First global NNPDF analysis

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“A first unbiased global NLO determination of parton distribution functions”
arXiv:1002.4407
Outline

1. Introduction
2. NNPDF method
3. NNPDF2.0: a global fit
4. Conclusions and outlook
Factorization Theorem \((Q^2 \gg \Lambda_{\text{QCD}}^2)\):

\[
\frac{d\sigma_H}{dX} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_f) f_b(x_2, \mu_f) \otimes \frac{d\hat{\sigma}_{ab}}{dX}(\alpha_s(\mu_r), \mu_r, \mu_f, x_1, x_2, Q^2)
\]

DGLAP equations:

\[
\frac{d}{dt} \begin{pmatrix} q \\ g \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix} + O(\alpha_s^2)
\]

PDFs and their associated uncertainties will play a crucial role in the full exploitation of the LHC physics potential.
Given a set of data points we must determine a set of functions with errors.

We need an error band in the space of functions, i.e. a probability density $P[f(x)]$ in the space of PDFs, $f(x)$. For an observable $F$ depending on PDFs:

$$\langle F[f(x)] \rangle = \int [Df] F[f(x)] P[f(x)]$$

### Standard approach

1. Choose a specific functional form
   $$q_i(x, Q_0^2) = A_i x^{b_i} (1 - x)^{c_i} (1 + ...)$$

2. Determine best-fit values of parameters which define the functions

3. Errors determined via gaussian linear error propagation and large tolerance
   $$\Delta \chi^2 \gg 1$$

### Issues

1. **Parametrization**: how can we know that it is flexible enough and does not introduce a theoretical bias?

2. **Large tolerance** $T = \sqrt{\Delta \chi^2}$ means that errors are blown up by
   $$S = \sqrt{\Delta \chi^2 / 2.7}$$

3. **Benchmark partons do not agree with global partons within errors**
Motivation
Historical overview

Monte Carlo representation of the probability measure in the space of functions

Use of neural network as redundant and unbiased parametrization

- Structure functions [hep-ph/0501067]
- Non-singlet PDF $q^- = u + d - (\bar{u} + \bar{d})$ [hep-ph/0701127]
- DIS global analysis: NNPDF1.0 [arXiv:0808.1231]
- Determination of the strange content: NNPDF1.2 [arXiv:0906.1958]
- Global (DIS+DY+JET) analysis: NNPDF2.0 [arXiv:1002.4407]

All sets are available in the LHAPDF interface
Motivation
What’s nice about NNPDF #1

1) Monte Carlo behaves in a statistically consistent way

- Generate a Monte Carlo ensemble in the space of data
- $N$ pseudo–data reproduce the probability distribution of the original experimental data.
- Precision of estimators scales as the size of the sample.

2) Results are shown to be independent of the parametrization and even stable upon the addition of independent PDFs parametrizations

$7x37$ pars $\rightarrow 7x31$ pars
Motivation
What’s nice about NNPDF #2

3) PDFs behave as expected upon the addition of new data
HERA-LHC benchmark: DIS data

4) Control on PDFs uncertainties: NuteV anomaly solved AND $V_{cs}$ precise determination at the same time
[NNPDF1.2 [arXiv:0906.1958]]
NNPDF approach

General scheme

\[ F_i^{\text{rep}}(k) = S_{i,N}^{(k)} F_i^{\text{exp}} \left( 1 + r_i^{(k)} \sigma_i^{\text{stat}} + \sum_{j=1}^{N_{\text{sys}}} r_i^{(k)} \sigma_{i,j}^{\text{sys}} \right) \]

\[ r_i^{(k)} = r_{i'}^{(k)} \text{ if } i \text{ and } i' \text{ correlated} \]
NNPDF approach

Main Ingredients

Monte Carlo determination of errors:

\[
\langle F[f(x)] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} F[f^{(k)}_{\text{net}}(x)]
\]

\[
\sigma_{F[f(x)]} = \sqrt{\langle F[f(x)]^2 \rangle - \langle F[f(x)] \rangle^2}
\]

Neural Networks as redundant and unbiased parametrisation of PDFs:

\[
\Sigma(x) \rightarrow \text{NN}_\Sigma(x) \quad 2-5-3-1 \quad 37 \text{ pars}
\]

\[
g(x) \rightarrow \text{NN}_g(x) \quad 2-5-3-1 \quad 37 \text{ pars}
\]

\[
V(x) \rightarrow \text{NN}_V(x) \quad 2-5-3-1 \quad 37 \text{ pars}
\]

\[
T_3(x) \rightarrow \text{NN}_T_3(x) \quad 2-5-3-1 \quad 37 \text{ pars}
\]

ΔS(x) ≡ \bar{d}(x) - \bar{u}(x) \rightarrow \text{NN}_\Delta(x) \quad 2-5-3-1 \quad 37 \text{ pars}

s^+(x) ≡ (s(x) + \bar{s}(x))/2 \rightarrow \text{NN}_{s^+}(x) \quad 2-5-3-1 \quad 37 \text{ pars}

s^-(x) ≡ (s(x) - \bar{s}(x))/2 \rightarrow \text{NN}_{s^-}(x) \quad 2-5-3-1 \quad 37 \text{ pars}

Dynamical stopping criterion in order to fit data and not statistical noise (259 pars).

* Divide data in two sets: training and validation.

* Minimisation is performed only on the training set. The validation \(\chi^2\) for the set is computed.

* When the training \(\chi^2\) still decreases while the validation \(\chi^2\) stops decreasing → STOP.
Data sets

NNPDF2.0 dataset

<table>
<thead>
<tr>
<th>OBS</th>
<th>Data sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_2^p$</td>
<td>NMC, SLAC, BCDMS</td>
</tr>
<tr>
<td>$F_2^d$</td>
<td>SLAC, BCDMS</td>
</tr>
<tr>
<td>$F_2^d / F_2^p$</td>
<td>NMC-pd</td>
</tr>
<tr>
<td>$\sigma_{NC}$</td>
<td>HERA-I AV, ZEUS-H2</td>
</tr>
<tr>
<td>$\sigma_{CC}$</td>
<td>HERA-I AV, ZEUS-H2</td>
</tr>
<tr>
<td>$F_{t}$</td>
<td>H1</td>
</tr>
<tr>
<td>$\sigma_{\nu}, \sigma_{\bar{\nu}}$</td>
<td>CHORUS</td>
</tr>
<tr>
<td>dimuon prod.</td>
<td>NuTeV</td>
</tr>
<tr>
<td>$d\sigma_{DY}/dM^2dy$</td>
<td>E605</td>
</tr>
<tr>
<td>$d\sigma_{DY}/dM^2dx_F$</td>
<td>E886</td>
</tr>
<tr>
<td>W asymmetry</td>
<td>CDF</td>
</tr>
<tr>
<td>Z rap. distr.</td>
<td>CDF, D0</td>
</tr>
<tr>
<td>incl. $\sigma$jet)</td>
<td>D0(cone) Run II</td>
</tr>
<tr>
<td>incl. $\sigma$(jet)</td>
<td>CDF($k_T$) Run II</td>
</tr>
</tbody>
</table>

- Kinematical cuts on DIS data
  $Q^2 > 2$ GeV$^2$
  $W^2 = Q^2(1 - x)/x > 12.5$ GeV$^2$

- No cuts on hadronic data

3477 data points

For comparison MSTW08 includes 2699 data points
NLO computation of hadronic observables too slow for parton global fits.

MSTW08 and CTEQ include Drell-Yan NLO as (local) K factors rescaling the LO cross section.

K-factor depends on PDFs and it is not always a good approximation.

- NNPDF2.0 includes full NLO calculation of hadronic observables.
- Use available fastNLO interface for jet inclusive cross-sections.[hep-ph/0609285]
- Built up our own FastKernel computation of DY observables.

Both PDFs evolution and double convolution sped up

Use high-orders polynomial interpolation

Precompute all Green Functions
Results

Statistical features: global $\chi^2$

<table>
<thead>
<tr>
<th>$\chi^2_{\text{tot}}$</th>
<th>1.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle E \rangle \pm \sigma_E$</td>
<td>$2.32 \pm 0.10$</td>
</tr>
<tr>
<td>$\langle E_{\text{val}} \rangle \pm \sigma_{E_{\text{val}}}$</td>
<td>$2.29 \pm 0.11$</td>
</tr>
<tr>
<td>$\langle \mathcal{E}<em>{\text{val}} \rangle \pm \sigma</em>{\mathcal{E}_{\text{val}}}$</td>
<td>$2.35 \pm 0.12$</td>
</tr>
<tr>
<td>$\langle TL \rangle \pm \sigma_{TL}$</td>
<td>$16175 \pm 6275$</td>
</tr>
<tr>
<td>$\langle \chi^2(k) \rangle \pm \sigma_{\chi^2}$</td>
<td>$1.29 \pm 0.09$</td>
</tr>
</tbody>
</table>
Results
Statistical features: individual experiments

Distribution of $\chi^2$ for sets

No obvious data incompatibility
Results
Comparison to data included in the fit
Reduction of uncertainties with respect to older NNPDF sets.

Small uncertainties due to excellent data compatibility.

Uncertainty larger than other groups when MSTW/CTEQ parametrizations are too restrictive.
Impact of modifications
A quantitative assessment

- A quantitative assessment is possible

\[
d(q_j) = \sqrt{\left\langle \frac{(\langle q_j \rangle_{(1)} - \langle q_j \rangle_{(2)})^2}{\sigma_1^2[q_j] + \sigma_2^2[q_j]} \right\rangle_{N_{\text{part}}}}
\]

\[
d(\sigma_j) = \sqrt{\left\langle \frac{(\langle \sigma_j \rangle_{(1)} - \langle \sigma_j \rangle_{(2)})^2}{\sigma_1^2[\sigma_j] + \sigma_2^2[\sigma_j]} \right\rangle_{N_{\text{part}}}}
\]

- Comparisons performed in NNPDF2.0 analysis
  - Start from NNPDF1.2
  - NNPDF1.2 vs. NNPDF1.2 + minimization/training improvements
  - Improved NNPDF1.2 vs. Improved NNPDF1.2 + \( t_0 \)-method
  - Fit to DIS dataset with H1/ZEUS data vs. Fit with HERA-I combined
  - Fit to DIS dataset vs. Fit to DIS+JET
  - Fit to DIS+JET vs. NNPDF2.0 final
Impact of modifications

HERA-I combined dataset

<table>
<thead>
<tr>
<th>Fit</th>
<th>NNPDF1.2</th>
<th>NNPDF1.2+IGA</th>
<th>NNPDF1.2+IGA+t0</th>
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<tr>
<td>$\chi^2_{\text{tot}}$</td>
<td>1.32</td>
<td>1.16</td>
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</tr>
<tr>
<td>$\langle E \rangle$</td>
<td>2.79</td>
<td>2.41</td>
<td>2.24</td>
<td>2.31</td>
</tr>
<tr>
<td>$\langle \chi^2(k) \rangle$</td>
<td>1.60</td>
<td>1.28</td>
<td>1.21</td>
<td>1.29</td>
</tr>
<tr>
<td>HERA-I</td>
<td>1.05</td>
<td>0.98</td>
<td>0.96</td>
<td>1.13</td>
</tr>
</tbody>
</table>

- HERA-I combined more precise.
- Quality of other data unchanged.
- Overall fit quality to the whole dataset is good
  - $\sigma^+_{\text{NC}}$ dataset has relatively high $\chi^2 \sim 1.3$
  - $\sigma^-_{\text{CC}}$ dataset has very low $\chi^2 \sim 0.55$
- Same $\chi^2$-pattern observed in the HERAPDF1.0 analysis
- Impact on PDFs, mainly Singlet and Gluon at small-$x$
Impact of modifications
Tevatron inclusive Jet data

<table>
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<tr>
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<th>NNPDF1.2+IGA+t₀</th>
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</tr>
<tr>
<td>CDFR2KT</td>
<td>1.10</td>
<td>0.95</td>
<td>0.78</td>
<td>0.91</td>
<td>0.79</td>
</tr>
<tr>
<td>D0R2CON</td>
<td>1.18</td>
<td>1.07</td>
<td>0.94</td>
<td>1.00</td>
<td>0.93</td>
</tr>
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- Tevatron Run-II inclusive jet data provide a valuable constrain on large-x gluon.
- No incompatibility.
- Run-I data not included but compatibility with the outcome of the fit has been checked.
Impact of modifications
Drell-Yan and Vector Boson production data

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<td>DYE866</td>
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<td>CDFFWASY</td>
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<td>1.85</td>
</tr>
<tr>
<td>CDFZRAP</td>
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<td>4.61</td>
<td>3.13</td>
<td>3.12</td>
<td>3.31</td>
<td>2.02</td>
</tr>
<tr>
<td>D0ZRAP</td>
<td>0.56</td>
<td>0.80</td>
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- Good description of fixed target Drell-Yan data (E605 proton and E886 proton and p/d ratio)
- Vector boson production at colliders (CDF W-asymmetry and Z rapidity distribution) harder to fit
- All valence-type PDF combinations are affected by these data
- Sizable reduction in the uncertainty of the strange valence (possible impact on NuTeV anomaly)
**Impact of modifications**

**Drell-Yan and Vector Boson production data**

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Conclusion and outlook

- **Monte Carlo** ensemble
  - Any statistical property of PDFs can be calculated using standard statistical methods.
  - No need of any tolerance criterion.

- **The Neural Network** parametrization
  - Small uncertainties come from the data, not from bias due to functional form.
  - Inconsistent data or underestimated uncertainties do not require a separate treatment and are automatically signalled by a larger value of the $\chi^2$.

- The NNPDF2.0 is the first unbiased global **NLO** fit [FastKernel].
- Same consistent statistical behaviour under addition of hadronic data.
- **No signs of incompatibility between data!!!!**
- Available on the common LHAPDF interface (http://projects.hepforge.org/lhapdf)
- The NNPDF2.X with FONLL (see Ref. ArXiv:1001.2312) with better treatment of heavy quarks mass will be soon available.
- The NNPDF2.Y **NNLO** fit is a work in progress.

**THANK YOU!!**
For some standard candle processes at LHC the uncertainty on PDFs is the dominant one.

- $\sigma(Z^0)$ at LHC: $\delta\sigma_{\text{PDG}} \sim 2\text{-}3\%$, $\delta\sigma_{\text{pert}} \sim 1\%$
- $\sigma(W^+, -)$ at LHC: $\delta\sigma_{\text{PDG}} \sim 3\text{-}4\%$
NNPDF approach
Ingredient #1: Monte Carlo Errors

Generate a $N_{\text{rep}}$ Monte Carlo sets of artificial data, or "pseudo-data" of the original $N_{\text{data}}$ data points

$$F_{i}^{(\text{art})(k)}(x_{p}, Q_{p}^{2}) \equiv F_{i,p}^{(\text{art})(k)}$$

$$i = 1, \ldots, N_{\text{data}}$$

$$k = 1, \ldots, N_{\text{rep}}$$

Multi-gaussian distribution centered on each data point:

$$F_{i,p}^{(\text{art})(k)} = S_{p,N}^{(k)} F_{i,p}^{\text{exp}} \left(1 + r_{p}^{(k)} \sigma_{p}^{\text{stat}} + \sum_{j=1}^{N_{\text{sys}}} r_{p,j}^{(k)} \sigma_{p,j}^{\text{sys}}\right)$$

If two points have correlated systematic uncertainties

$$r_{p,j}^{(k)} = r_{p',j}^{(k)}$$

Correlations are properly taken into account.
NNPDF approach
Ingredient \#1: Monte Carlo Errors

\[
\langle \mathcal{F}[f(x)] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{F}[f^{(k)}(\text{net})(x)] \\
\sigma_{\mathcal{F}[f(x)]} = \sqrt{\langle \mathcal{F}[f(x)]^2 \rangle - \langle \mathcal{F}[f(x)] \rangle^2}
\]

Even though individual replicas may fluctuate significantly, average quantities such as central values and error bands are smooth inasmuch as stability is reached due to the dimension of the ensemble increasing.
NNPDF approach

Ingredient 2: Neural Network as unbiased parametrization

Each independent PDF at the initial scale $Q_0^2 = 2\text{GeV}^2$ is parameterized by an individual NN.

* Each neuron receives input from neurons in preceding layer.
* Activation determined by weights and thresholds according to a non linear function:

$$\xi_i = g\left(\sum_j \omega_{ij} \xi_j - \theta_i\right), \quad g(x) = \frac{1}{1 + e^{-x}}$$

In a simple case (1-2-1) we have,

$$\xi_1^{(3)} = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - \xi_1^{(1)} \omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - \xi_1^{(1)} \omega_{21}^{(1)}}}}}$$

...Just a convenient functional form which provides a redundant and flexible parametrization.

We want the best fit to be independent of any assumption made on the parametrization.
NNPDF approach
Ingredient #2: Neural Network as unbiased parametrization

Basis set: \( Q_0^2 = 2 \text{ GeV}^2 \)

Singlet: \( \Sigma(x) \) \( \longrightarrow \text{NN}_\Sigma(x) \) 2-5-3-1 37 pars
Gluon: \( g(x) \) \( \longrightarrow \text{NN}_g(x) \) 2-5-3-1 37 pars
Total valence: \( V(x) \equiv u_V(x) + d_V(x) \) \( \longrightarrow \text{NN}_V(x) \) 2-5-3-1 37 pars
Non-singlet triplet: \( T_3(x) \) \( \longrightarrow \text{NN}_{T_3}(x) \) 2-5-3-1 37 pars
Sea asymmetry: \( \Delta_S(x) \equiv \bar{d}(x) - \bar{u}(x) \) \( \longrightarrow \text{NN}_{\Delta}(x) \) 2-5-3-1 37 pars
Total strangeness: \( s^+(x) \equiv (s(x) + \bar{s}(x))/2 \) \( \longrightarrow \text{NN}_{(s^+)}(x) \) 2-5-3-1 37 pars
Strangeness valence: \( s^-(x) \equiv (s(x) - \bar{s}(x))/2 \) \( \longrightarrow \text{NN}_{(s^-)}(x) \) 2-5-3-1 37 pars

259 parameters
NNPDF approach
Ingredient #3: Training and dynamical stopping

Our fitting strategy is very different from that used by other collaborations: instead of a set of basis functions with a small number of parameters, we have an unbiased basis of functions parameterized by a very large and redundant set of parameters.

CTEQ, MSTW, AL

\( \mathcal{O}(20) \) parm

NNPDF

\( \mathcal{O}(200) \) parm

Not trivial because ...

A redundant parametrization might adapt not only to physical behavior but also to random statistical fluctuations of data.

Ingredients of fitting procedure

- Flexible and redundant parametrization
- Genetic Algorithm minimization
- Dynamical stopping criterion: cross-validation technique
**NNPDF approach**

**Ingredient #3: Dynamical stopping**

* GA is monotonically decreasing by construction.
* The best fit is not given by the absolute minimum.
* The best fit is given by an optimal training beyond which the figure of merit improves only because we are fitting statistical noise of the data.

**Cross-validation method**

* Divide data in two sets: training and validation.
* Random division for each replica ($f_t = f_v = 0.5$).
* Minimisation is performed only on the training set. The validation $\chi^2$ for the set is computed.
* When the training $\chi^2$ still decreases while the validation $\chi^2$ stops decreasing $\rightarrow$ STOP.
Motivation

What’s nice about NNPDF #2

Results are statistically independent of the choice of the parametrization

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Extrapolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma(x, Q^2_0)$</td>
<td>$5 \times 10^{-4} \leq x \leq 0.1$</td>
<td>$10^{-5} \leq x \leq 10^{-4}$</td>
</tr>
<tr>
<td>$\langle d[f] \rangle$</td>
<td>0.62</td>
<td>0.88</td>
</tr>
<tr>
<td>$\langle d[\sigma] \rangle$</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>$g(x, Q^2_0)$</td>
<td>$5 \times 10^{-4} \leq x \leq 0.1$</td>
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</tr>
<tr>
<td>$\langle d[f] \rangle$</td>
<td>1.07</td>
<td>0.87</td>
</tr>
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<td>0.86</td>
<td>0.78</td>
</tr>
<tr>
<td>$T_3(x, Q^2_0)$</td>
<td>$0.05 \leq x \leq 0.75$</td>
<td>$10^{-3} \leq x \leq 10^{-2}$</td>
</tr>
<tr>
<td>$\langle d[f] \rangle$</td>
<td>1.00</td>
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<tr>
<td>$\langle d[\sigma] \rangle$</td>
<td>1.24</td>
<td>1.61</td>
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<tr>
<td>$V(x, Q^2_0)$</td>
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<tr>
<td>$\langle d[f] \rangle$</td>
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<td>0.90</td>
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<td>$\langle d[\sigma] \rangle$</td>
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<tr>
<td>$\Delta S(x, Q^2_0)$</td>
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<td>$\langle d[f] \rangle$</td>
<td>0.84</td>
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<td>$\langle d[\sigma] \rangle$</td>
<td>1.02</td>
<td>1.12</td>
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</table>

$37$ pars $\rightarrow$ $31$ pars
Motivation

What’s nice about NNPDF #2

Results are statistically independent of the choice of the parametrization

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</tbody>
</table>

37 pars $\rightarrow$ 31 pars
Motivation

What's nice about NNPDF #3

- PDFs are stable upon the addition of new independent PDFs parametrizations

- **NNPDF1.0**: flavor assumptions, symmetric strange sea proportional to non strange sea according to $C_s \sim 0.5$ suggested by neutrino DIS data.

  $$ s(x) = \bar{s}(x) \quad \bar{s}(x) = \frac{C_s}{2}(\bar{u}(x) + \bar{d}(x)) $$

- **NNPDF1.1**: independent parametrization of the strange content of the nucleon.

  - Total strangeness: $s^+(x) \equiv (s(x) + \bar{s}(x))/2 \rightarrow NN_{(s^+)}(x)$ 2-5-3-1 37 pars
  - Strangeness valence: $s^-(x) \equiv (s(x) - \bar{s}(x))/2 \rightarrow NN_{(s^-)}(x)$ 2-5-3-1 37 pars

- Added two unconstrained PDFs.

  - 185 $\rightarrow$ 259 parameters

- Only strange (and $\Sigma$) affected. Gluon and statistical features remain unchanged.
PDFs are stable upon the addition of new independent PDFs parametrizations.

- Larger in extrapolation region due to more flexibility (+ 60 pars).
- Same $\chi^2$ and statistical features of the fit. Same gluon shape and error band.
Define second momentum of PDFs $f$: $[F] = \int_0^1 dx \times f(x, Q^2)$.

Discrepancy $\geq 3\sigma$ between indirect and direct determination from NuTeV measurement assuming $[S^-] = 0$ and isospin symmetry.

**EW fit**

$$\sin^2 \theta_W = 0.2223 \pm 0.0002$$

**NuTeV**

$$\sin^2 \theta_W = 0.2276 \pm 0.0014$$

$$\delta_s \sin^2 \theta_W \sim -0.240 \frac{[S^-]}{[Q^-]}$$

$$\delta_s \sin^2 \theta_W = -0.0005 \pm 0.0096^{\text{PDFs}} \pm \text{sys}$$
Motivation
What's nice about NNPDF #5

Control on PDFs uncertainties: NuteV anomaly solved AND precision studies at the same time

CKM fit

\[ V_{cs} = 0.97334 \pm 0.00023, \quad \Delta V_{cs} \sim 0.02\% \]

Direct Determination

\[ V_{cs} = 1.04 \pm 0.06, \quad \Delta V_{cs} \sim 6\% \quad \text{D/B decays} \]
\[ V_{cs} > 0.59 \quad \text{DIS fit} \]

NNPDF1.2 analysis

\[ V_{cs} = 0.97 \pm 0.07, \quad \Delta V_{cs} \sim 6\% \]
DIS data are insufficient to determine accurately PDFs.

Flavor decomposition of quark–antiquark sea and large–$x$ gluon distribution.

\[
R^{pd} = \frac{d\sigma^d/dM^2/dx_F}{d\sigma^p/dM^2/dx_F} \propto (1 + \bar{d}/\bar{u})
\]

\[
A^W = \frac{d\sigma^+/dy - d\sigma^-/dy}{d\sigma^+/dy - d\sigma^-/dy} \propto \frac{u\bar{d} - d\bar{u}}{u\bar{d} + d\bar{u}}
\]

NNPDF2.0 includes most of the available hadronic data.
GLOBAL: Includes fixed target Drell-Yan, Tevatron jets and weak bosons data.

Improvements in training/stopping
  - Target Weighted Training
  - Improved stopping

Improved treatment of normalization errors ($t_0$ method)
  - For details see [arXiv:0912.2276]

Fast DGLAP evolution based on higher-order interpolating polynomials

NLO: Fast computation of Drell-Yan observables based on higher-order interpolating polynomials.

Enforced positivity constraints
New Features

FastKernel

- New strategy to solve DGLAP evolution equation
- Implementation benchmarked against the Les Houches tables
- Gain in speed by a factor 30 (for a fit to 3000 datapoints)

<table>
<thead>
<tr>
<th>x (50 pts)</th>
<th>$e_{\text{rel}}(u_{\nu})$</th>
<th>$e_{\text{rel}}(\Sigma)$</th>
<th>$e_{\text{rel}}(g)$</th>
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<tr>
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<td>$1.0 \cdot 10^{-3}$</td>
<td>$8.0 \cdot 10^{-4}$</td>
<td>$2.8 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

- Drell–Yan fast computation exploits linear interpolation
- Accuracy below 1% for all points included in the fit
- Increasing number of points in the grid one can improve accuracy;

A truly NLO analysis

Maria Ubiali (University of Edinburgh & Université Catholique de Louvain)

First global NNPDF analysis

Rencontres de Moriond, QCD session
## Impact of modifications

<table>
<thead>
<tr>
<th>Fit</th>
<th>NNPDF1.2</th>
<th>NNPDF1.2+IGA</th>
<th>NNPDF1.2+IGA+t₀</th>
<th>2.0 DIS</th>
<th>2.0 DIS+JET</th>
<th>NNPDF2.0</th>
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<td>1.20</td>
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<td>(\langle E \rangle)</td>
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<td>2.31</td>
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</tbody>
</table>
Impact of modifications
HERA-I combined dataset

Distance between central values

Distance between PDF uncertainties

Maria Ubiali (University of Edinburgh & Université Ca
First global NNPDF analysis
Rencontres de Moriond, QCD session 42 / 53
Impact of modifications
Drell-Yan and Vector Boson production data

Distance between central values

Distance between PDF uncertainties

Maria Ubiali (University of Edinburgh & Université Catholique de Louvain)
First global NNPDF analysis
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Impact of modifications
Tevatron inclusive Jet data

Distance between central values
NNPDF 2.0-DIS vs. 2.0-DIS+JET

Distance between PDF uncertainties
NNPDF 2.0-DIS vs. 2.0-DIS+JET

Maria Ubiali (University of Edinburgh & Université C)
We want the fitting procedure to explore only the subspace of acceptable physical solutions.

- We want $F_L$ positive.
- Dimuon cross-section positive.
- Momentum and valence sum rules.

Modify the training function with the addition of a Lagrangian multiplier:

$$E^{(k)} \rightarrow E^{(k)} - \lambda_{\text{pos}} \sum_{l=1}^{N_{\text{dat, pos}}} \Theta(F_l^{(\text{net})(k)}) F_l^{(\text{net})(k)}$$

- $N_{\text{dat, pos}}$ : number of pseudodata points used to implement positivity constraints.
- $\lambda_{\text{pos}}$ : associated Lagrangian multiplier ($10^{10}$)
Results

Impact of positivity

Maria Ubiali (University of Edinburgh & Université Catholique de Louvain)

First global NNPDF analysis

Rencontres de Moriond, QCD session
Results

Gaussian behaviour
Some phenomenology
The proton strangeness revisited

![Graph showing distributions for NNPDF1.2 and NNPDF2.0]

Determinations of the weak mixing angle $\sin^2 \theta_W$

- NuTeV01
- NuTeV01
- NuTeV01
- EW fit
- NNPDF1.2 [S]
- NNPDF2.0 [S]

$$K_S = \begin{cases} 0.71^{+0.19}_{-0.31}^{\text{stat}} \pm 0.26^{\text{syst}} & \text{(NNPDF1.2)} \\ 0.503 \pm 0.075^{\text{stat}} & \text{(NNPDF2.0)} \end{cases}$$

$$R_S = \begin{cases} 0.006 \pm 0.045^{\text{stat}} \pm 0.010^{\text{syst}} & \text{(NNPDF1.2)} \\ 0.019 \pm 0.008^{\text{stat}} & \text{(NNPDF2.0)} \end{cases}$$

- Uncertainty reduced by addition of DY data
- Striking agreement with EW fits

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First global NNPDF analysis
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Results
Parton Luminosities

\[ \Phi_{gg}(M_X^2) = \frac{1}{s} \int_{\tau}^{1} \frac{dx_1}{x_1} \, g\left(x_1, M_X^2\right) \, g\left(\frac{\tau}{x_1}, M_X^2\right) \]

\[ \Phi_{gq}(M_X^2) = \frac{1}{s} \int_{\tau}^{1} \frac{dx_1}{x_1} \left[ g\left(x_1, M_X^2\right) \, \Sigma\left(\frac{\tau}{x_1}, M_X^2\right) + (1 \rightarrow 2) \right] \]

\[ \Phi_{qq}(M_X^2) = \frac{1}{s} \int_{\tau}^{1} \frac{dx_1}{x_1} \sum_{i=1}^{N_f} \left[ q_i\left(x_1, M_X^2\right) \bar{q}_i\left(\frac{\tau}{x_1}, M_X^2\right) + (1 \rightarrow 2) \right] \]
Greater sensitivity to $\alpha_s$ than NNPDF1.2
Greater NLO corrections for Drell-Yan observables.
Some phenomenology
LHC standard candles

- W$^+$ Cross Section at the LHC [MCFM]
- Z$^0$ Cross Section at the LHC [MCFM]
- W Cross Section at the LHC [MCFM]
- H Cross Section at the LHC [gg fusion, $M_H=120$ GeV, MCFM]
- ttbar Cross Section at the LHC [MCFM]

- HQ treatment?
- Impact of K factor approximation?
- Impact of rigid parametrization?
Some applications of the NNPDF technique

Predictions on future experimental constraints on PDFs.

- Generate LHeC pseudo-data.
- Add them to the data set.
- Fit them (or reweight)

LHeC Linac(150 GeV)-Ring, Scenario E

- Same settings: no tuning
- High predictivity.

Maria Ubiali (University of Edinburgh & Université Ca

First global NNPDF analysis

Rencontres de Moriond, QCD session 52 / 53
Impact of modifications

To conclude...

* Precise HERA data
  \( \rightarrow \) Small × gluon & singlet

* Tevatron W asymmetry data
  \( \rightarrow \) Small × flavor separation

* Fixed Target DIS data, Drell-Yan, neutrino inclusive
  \( \rightarrow \) Small × flavor separation

* Neutrino dimuon
  \( \rightarrow \) Strangeness

* Tevatron jets
  \( \rightarrow \) Large × gluon

No signs of tension between datasets included in the analysis!!!