$h_c$ production in hadron collisions

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In this work, we calculate the $h_c(1P_1)$ production rate at the LHC to leading order of the strong coupling constant, for both color-singlet and -octet mechanisms. Numerical results show that a considerable number of $h_c$ events with moderate transverse momentum $p_T$ will be produced in the early run of the LHC. For $h_c$ production with large transverse momentum, the fragmentation function of gluon splitting into $h_c$ is calculated. The analytic expression is given and the fragmentation probability is found to be $7.1 \times 10^{-7}$.

1 Introduction

Since the first charmonium, the $J/\psi$, was discovered thirty years ago, much effort has been made to explore it and its higher excited states in both theoretical and experimental aspects. These studies have provided deep insights into the heavy quark-antiquark bound states and ideal opportunities to understand both perturbative and non-perturbative properties of quantum chromodynamics (QCD). Although much progress has been made, there are still many unsolved problems left in quarkonium physics. For instance, in the charmonium sector, the $c\bar{c}$ mass spectrum of the naive quark model prediction has not been completely confirmed in experiment yet. Below the open charm threshold, all expected charmonia have been identified in recent years, but experimental measurements of the physical natures of P-wave spin-singlet charmonium, $h_c(1P_1)$, are quite limited, partly due to the low production rate in leptonic collision and complicated background in hadronic collision. In hadron-hadron collision, the $1P_1$ state can be formed directly in many ways. The goal of this work is to analyze the $h_c$ production at hadron colliders, e.g. the Large Hadron Collider (LHC).

The rest of this paper is organized as follows: in section 2, $h_c$ production rate at LHC is calculated; in section 3, we give out the analytical expression for the fragmentation function of gluon splitting into $h_c$; in section 4, a brief conclusion of our results is given.

2 $h_c$ production at the LHC

To obtain more knowledge of the nature of $h_c$, a key point for experimentalists is to obtain enough $h_c$ data. The newly run collider LHC may supply a good opportunity to study quarkonium physics, including $h_c$. With a luminosity of about $10^{32} \sim 10^{34} \text{cm}^{-2}\text{s}^{-1}$ and a center of mass...
energy of 10 ∼ 14 TeV, the LHC will produce copious charmonium data, which in principle will enable the measurement of $h_c$ state precisely. In the following we evaluate the $h_c$ production rate at the LHC.

The differential cross section for $h_c$ hadroproduction is formulated in a standard way,

$$\frac{d\sigma}{dp_T}(pp \to h_c + X) = \sum_{a,b} \int dx_a dy f_a/p(x_a) f_b/p(x_b) \frac{4p_T x_a x_b}{2x_a - \bar{x}_T} \frac{d\hat{s}}{d\hat{t}}(a + b \to h_c + X),$$

where $f_a/p$ and $f_b/p$ denote the parton densities; $s$, $t$, and $u$ are Mandelstam variables at the parton level; $y$ stands for the rapidity of produced $h_c$; $\bar{x}_T = 2y/\sqrt{S}$ with $m_T = \sqrt{M^2 + p_T^2}$; and the capital $\sqrt{S}$ and $M$ denote the total energy of incident beam and the mass of $h_c$, respectively. To leading order and with moderate transverse momentum, the dominant partonic sub-processes for $h_c$ hadroproduction evidently include

$$g + g \rightarrow h_c(1S^{[8]}_0) + g,$$  

$$g + q(\bar{q}) \rightarrow h_c(1S^{[8]}_0) + q(\bar{q}),$$  

$$q + \bar{q} \rightarrow h_c(1S^{[8]}_0) + g,$$  

$$g + g \rightarrow h_c(1P^{[1]}_1) + g,$$  

$$g + c(\bar{c}) \rightarrow h_c(1P^{[1]}_1) + c(\bar{c}),$$

where the first three represent the $h_c$ production processes in the color-octet model (COM)\(^1,2\), while the last two are through color-singlet model (CSM)\(^3,4,5,6,7\).

In our numerical evaluation, the input parameters are taken as follows: $\sqrt{S} = 14$ TeV, $m_c = M/2 = 1.75$ GeV, the value of the color-singlet matrix element $\langle 0|\mathcal{O}_{c}^{h_c}(1P_1)|0 \rangle = 0.32$ GeV\(^4\), the value of the color-octet matrix element $\langle 0|\mathcal{O}_{S}^{h_c}(1S_0)|0 \rangle = 9.8 \times 10^{-3}$ GeV\(^3\), and the pseudorapidity cut $|\eta(h_c)| < 2.2$ is enforced according to the LHC experimental environment, and the CTEQ5L\(^8\) parton distribution function is employed. The numerical results of the integrated cross section for different $p_T$ lower bounds are given in Figure 1. From the figure, it can be found that the contribution from COM is about two orders of magnitude larger than that from CSM in almost every transverse momentum region. Among the three color-octet processes, the contribution from process (2) dominates over the other two. Of the two color-singlet processes, the yield from process (6) overshoots that from process (5) in the large transverse momentum region, in spite of the suppression of the extrinsic charm distribution. Because of the big gap between the yields from the color-singlet and color-octet, one result of this calculation is that the experimental measurement may tell whether the color-octet estimate of $h_c$ production is reliable or not. The calculation details and the possibility of detecting $h_c$ at the LHC can be found elsewhere\(^9\).

3 The gluon fragmentation to $h_c$

It is well-known that the fragmentation mechanism dominates the heavy quarkonium hadroproduction at large transverse momentum\(^10,11\). It is hence important to obtain the corresponding fragmentation function in order to properly estimate the production rate of a specific charmonium state. Fortunately it was found that these fragmentation functions for heavy quarkonium production are analytically calculable by virtue of perturbative QCD, with limited universal (phenomenological) parameters.

In the framework of non-relativistic QCD, the fragmentation function of a virtual gluon splitting into heavy quarkonium $H$ reads

$$D_{g \to H}(z, \mu) = \sum_n d_n(z, \mu) \langle 0|\mathcal{O}_n^{H}|0 \rangle.$$  

(7)
Here, $n$ is the color-spin-orbital quantum number of the heavy quark pair with null relative momentum. The short-distance coefficient $d_n(z, \mu)$, describing the production of heavy quark pair with appropriate quantum number $n$, is pQCD calculable. $O_n^{H}$ are local four-fermion operators in NRQCD, and their vacuum expectation values are proportional to the probabilities of heavy quark pairs with quantum number $n$ hadronizing into quarkonium states. According to NRQCD, in (7) the short distance sector $d_n(z, \mu)$ can be computed order-by-order in strong coupling $\alpha_s(2m_c)$, and the long distance sector, the matrix elements, can be expanded in series of the typical relative velocity $v$ of heavy quarks inside heavy quarkonium $^{14,15}$. In principle the fragmentation function is calculable to any order in $\alpha_s$ and $v$ as desired; in practice, normally the NLO results are enough for phenomenological aim. It should be mentioned that in recently there are discussions about whether the definition of fragmentation function beyond NLO in strong coupling expansion is complete or not in the framework of NRQCD $^{16,17}$, whereas it has no influence on our study in this work.

For process of gluon splitting into $h_c$, there are two classes of processes at leading order of $\alpha_s$ and $v$. One is the color-singlet process, in which $c\bar{c}$ pair is in spin-singlet, color-singlet and P-wave state (denoted by $1P_1^{(1)}$); the other is color-octet process, in which the quark pair is in spin-singlet, color-octet and S-wave state($1S_0^{(8)})$. The full fragmentation function for $g \to h_c$ then composes of color-singlet and color-octet terms, like

$$D_{g\to h_c}(z, \mu) = d_1(z, \Lambda)\langle 0|O_1^{hc}(1P_1)|0\rangle + d_8(z)\langle 0|O_8^{hc}(1S_0)|0\rangle(\Lambda), \quad (8)$$

The factorization scale $\Lambda$ is introduced to separate the effect at short distance of order $1/m_c$ from the one at long distance in order of heavy quarkonium radius $1/(m_c v)$. $\langle 0|O_1^{hc}(1P_1)|0\rangle$ and $\langle 0|O_8^{hc}(1S_0)|0\rangle$ are matrix elements of NRQCD operators, scaling as $m_c^2 v$ and $m_c^2 v^5$ according to velocity-scaling rules in NRQCD. Their dependence on $\Lambda$ can be obtained by renormalization group equations. To leading order of $\alpha_s$ they are $^{15}$,

$$\Lambda \frac{d}{d\Lambda} \langle 0|O_1^{hc}(1P_1)|0\rangle = 0$$
$$\Lambda \frac{d}{d\Lambda} \langle 0|O_8^{hc}(1S_0)|0\rangle = \frac{4C_F\alpha_s}{3N_c\pi m_c^2} \langle 0|O_1^{hc}(1P_1)|0\rangle. \quad (9)$$

The calculation for $d_1(z, \Lambda)$ and $d_8(z)$ is straightforward in perturbative QCD, the results
are
\begin{align}
d_t(z, \Lambda) &= f(z) + \frac{5\alpha_s^2(\mu)}{162\pi m_c^3} \left\{ - \left[ -2z^2 + 3z + 2(1-z) \ln(1-z) \right] \ln \frac{\Lambda}{m_c} + w(z) \right\}, \\
d_8(z) &= \frac{5\alpha_s^2(\mu)}{96m_c^3} \left[ -2z^2 + 3z + 2(1-z) \ln(1-z) \right].
\end{align}

The functions \( f(z) \) and \( w(z) \) are \( \Lambda \)-independent, their analytical expressions are shown in Appendix. With the Eq.9, one can easily check the fragmentation function (8) are scale-independent,
\[ \frac{d}{d\Lambda} D_{g \to h_c}(z, \Lambda) = 0. \]

Integrating the fragmentation function \( D_{g \to h_c}(z, 2m_c) \) over the momentum fraction \( z \), we get the fragmentation probability,
\[ \int_0^1 dz \ D_{g \to h_c}(z, 2m_c) = \frac{5\alpha_s^2(2m_c)}{96m_c^3} \left\{ \frac{(-54.5)\alpha_s(2m_c)}{216\pi m_c^2} \langle O_5^{h_c} \rangle + (0.33)\langle O_8^{h_c} \rangle(m_c) \right\}. \]

To make the total probability positive, the lower bound for the magnitude of color-octet matrix element at factorization scale \( \Lambda = m_c \) is set to be
\[ \langle O_8^{h_c} \rangle(m_c) > \frac{3\alpha_s(2m_c)}{4\pi m_c^2} \langle O_4^{h_c} \rangle. \]

For numerical evaluation, the input parameters are taken as follows: \( m_c = m_{h_c}/2 = 1.78 \text{GeV}, \alpha_s(2m_c) = 0.26, \langle 0|O_4^{h_c}(1)P_\ell|0 \rangle = 0.32\text{GeV}^5 \), which enables \( \langle 0|O_8^{h_c}(1)S_0|0 \rangle(m_c) > 6.3 \times 10^{-3} \text{GeV}^3 \), \( \langle 0|O_8^{h_c}(1)S_0|0 \rangle(m_c) = 9.8 \times 10^{-3} \text{GeV} \). With above inputs the fragmentation probability in Eq.(13) is found to be about \( 7.1 \times 10^{-7} \). Compared with the probabilities of gluon fragmenting into \( \chi_c(0), \chi_c(1) \), and \( \chi_c(2) \), which are \( 4.0 \times 10^{-4}, 1.8 \times 10^{-4} \) and \( 2.4 \times 10^{-4} \), the value of gluon fragmentation to \( h_c \) is smaller by two to three orders, and it is even less than the color-singlet probability of process \( g^* \to J/\psi gg \) by an order \( 14 \). The \( z \) dependence of the fragmentation function at \( \mu = m_c \) and \( \Lambda = m_c \) is shown in Figure 2.

4 Conclusions

In conclusion, we have evaluated the \( h_c \) direct production rate at the LHC, where the \( h_c \) indirect yields are much less than the direct ones according to a similar analysis for \( h_c \) production at HERA-b. Our calculation is performed to leading order of the strong coupling constant \( \alpha_s \) and to second order in the relative velocity \( \nu \) expansion. Both color-singlet and -octet production schemes are taken into account in this work. We find that there will be enough \( h_c \) yields at the LHC for a precise measurement on the nature of this P-wave spin singlet. Although as usual the high order corrections may induce some uncertainties in the calculation, as an order-of-magnitude estimate our results should hold. Due to the large discrepancy between predictions from the color-singlet and color-octet schemes, the experimental measurement of the \( h_c \) production rate at the LHC may tell to what degree the color-octet mechanism plays a role in charmonium production as well.

At the region of large transverse momentum, the dominant mechanism for heavy quarkonium hadron production is fragmentation, hence the fragmentation function is important for \( h_c \) production. We have computed in this work the fragmentation function of gluon to P-wave spin singlet quarkonium \( h_c \). While both color-singlet and -octet processes are taken into account, the analytic expression is independent of factorization scale. It is found that the fragmentation probability of a high energy virtual gluon splitting into \( h_c \) is about \( 7.1 \times 10^{-7} \). Finally, it is worthy to note that result in this work can be readily applied to the study of \( h_b \) physics.
Figure 2: The fragmentation function $D_{q\to h_c}(z,\mu)$ as a function of $z$ for $\mu = 2m_c$ and $\Lambda = m_c$, the dotted line is the contribution from color-singlet process, the dash-dotted line is the contribution from color-octet process and the solid line is the total result.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (NSFC) under the grants 10935012, 10928510, 10821063 and 10775179, by the CAS Key Projects KJCX2-yw-N29 and H92A0200S2.

Appendix

$$w(z) = 2(1 - z) \ln^2(1 - z) + [-2 - 3(-2 + z) + 3(-1 + z) \ln z] \ln(1 - z) + \frac{1}{6} [2\pi^2(1 - z) + 3(7 + 8z) + 3(-3 + 2z) \ln z] - 2(1 - z) L_2(1 - z) - (1 - z) L_2(z),$$

and

$$f(z) = \frac{5a_s^2(\mu)}{20736\pi m_c^2} \int_0^{\ln(1+y)/2} \frac{dy}{(1+y)^4(y-z)^2} \left\{ \frac{1}{(1-r)^2} - 4g + 2z^2 \right\} h(y, r)$$

$$+ \frac{1}{(y-r)^2} \sum_{i=0}^2 z^i \left( (r-y)f_i(y, r) + \frac{g_i(y, r)}{\sqrt{y-r}} \ln \frac{y-r + \sqrt{y-r}}{y-r - \sqrt{y-r}} \right)$$

with functions $w(z)$, $h(y, r)$, $f_i(y, r)$ and $g_i(y, r)$ read

$$h(y, r) = (1 + 11r - 5r^2 + r^3)(1 - 20r + 6r^2 - 4r^3 + r^4) + (2 - 20r + 606r^2 - 280r^3 + 94r^4 - 20r^5 + 2r^6) y$$

$$+ (4 + 468r - 792r^2 + 232r^3 - 44r^4 + 4r^5) y^2 + (8 - 1120r + 560r^2 - 96r^3 + 8r^4) y^3$$

$$+ (-240 + 1072r - 208r^2 + 16r^3) y^4 + (544 - 448r + 32r^2) y^5 + (-448 + 64r) y^6 + 128y^7,$$

$$f_0(y, r) = r^3(36 - 15r + 25r^2 - 57r^3 + 59r^4 - 97r^5 + 23r^6 - 77r^7 + r^8)$$

$$+ (-164r^3 + 54r^4 + 80r^5 - 34r^6 + 344r^7 - 2r^8 + 12r^9 - 2r^{10}) y$$

$$+ (-82r^2 + 274r^3 - 374r^4 - 46r^5 - 746r^6 + 26r^7 - 14r^8 + 2r^9) y^2$$

$$+ (-4r + 488r^2 + 88r^3 + 652r^4 + 824r^5 - 564r^6 - 108r^7) y^3$$

$$+ (1 + 3r^2 - 1277r^2 - 637r^3 - 877r^4 + 1947r^5 + 553r^6 + r^7) y^4$$

$$+ (2 - 160r + 1734r^2 + 488r^3 - 2794r^4 - 1032r^5 + 66r^6) y^5.$$
\[-2y_2f(y, r),\]
\[f_1(y, r) = 2 \left[ r^4 (6 + 8r - 7r^2 + 33r^3 - 24r^4 + 46r^5 - 7r^6 + r^7) \right.\]
\[+ \left. (16r^2 + 96r^3 + 56r^4 + 2r^5 - 206r^6 - 10r^7 - 2r^8) \right] y^2\]
\[+ \left. (16r - 148r^2 - 248r^3 - 74r^4 + 502r^5 + 110r^6 + 2r^7) \right] y^3\]
\[+ \left. (124r + 470r^2 + 286r^3 - 614r^4 - 234r^5 + 16r^6) \right] y^4\]
\[+ \left. (7 - 285r - 592r^2 + 320r^3 + 151r^4 - 99r^5) y^5 + \left( 34 + 336r + 204r^2 + 128r^3 + 242r^4 \right) y^6\]
\[+ \left. (8 - 220 + 340r^2 - 316r^3 + 112 + 232r + 248r^2 \right) y^7 + \left( -96 - 12r \right) y^8 + 32y^9 \right],\]
\[g_0(y, r) = 2 \left[ r^4 (12 - 8r - 8r^2 + 6r^3 - 8r^4 + 16r^5 + 56r^6 + 8r^7 + 12r^8 + 20r^9 + 4r^10 + 66r^11 + 7r^{10}) y^2\right.\]
\[+ (28r^3 - 111r^4 - 94r^5 - 148r^6 - 34r^7 - 213r^8 - 20r^9) y^2\]
\[+ \left. (14r^3 + 193r^4 + 146r^5 - 150r^6 + 356r^7 + 111r^8 - 26r^9) y^3\right.\]
\[+ \left. (14r^2 - 297r^3 - 232r^4 - 120r^5 - 330r^6 - 695r^7 + 182r^8) y^4\right.\]
\[+ \left. (102r^2 - 166r^3 + 452r^4 + 152r^5 - 160r^6 - 498r^7 + 16r^8) y^5\right.\]
\[+ \left. (608r^2 - 590r^3 - 662r^4 + 193r^5 + 614r^6 - 124r^7) y^6\right.\]
\[+ \left. (1160r^2 + 1152r^3 - 896r^4 - 112r^5 + 406r^6) y^7\right.\]
\[+ \left. (8r - 1152r^2 - 216r^3 - 696r^4 - 744r^5) y^8 + \left( -104r + 360r^2 + 864r^3 + 816r^4 \right) y^9\right.\]
\[+ \left. (8 + 216r - 376r^2 - 520r^3) y^{10} + \left( -16 - 48r + 160r^2 \right) y^{11} \right],\]
\[g_1(y, r) = 2 \left[ r^4 (3 - 5r + 8r^2 + 3r^3 + 13r^4 + 16r^5 - 10r^3 - 14r^4 + 11r^5 + 4r^6 + 84r^7 + 138r^8 + 11r^9) y^2\right.\]
\[+ \left. (10r^2 + 11r^3 + 114r^4 + 60r^5 - 196r^6 - 495r^7 - 76r^8) y^2\right.\]
\[+ \left. (16r^2 - 183r^3 - 492r^4 + 86r^5 + 940r^6 + 201r^7 - 8r^8) y^3\right.\]
\[+ \left. (16r + 175r^2 + 796r^3 + 640r^4 - 914r^5 - 215r^6 + 62r^7) y^4\right.\]
\[+ \left. (96r - 688r^2 - 172r^3 + 108r^4 - 76r^5 - 204r^6) y^5\right.\]
\[+ \left. (218r + 922r^2 + 562r^3 + 502r^4 + 372r^5) y^6 + \left( 200r + 472r^2 + 568r^3 + 408r^4 \right) y^7\right.\]
\[+ \left. (4 + 36r + 268r^2 + 260r^3) y^8 - (8 - 8r - 80r^2) y^9 \right].\]

References