FROM Ω^- TO Ω_b

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I discuss several recent highly accurate theoretical predictions for masses of baryons containing the b quark, especially Ω_b (ssb) very recently reported by CDF. I also point out an approximate effective supersymmetry between heavy quark baryons and mesons and provide predictions for the magnetic moments of Λ_c and Λ_b. Proper treatment of the color-magnetic hyperfine interaction in QCD is crucial for obtaining these results.

1 Introduction

QCD describes hadrons as valence quarks in a sea of gluons and $\bar{q}q$ pairs. At distances above ~ 1 GeV^{-1} quarks acquire an effective *constituent mass* due to chiral symmetry breaking. A hadron can then be thought of as a bound state of constituent quarks. In the zeroth-order approximation the hadron mass M is then given by the sum of the masses of its constituent quarks m_i, $M = \sum_i m_i$. The binding and kinetic energies are "swallowed" by the constituent quarks masses. The first and most important correction comes from the color hyper-fine (HF) chromo-magnetic interaction,

$$M = \sum_i m_i + V_{i<j}^{HF(QCD)}; \qquad V_{ij}^{HF(QCD)} = v_0\,(\vec{\lambda}_i \cdot \vec{\lambda}_j)\,\frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j}\,\langle\psi|\delta(r_i - r_j)|\psi\rangle \tag{1}$$

where v_0 gives the overall strength of the HF interaction, $\vec{\lambda}_{i,j}$ are the $SU(3)$ color matrices, $\sigma_{i,j}$ are the quark spin operators and $|\psi\rangle$ is the hadron wave function. This is a contact spin-spin interaction, analogous to the EM hyperfine interaction, which is a product of the magnetic moments,

$$V_{ij}^{HF(QED)} \propto \vec{\mu}_i \cdot \vec{\mu}_j = e^2\,\frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \tag{2}$$

in QCD, the $SU(3)_c$ generators take place of the electric charge. From eq. (1) many very accurate results have been obtained for the masses of the ground-state hadrons. Nevertheless, several caveats are in order. First, this is a low-energy phenomenological model, still awaiting a rigorous derivation from QCD. It is far from providing a complete description of the hadronic spectrum, but it provides excellent predictions for mass splittings and magnetic moments. The crucial assumptions of the model are: (a) HF interaction is considered as a perturbation which does not change the wave function; (b) effective masses of quarks are the same inside mesons and baryons; (c) there are no 3-body effects.

2 Quark masses

Table I shows the quark mass differences obtained from mesons and baryons [1]. The mass difference between two quarks of different flavors denoted by i and j are seen to have the same value to a good approximation when they are bound to a "spectator" quark of a given flavor.

On the other hand, Table 1 shows clearly that *constituent quark mass differences depend strongly on the flavor of the spectator quark*. For example, $m_s - m_d \approx 180$ MeV when the spectator is a light quark but the same mass difference is only about 90 MeV when the spectator is a b quark.

Since these are *effective masses*, we should not be surprised that their difference is affected by the environment, but the large size of the shift is quite surprising and its quantitative derivation from QCD is an outstanding challenge for theory. Let us now discuss the HF splitting in mesons and baryons. We have

$$M(K^*)-M(K) = 4v_0\,\frac{\vec{\lambda}_u\cdot\vec{\lambda}_s}{m_u m_s}\,\langle\psi|\delta(r)|\psi\rangle, \qquad (3)$$

$$M(\Sigma^*) - M(\Sigma) = 6v_0\,\frac{\vec{\lambda}_u\cdot\vec{\lambda}_s}{m_u m_s}\,\langle\psi|\delta(r_{rs})|\psi\rangle \qquad (4)$$

Table I. Quark mass differences.

observable	baryons		mesons				Δm_{Bar}	Δm_{Mes}
			$J=1$		$J=0$			
	B_i	B_j	$\mathcal{V}_i$	$\mathcal{V}_j$	$\mathcal{P}_i$	$\mathcal{P}_j$	MeV	MeV
$\langle m_s - m_u\rangle_d$	sud	uud	$s\bar{d}$	$u\bar{d}$	$s\bar{d}$	$u\bar{d}$	177	179
	Λ	N	K^*	ρ	K	π		
$\langle m_s - m_u\rangle_c$			$c\bar{s}$	$c\bar{u}$	$c\bar{s}$	$c\bar{u}$		103
			D_s^*	D_s^*	D_s	D_s		
$\langle m_s - m_u\rangle_b$			$b\bar{s}$	$b\bar{u}$	$b\bar{s}$	$b\bar{u}$		91
			B_s^*	B_s^*	B_s	B_s		
$\langle m_c - m_u\rangle_d$	cud	uud	$c\bar{d}$	$u\bar{d}$	$c\bar{d}$	$u\bar{d}$	1346	1360
	Λ_c	N	D^*	ρ	D	π		
$\langle m_c - m_u\rangle_c$			$c\bar{c}$	$u\bar{c}$	$c\bar{c}$	$u\bar{c}$		1095
			ψ	D^*	η_c	D		
$\langle m_c - m_s\rangle_d$	cud	sud	$c\bar{d}$	$s\bar{d}$	$c\bar{d}$	$s\bar{d}$	1169	1180
	Λ_c	Λ	D^*	K^*	D	K		
$\langle m_c - m_s\rangle_c$			$c\bar{c}$	$s\bar{c}$	$c\bar{c}$	$s\bar{c}$		991
			ψ	D_s^*	η_c	D_s		
$\langle m_b - m_u\rangle_d$	bud	uud	$b\bar{d}$	$u\bar{d}$	$b\bar{d}$	$u\bar{d}$	4685	4700
	Λ_b	N	B^*	ρ	B	π		
$\langle m_b - m_u\rangle_s$			$b\bar{s}$	$u\bar{s}$	$b\bar{s}$	$u\bar{s}$		4613
			B_s^*	K^*	B_s	K		
$\langle m_b - m_s\rangle_d$	bud	sud	$b\bar{d}$	$s\bar{d}$	$b\bar{d}$	$s\bar{d}$	4508	4521
	Λ_b	Λ	B^*	K^*	B	K		
$\langle m_b - m_c\rangle_d$	bud	sud	$b\bar{d}$	$c\bar{d}$	$b\bar{d}$	$c\bar{d}$	3339	3341
	Λ_b	Λ_c	B^*	D^*	B	D		
$\langle m_b - m_c\rangle_s$			$b\bar{s}$	$c\bar{s}$	$b\bar{s}$	$c\bar{s}$		3328
			B_s^*	D_s^*	B_s	D_s		

we can then use eqs. (3) and (4) to compare the quark mass ratio obtained from mesons and baryons:

$$\begin{aligned}(m_c/m_s)_{Bar} &= (M_{\Sigma^*} - M_\Sigma)/(M_{\Sigma_c^*} - M_{\Sigma_c}) = 2.84\\ (m_c/m_s)_{Mes} &= (M_{K^*} - M_K/(M_{D^*} - M_D) = 2.81\end{aligned} \qquad (5)$$

Similarly, $(m_c/m_u)_{Bar} = 4.36$ and $(m_c/m_u)_{Mes} = 4.46$. We find the same value from mesons and baryons $\pm 2\%$. We can write down an analogous relation for hadrons containing the b quark instead of the s quark, obtaining the prediction for splitting between Σ_b and Λ_b:

$$\frac{M_{\Sigma_b}-M_{\Lambda_b}}{M_\Sigma - M_\Lambda} = \frac{(M_\rho - M_\pi)-(M_{B^*}-M_B)}{(M_\rho - M_\pi)-(M_{K^*}-M_K)} = 2.51 \qquad (6)$$

yielding $M(\Sigma_b) - M(\Lambda_b) = 194\,\mathrm{MeV}$ [1,2], to be compared with the isospin average of the recent measurement by CDF[3], $M(\Sigma_b) - M(\Lambda_b) = 192 \pm 2.3$ MeV. There is also the prediction for the spin splittings, $M(\Sigma_b^*) - M(\Sigma_b) = [M(B^*) - M(B)] \cdot [M(\Sigma^*) - M(\Sigma)]/[M(K^*) - M(K)] = 22\,\mathrm{MeV}$, to be compared with 21 MeV from the isospin-average of CDF measurements [3]. The challenge is to understand how and under what assumptions one can derive from QCD the very simple model of hadronic structure at low energies which leads to such accurate predictions.

3 Effective Meson-Baryon SUSY

Some of the results described above can be understood [2] by observing that in the hadronic spectrum there is an approximate effective supersymmetry between mesons and baryons related by replacing a light antiquark by a light diquark. This supersymmetry transformation goes beyond the simple constituent quark model. It assumes only a valence quark of flavor i with a model independent structure bound to "light quark brown muck color antitriplet" of model-independent structure carrying the quantum numbers of a light antiquark or a light diquark. The mass difference between the meson and baryon related by this transformation has been shown [4] to be independent of the quark flavor i for all four flavors (u, s, c, b) when the contribution of the hyperfine interaction energies is removed as denoted by the "$\sim$" overscript. For the two cases of spin-zero $S = 0$ and spin-one $S = 1$ diquarks [4] (all masses differences in MeV):

$$M(N) - \tilde{M}(\rho){=}323{\approx}M(\Lambda) - \tilde{M}(K^*){=}321{\approx}M(\Lambda_c) - \tilde{M}(D^*){=}312{\approx}M(\Lambda_b) - \tilde{M}(B^*){=}310$$

$$\tilde{M}(\Delta) - \tilde{M}(\rho){=}518{\approx}\tilde{M}(\Sigma) - \tilde{M}(K^*){=}526{\approx}\tilde{M}(\Sigma_c) - \tilde{M}(D^*){=}524{\approx}\tilde{M}(\Sigma_b) - \tilde{M}(B^*){=}512 \quad (7)$$

4 Magnetic Moments of Heavy Quark Baryons

In Λ, Λ_c and Λ_b baryons the light quarks are coupled to spin zero. Therefore the magnetic moments of these baryons are determined by the magnetic moments of the s, c and b quarks, respectively. The latter are proportional to the chromomagnetic moments which determine the hyperfine splitting in baryon spectra. We can use this fact to predict the Λ_c and Λ_b baryon magnetic moments by relating them to the hyperfine splittings in the same way as given in the original prediction [5] of the Λ magnetic moment,

$$\mu_\Lambda = -\frac{\mu_p}{3} \cdot \frac{M_{\Sigma^*} - M_\Sigma}{M_\Delta - M_N} = -0.61\,\text{n.m.} \qquad (\text{EXP} = -0.61\,\text{n.m.}) \tag{8}$$

We obtain $\mu_{\Lambda_c} = -2\mu_\Lambda \cdot \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{\Sigma^*} - M_\Sigma} = 0.43$ n.m.; $\mu_{\Lambda_b} = \mu_\Lambda \cdot \frac{M_{\Sigma_b^*} - M_{\Sigma_b}}{M_{\Sigma^*} - M_\Sigma} = -0.067\,\text{n.m.}$ We view these predictions as a challenge for the experimental community.

5 Predicting the Mass of b-Baryons

The Ξ_Q baryons quark content is Qsd or Qsu. They can be obtained from "ordinary" Ξ (ssd or ssu) by replacing one of the s quarks by a heavier quark $Q = c, b$. There is one important difference, however. In the ordinary Ξ, Fermi statistics dictates that two s quarks must couple to spin-1, while in the ground state of Ξ_Q the (sd) and (su) diquarks have spin zero. Consequently, the Ξ_b mass is given by the expression: $\Xi_q = m_q + m_s + m_u - 3v\langle\delta(r_{us})\rangle/m_u m_s$. The Ξ_b mass can thus be predicted using the known Ξ_c baryon mass as a starting point and adding the corrections due to mass differences and HF interactions:

$$\Xi_b = \Xi_c + (m_b - m_c) - 3v\left(\langle\delta(r_{us})\rangle_{\Xi_b} - \langle\delta(r_{us})\rangle_{\Xi_c}\right)/(m_u m_s) \tag{9}$$

Since the Ξ_Q baryon contains a strange quark, and the effective constituent quark masses depend on the spectator quark, the optimal way to estimate the mass difference $(m_b - m_c)$ is from mesons which contain both s and Q quarks: $m_b - m_c = \frac{1}{4}(3B_s^* + B_s) - \frac{1}{4}(3D_s^* + D_s) = 3324.6 \pm 1.4$. On the basis of these results we predicted [7] $M(\Xi_b) = 5795 \pm 5$ MeV. Our paper was submitted on June 14, 2007. The next day CDF announced the result, $M(\Xi_b) = 5792.9 \pm 2.5 \pm 1.7$ MeV, following up on an earlier D0 measurement, $M(\Xi_b) = 5774 \pm 11 \pm 15$ MeV.

Using methods similar to these it is possible to make predictions for many other ground-state and excited baryons containing the b quark [11].

Mass of the Ω_b: For the spin-averaged Ω_b mass we have $\frac{1}{3}(2M(\Omega_b^*) + M(\Omega_b)) = \frac{1}{3}(2M(\Omega_c^*) + M(\Omega_c)) + (m_b - m_c)_{B_s - D_s} = 6068.9 \pm 2.4$ MeV. For the HF splitting we obtain

$$M(\Omega_b^*) - M(\Omega_b) = (M(\Omega_c^*) - M(\Omega_c))\frac{m_c}{m_b}\frac{\langle\delta(r_{bs})\rangle_{\Omega_b}}{\langle\delta(r_{cs})\rangle_{\Omega_c}} = 30.7 \pm 1.3 \text{ MeV} \tag{10}$$

leading to the following predictions: Ω_b=6052.1±5.6 MeV; Ω_b^*=6082.8±5.6 MeV. Fig. 1 shows a comparison of our predictions for the masses of Σ_b, Ξ_b and Ω_b baryons with the data.

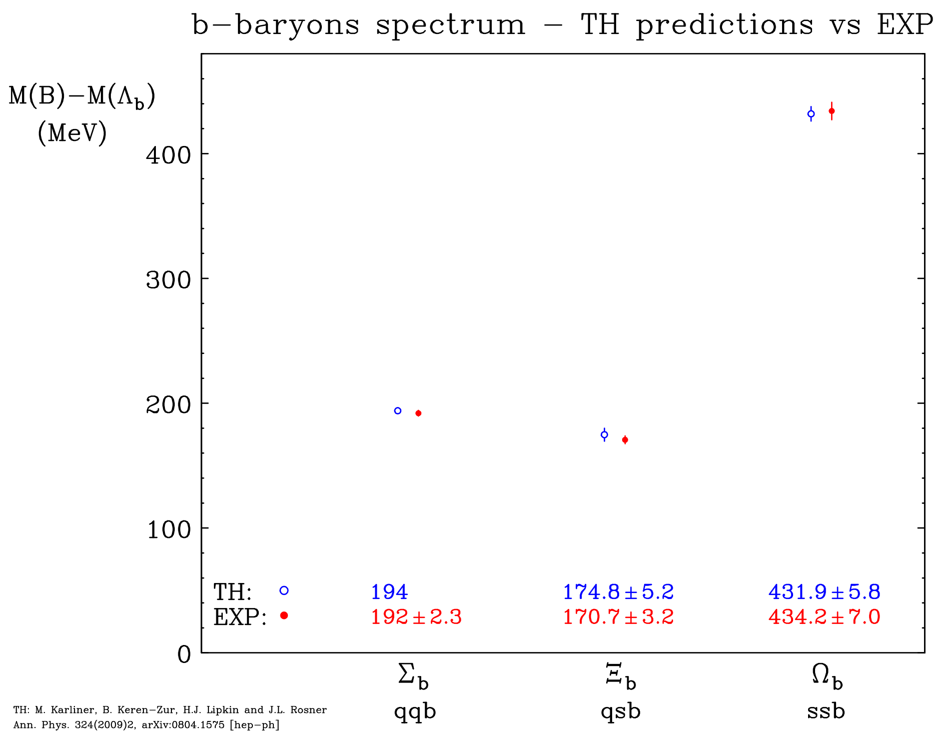

Fig. 1. Masses of b-baryons – comparison of theoretical predictions[7,11] with experiment.

The sign in our prediction $M(\Sigma_b^*) - M(\Sigma_b) < M(\Omega_b^*) - M(\Omega_b)$ appears to be counterintuitive, since the color hyperfine interaction is inversely proportional to the quark mass. This reversed inequality is not predicted by other recent approaches[12,13,14]. However the reversed inequality is also seen in the corresponding charm experimental data, $M(\Sigma_c^*) - M(\Sigma_c)(= 64.3 \pm 0.5\text{MeV}) < M(\Omega_c^*) - M(\Omega_c)(= 70.8 \pm 1.5\text{MeV})$. It is of interest to follow this clue theoretically and experimentally. Additional predictions for some excited states of b-baryons are given in Ref. [7,11].

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