Study light scalar meson property from heavy meson decays

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In the SU(3) symmetry limit, the ratio $R \equiv \frac{B(D^+ \to f_0^0 l^+ \nu) + B(D^+ \to \sigma l^+ \nu)}{B(D^+ \to a_0^0 l^+ \nu)}$ is equal to 1 if the scalar mesons are $\bar{q}q$ states, while it is 3 if these mesons are tetraquark states. This ratio provides a model-independent way to distinguish the descriptions for light scalar mesons. It also applies to the $B^- \to S l^\nu$ and $B^0 \to J/\psi(\eta_c)S$ decays. The SU(3) symmetry breaking effect is found to be under control, which will not spoil our method. The branching fractions of the $D^+ \to Sl^\nu$, $B^- \to Sl^\nu$ and $B^0 \to J/\psi(\eta_c)S$ decays roughly have the order $10^{-3}$, $10^{-4}$ and $10^{-6}$, respectively. The B factory experiments and ongoing BEPC-II experiments are able to measure these channels and accordingly to provide the detailed information of the scalar meson inner structure.

In spite of the striking success of QCD theory for strong interaction, the underlying structure of the light scalar mesons is still under controversy\textsuperscript{1,2,3}. To understand the internal structure of scalar mesons is one of the most interesting topics in hadron physics for several decades. Irrespective of the dispute on the existence of $\sigma$ and $\kappa$ mesons, scalar mesons have been identified as ordinary $\bar{q}q$ states, four-quark states or meson-meson bound states or even those supplemented with a scalar glueball. Due to the unknown nonperturbative properties of QCD, there is almost no model-independent way to effectively solve these old puzzles.

Most of the studies up to now, concentrated on the decay property of scalar mesons. It is interesting to study the production property of the scalar mesons, especially the production from heavy meson decays. At present, there are many experimental studies on the production of scalar mesons in nonleptonic $D$ decays. For example, branching ratios of $D^+ \to \sigma \pi^+$ and $D^+ \to f_0 \pi^+$ have the order of $10^{-3}$ and $10^{-4}$, respectively\textsuperscript{4}. On the theoretical side compared with nonleptonic $D$ decays, semileptonic $D^+(B^+) \to Sl^\nu$ decays only contain one scalar meson in the final state, where the heavy quark effective theory can be used. This could be the better candidate to probe different structure scenarios of scalar mesons.

In this work, we will only focus on the two-quark and the four-quark scenarios for scalar mesons. We propose a model-independent way to distinguish these two descriptions through the semileptonic $B^- \to Sl^\nu$ and/or $D^+ \to Sl^\nu$ decays, where $S$ denotes a scalar meson among $a_0(980)$, $f_0(980)$ and $\sigma$\textsuperscript{5}. These two kinds of decays are clean as they do not receive much pollution from the strong interactions. In $B$ decays, the lepton pair can also be replaced by a charmonium state since they share the same properties in the flavor SU(3) space. For example, the $\bar{B}^0 \to J/\psi(\eta_c)S$ decays are probably much easier for the experiments to observe.

A number of scalar mesons have been discovered experimentally. Among them, there are 9 mesons below or near 1 GeV, which form a nonet consisting of $\sigma, \kappa, f_0(980)$ and $a_0(980)$. Hereafter we will use $f_0$ and $a_0$ to abbreviate $f_0(980)$ and $a_0(980)$ for simplicity. In the $\bar{q}q$
picture, scalar mesons are viewed as P-wave states, whose flavor wave functions are given by

\[ |\sigma\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \equiv |\bar{n}n\rangle, \quad |f_0\rangle = |\bar{s}s\rangle, \quad |a_0^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle), \quad |a_0^-\rangle = |\bar{u}d\rangle, \quad |a_0^+\rangle = |\bar{d}u\rangle. \]

\[ |\kappa^-\rangle = |\bar{s}s\rangle, \quad |\kappa^0\rangle = |\bar{s}\bar{s}\rangle, \quad |\kappa^0\rangle = |\bar{s}\bar{d}\rangle, \quad |\kappa^+\rangle = |\bar{s}u\rangle. \]  

(1)

In this picture, \( f_0 \) is mainly made up of \( \bar{s}s \), which is supported by the large production rates in \( J/\psi \to \phi f_0 \) and \( \phi \to f_0 \gamma \) decays. Meanwhile, the experimental data also indicate the nonstrange component of \( f_0 \): the branching fraction of \( J/\psi \to \omega f_0 \) is comparable with that of \( J/\psi \to \phi f_0 \).

To accommodate with the experimental data, \( f_0 \) is supposed to be the mixture of \( \bar{n}n \) and \( \bar{s}s \) as

\[ |f_0\rangle = |\bar{s}s\rangle \cos \theta + |\bar{n}n\rangle \sin \theta, \quad |\sigma\rangle = -|\bar{s}s\rangle \sin \theta + |\bar{n}n\rangle \cos \theta. \]

With various available experimental data, the mixing angle \( \theta \) is constrained as \( 25^\circ < \theta < 40^\circ, 140^\circ < \theta < 165^\circ \).

The classical \( \bar{q}q \) picture meets with several difficulties. In particular it is difficult to explain the fact that the strange meson \( \kappa \) is lighter than the isotriplet mesons \( a_0 \), and the isosinglet meson \( f_0 \) has a degenerate mass with \( a_0 \), since \( s \) quark is expected to be heavier than \( u/d \) quark. Inspired by these difficulties, other candidate scenarios are proposed. In Ref. 8, scalar mesons are identified as diquark-diquark states. In the SU(3) flavor space, the two quarks can form two multiplets as \( 3 \otimes 3 = \bar{3} \oplus 6 \), while the other two antiquarks reside in \( 3 \) or \( \bar{6} \) multiplets. The diquark in a scalar meson is taken to be totally antisymmetric for all quantum numbers, color antitriplet, flavor antitriplet, spin 0. The lightest \( q^2 (\bar{q})^2 \) states make a flavor nonet, whose internal structure is given as:

\[ |\sigma\rangle = \bar{u}\bar{d}d, \quad |f_0\rangle = |\bar{n}\bar{s}s\rangle, \quad |a_0^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle)|\bar{s}s\rangle, \quad |a_0^\pm\rangle = |\bar{u}d\bar{s}s\rangle. \]

\[ |\kappa^+\rangle = |\bar{s}\bar{d}\bar{d}\rangle, \quad |\kappa^0\rangle = |\bar{s}\bar{d}\bar{u}\rangle, \quad |\kappa^0\rangle = |\bar{s}\bar{d}\bar{u}\rangle, \quad |\kappa^-\rangle = |\bar{s}\bar{u}\bar{d}\rangle. \]  

(2)

Taking the mixing into account, the isosinglet mesons are expressed as

\[ |f_0\rangle = |\bar{n}\bar{s}s\rangle \cos \phi + |\bar{u}\bar{u}\bar{d}\rangle \sin \phi, \quad |\sigma\rangle = -|\bar{n}\bar{s}s\rangle \sin \phi + |\bar{u}\bar{u}\bar{d}\rangle \cos \phi, \]

where the \( \phi \) between \( f_0 \) and \( \sigma \) meson is constrained as \( \phi = (174.6_{-3.4}^{+3.4})^\circ \).

The Feynman diagrams for \( D^+ \to St^+ \nu \) decays and \( B \to J/\psi (\eta_c) S \) decays in two different pictures are given in Fig. 1. The diagrams (a,c) are for the two-quark scenario, while the diagrams (b,d) are for the four-quark scenario. If a scalar meson is made of \( \bar{q}q \), the light quark is generated from the electroweak vertex and the antiquark \( \bar{d} \) serves as a spectator. Thus only the component \( \bar{d}d \) contributes to semileptonic \( D \) decays. In the SU(3) symmetry limit, decay amplitudes of \( D \to f_0(\sigma)\ell^+\nu \) channels under the \( \bar{q}q \) picture have the following relation:

\[ A(D^+ \to f_0\ell^+\nu) = -\sin \theta \hat{A}, \quad A(D^+ \to \sigma \ell^+\nu) = -\cos \theta \hat{A}, \]

(3)
where the transition amplitude $\hat{A}$ is defined as $\hat{A} \equiv A(D^+ \to a_0^1 l^+ \nu)$. This leads to

$$B(D^+ \to a_0^1 l^+ \nu) = B(D^+ \to f_0 l^+ \nu) + B(D^+ \to a_1^0 l^+ \nu).$$

(4)

One may worry about the accuracy of our results because of the possible large QCD scattering effect. However if we use the hadron picture, we can still get the same result. The $d \bar{d}$ pair produced from the weak interaction in Fig.1(a) can form isospin 0 and isospin 1 states with the ratio of 1:1. Although the scattering can mix between states, the non-perturbative QCD interactions conserve the isospin. Therefore the sum of production rates of isospin 0 states on the right hand side of eq.(4) is always equal to production rates of the isospin 1 states on the left hand side of eq.(4). The isospin breaking effect in strong interaction is negligible.

If a scalar meson is composed of four quarks, besides the light quark from the electroweak interaction, another $\bar{q}q$ pair is generated from the QCD vacuum. The decay amplitudes are given as

$$A(D^+ \to f_0 l^+ \nu) = - (\cos \phi + \sqrt{2} \sin \phi) \hat{A}, \quad A(D^+ \to a_1^0 l^+ \nu) = (\sin \phi - \sqrt{2} \cos \phi) \hat{A},$$

(5)

which gives

$$B(D^+ \to a_0^1 l^+ \nu) = \frac{1}{3} [B(D^+ \to f_0 l^+ \nu) + B(D^+ \to a_1^0 l^+ \nu)].$$

It is meaningful to define the ratio of partial decay widths

$$R \equiv \frac{B(D^+ \to f_0 l^+ \nu) + B(D^+ \to a_1^0 l^+ \nu)}{B(D^+ \to a_0^1 l^+ \nu)}.$$  

(6)

Clearly, the ratio is 1 for the two-quark description, while it is 3 for the four-quark description of scalar mesons. Similarly for semileptonic $B \to Sl \nu$ decays, the charm quark in Fig.1 is replaced by a bottom quark and the $\bar{d}$ quark is replaced by a $\bar{u}$ quark, while leptons are replaced by their charge conjugates. We have the same sum rules

$$R = \frac{B(B^- \to f_0 l^- \bar{\nu}) + B(B^- \to a_1^0 l^- \bar{\nu})}{B(B^- \to a_0^1 l^- \bar{\nu})} = \begin{cases} 1 & \text{two quark} \\ 3 & \text{tetra-quark} \end{cases}.$$  

(7)

The semileptonic $D/B$ decays are clean, which do not receive much pollution from the strong interaction. But since the neutrino is identified as missing energy, the efficiency to detect these channels may be limited. The lepton pair can also be replaced by some other SU(3) singlet systems such as a $J/\psi$ or $\eta_c$ meson. Replacing the lepton pair by the $J/\psi$ and replacing $B^-$ by a $\bar{B}^0$ state (a different spectator antiquark will not change the relation) in Eq. (7), one can easily obtain the similar sum rules for the branching fractions

$$R = \frac{B(\bar{B}^0 \to f_0 J/\psi(\eta_c)) + B(\bar{B}^0 \to a_1^0 J/\psi(\eta_c))}{B(\bar{B}^0 \to a_0^1 J/\psi(\eta_c))} = \begin{cases} 1 & \text{two quark} \\ 3 & \text{tetra-quark} \end{cases}.$$  

(8)

Since we used SU(3) symmetry to obtain these relations, it is necessary to estimate the size of SU(3) symmetry breaking effects. For example, the isospin singlet scalar mesons have different masses, which can change the phase space in the semileptonic $D/B$ decays. Fortunately, this SU(3) breaking effect can be well studied, which almost does not depend on the internal structure of scalar mesons or the strong interactions. The mass of $f_0$ is well measured but the mass of $\sigma$ meson has large uncertainties $m_\sigma = (0.4 - 1.2)$ GeV. This big range of masses indeed induces large differences to $D$ decays, since the $D$ meson mass is only 1.87GeV. The branching ratio of the semileptonic decay is affected by a factor of $(0.31 - 5.4)$ depending on the mass of the $\sigma$ meson. Therefore the sum rule in eq.(6) is not good unless the $\sigma$ meson mass is well
measured. But in $B$ meson decays, the sum rule in eq.(6) will not be affected sizably, since the 
$\sigma$ meson mass is negligible compared with the large $B$ meson mass. Numerically, this correction 
factor in $B$ decays is $(0.9 - 1.1)$.

Another SU(3) breaking effect comes from the decay form factors of various scalar mesons. 
In the two-quark scenario, only the $\bar{d}d$ component contributes to the form factors shown in 
the diagram (a) of Fig.1. The SU(3) symmetry breaking effect to the form factors is thus negligible. 
In the four-quark scenario, there are $\bar{u}udd$ component in $f_0$ and $\sigma$ meson state, which is different 
from the internal structure of $a_0$ (with a pair of $s\bar{s}$). From the diagram (b) of Fig.1, one can see 
that it would be easier to produce the $\bar{u}u$ quark pair from the vacuum than the $s\bar{s}$ quark pair 
since the $\bar{u}u$ quark pair is lighter. The SU(3) symmetry breaking effects may make the form 
factor of $D/B \to \sigma$ and $D/B \to f_0$ larger than that of $D/B \to a_0$. It will make the ratio $R$ 
larger than 3 in the four-quark scenario. Thus this SU(3) symmetry breaking effects in the form 
 factors will not spoil our method but it will instead improve its applicability.

In heavy meson D/B decays, there is an advantage to apply heavy quark effective theory. 
Unlike non-leptonic decays, the SU(3) breaking effects in semi-leptonic heavy meson decays is 
quite small, which is guaranteed by the heavy quark symmetry. The size of the SU(3) breaking 
effect could be roughly estimated by the mass difference between $u/d$ and $s$ quarks, whose magnitude is

$$
\frac{m_s - m_{u/d}}{\sqrt{m_{D/B}\Lambda}} \sim \begin{cases} 
0.3 & \text{for } D \\
0.1 & \text{for } B
\end{cases}
$$

(9)

where $\sqrt{m_{D/B}\Lambda}$ denotes the typical scale in the form factors, and $\Lambda$ is hadronic scale. Clearly, 
the $B$ decays suffer less pollution from the SU(3) symmetry breaking effect. Even if in the $D$ 
meson case, the SU(3) breaking effect of 30% can not pollute the clear difference between 1 and 
3 of ratio $R$ in eq.(7).

If the mixing angle is close to $\theta = 0^\circ$ or $\theta = 90^\circ$ in the two-quark picture ($\phi = 54.7^\circ$ or 
$\phi = 144.7^\circ$ in four-quark scenario), either $\sigma$ or $f_0$ meson has small production rates but the 
other one should have large production rates. Neglecting the highly suppressed channel, the 
ratio defined in eq.(6,7) can still distinguish the two different scenarios for scalar mesons. If the 
mixing angle is modest, i.e. it is not close to the values discussed in the above paragraph, all 
three $D^+ \to Sl^+\nu$ channels would have similar branching ratios in magnitude. The branching 
ratio of the semileptonic $D_s \to f_0$ decay is measured as

$$
B(D_s \to f_0) \times B(f_0 \to \pi^+\pi^-) = (2.0 \pm 0.3 \pm 0.1) \times 10^{-3}.
$$

Thus as an estimation, branching ratios for the cascade $D^+ \to Sl^+\nu$ 
decays are expected to have the order $\frac{1}{\sqrt{2}} \times 2 \times 10^{-3} \sim 1 \times 10^{-4}$. The luminosity of BES-III 
experiment at BEPC II in Beijing is designed as $3 \times 10^{32}$ cm$^{-2}$ s$^{-1}$. This experiment, starting 
running since summer 2008, will accumulate 30 million $D\bar{D}$ pair per running year. Even we 
assume the detect efficiency is only 20%, there will be 600 events per running year. It is very 
likely to observe these decay channels.

As for the $B$ decays, the branching ratio of $B \to Sl\bar{\nu}$ can be estimated utilizing the $B \to \rho l\bar{\nu}$ 
and $D_s^+ \to \phi l^+\nu$ decays. If the mixing angle is moderate, the branching ratio can be estimated 
using heavy quark symmetry as

$$
B(B \to f_0l\bar{\nu}) \sim B(B \to \rho l\bar{\nu}) \frac{B(D_s \to f_0l\bar{\nu})}{B(D_s \to \phi l\bar{\nu})} \sim 10^{-4} \times \frac{10^{-3}}{10^{-2}} = 10^{-5}.
$$

(10)

Such a large branching fraction offers a great opportunity for distinguishing the descriptions. 
Even if the present $B$ factory does not observe these channels, it is easy for the forthcoming 
Super B factory to measure these channels. Although $B \to J/\psi S$ are hadronic decays, the 
hadronic uncertainties are mostly canceled in the sum rule of ratios. The branching fraction is
expected to have the order

\[ \mathcal{B}(B \to f_0 J/\psi) \sim \mathcal{B}(\bar{B} \to \rho^0 J/\psi) \frac{\mathcal{B}(D_s \to f_0 l \bar{\nu})}{\mathcal{B}(D_s \to \phi l \bar{\nu})} \sim 10^{-5} \times \frac{10^{-3}}{10^{-2}} = 10^{-6}. \]  

(11)

On experimental side, the \( J/\psi \) is easily detected through a lepton pair \( l^+ l^- \) and thus this mode may be more useful.

In conclusion, we have investigated the possibility to distinguish the two-quark and four-quark picture for light scalar mesons. The semileptonic \( D/B \to S l \bar{\nu} \) decays and the nonleptonic \( B \to J/\psi(\eta_c) S \) decays are discussed in detail. These decay channels have a large potential to be measured on the ongoing BES-III and the forthcoming Super B experiments. With the same quantum number as the vacuum, the lightest scalar mesons are very complicated in nature. It is likely that these scalar mesons are neither pure 2-quark nor 4-quark states. However, our method is at least helpful to rule out one of the possibility. If the ratio \( R \) were 1, the pure 4-quark picture is likely to be ruled out but if the ratio \( R \) were 3 the pure 2-quark picture is likely to be excluded. Our proposed method provides a unique role to uncover the internal structures of scalar mesons and help to solve the old puzzles.

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