the above equation.
- Amazingly, this is still an open problem
Introduction

- For low multiplicity processes \((N < 5)\) the method of choice is the Passarino-Veltman (PV) reduction.
- PV-based approaches are highly optimized and deliver important physics results (MCFM, ttH, Wbb, Hjj, ttj, WWj, etc.).
- Two problems with PV reduction:
  - increase in the number of Feynman graphs
  - divisions by tiny numbers (Gram determinants)
- These problems prevented straightforward application of PV-based techniques to more complex \((N > 5)\) processes.
Introduction

- In recent years new ideas for NLO computations appeared; generalized unitarity is one of them

- Key observations:
  - unitarity constrains reduction coefficients
  - tree-level amplitudes are involved in the constraint

\[
\text{Im}\left(A^{1-\text{loop}}\right) \propto \sum \left|A^{\text{tree}}\right|^2 \quad \quad A^{1-\text{loop}} = \sum c_j I_j
\]

\[
\sum c_j \text{Im}(I_j) \propto \sum \left|A^{\text{tree}}\right|^2
\]

- We know how to use such constraints efficiently
Introduction

- Important steps include
  - The idea introduced by Bern, Dixon, Kosower
  - Cuts w.r.t. loop momentum give (box) coefficients directly (Cachazo, Britto, Feng)
  - Ossola-Pittau-Papadopoulos (OPP) tensor integral reduction technique
  - The OPP procedure meshes well with unitarity (Ellis, Kunszt, Giele)
  - D-dimensional unitarity (Giele, Kunszt, K.M.)
Any one-loop amplitude can be written as

\[ A^{1 \text{ loop}} = \int \frac{dk^D}{(2\pi)^D} \frac{\text{Num}_D(k, p)}{\prod_i d_i} = \sum c_j I_j \]

Ossola, Pittau and Papadopoulos told us how to compute coefficients \( c_j \) by considering such loop momenta for which sets of inverse propagators vanish.

Typically, a set of inverse propagators vanish for complex loop momenta.
Overview of generalized unitarity

- Inverse propagators vanish when particles go on the mass shell → one-loop integrand factorizes into a product of on-shell tree amplitudes

\[
\text{Num}_D(k, p) \rightarrow \prod_i A^{\text{tree}}_i(p), \{k_i\}
\]

- Those on-shell amplitudes are conventional; the only peculiarity is that external momenta are complex.

- Apart from that, we see that on-shell tree amplitudes is all that is needed for one-loop computations
Overview of generalized unitarity

- QCD needs regularization/renormalization and this leads to peculiarities
- Dimensional regularization shifts the number of dimensions from 4 to D, so it seems natural to deal with unitarity in D dimensions
- Unfortunately, dimensional regularization requires treating dimensionality of space-time D as a complex parameter → hard for numerical implementation
- Fortunately, if one understands how D-dependence arises, one can design a QFT which operates with integer-dimensional spaces and is "dual" to a QFT regularized dimensionally
Overview of generalized unitarity

- It turns out that this ``dual'' theory is a QFT in 6 or 8 dimensions, with momenta restricted to 5-dim subspace and all external particles to 4-dim subspace. Tree-level S-matrix of such theory is all that is required for one-loop computations.

- The price of calculating amplitudes in D rather than in 4 dimensions is only a factor of two in computing time.

- Berends-Giele recurrence relations are employed to compute tree level matrix elements in arbitrary D and for complex momenta.

- Spinors for fermions and polarization vectors for gluons in D=6,8 are constructed explicitly.
Implementation

• A number of attempts to employ unitarity and OPP ideas for one-loop computations (Blackhat, OPP, Lazopoulos, Giele & Winter)

• FORTRAN 90 program Rocket

• Currently, Rocket can compute the following one-loop amplitudes
  • N-gluon scattering amplitudes
  • two quark (massless and massive) + N-gluon scattering amplitudes
  • $W$ boson + two quarks + N-gluons
  • $W$ boson + four quarks + 1 gluon
  • $tt+N_{gluons}$, $ttqq+N$ gluons (Schulze)
Implementation

- Numerical implementation requires speed and stability. Speed seems to be marginally acceptable to handle $2 \rightarrow 5$ processes.
- Numerical stability is a mild issue, treated with computations in quadrupole precision.
First physics: W+3 jets

- Proof of concept that new methods for NLO QCD computations can compete with traditional methods
- NLO QCD corrections to W + 3 jets to be a case worth exploring because
  - relevant for phenomenology (Tevatron measurements, background to tt, single top, Higgs searches, SUSY searches)
  - large number (1480) of diagrams so traditional methods have a hard time
- All one-loop amplitudes required for this calculation are implemented in Rocket
First physics: $W+3$ jet

- We simplify the problem by
  - working at large $N_c$
  - keeping only two-quark channels ($qqW^+gluons$)
- These are 10-30 percent approximations, so phenomenology is rather preliminary
- Virtual corrections are computed using a grid determined from the leading order computation
- Dipole subtraction for real emission corrections is employed
First physics: $W^+ + 3$ jets

- $W^+$ production cross section at the LHC

- Note very strong dependence of the LO cross-section on ren/fact scales and the dependence of $K$-factor on the total transverse energy
Conclusions

• Generalized D-dim unitarity is a very robust method for one-loop computations

• A significant number of one-loop virtual amplitudes are implemented in F90 program Rocket

• It is (almost) possible to compute pp → 4jets, pp → tt+j, pp → V+3jet through NLO QCD using building blocks already available in Rocket

• Important issue is an automation of real emission corrections and the corresponding subtraction terms

• First results for W+3 jet are encouraging.