h_c production in hadron collisions

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In this work, we calculate the $h_c(^1P_1)$ production rate at the LHC to leading order of the strong coupling constant, for both color-singlet and -octet mechanisms. Numerical results show that a considerable number of h_c events with moderate transverse momentum p_T will be produced in the early run of the LHC. For h_c production with large transverse momentum, the fragmentation function of gluon splitting into h_c is calculated. The analytic expression is given and the fragmentation probability is found to be 7.1×10^{-7}.

1 Introduction

Since the first charmonium, the J/ψ, was discovered thirty years ago, much effort has been made to explore it and its higher excited states in both theoretical and experimental aspects. These studies have provided deep insights into the heavy quark-antiquark bound states and ideal opportunities to understand both perturbative and non-perturbative properties of quantum chromodynamics(QCD). Although much progress has been made, there are still many unsolved problems left in quarkonium physics. For instance, in the charmonium sector, the $c\bar{c}$ mass spectrum of the naive quark model prediction has not been completely confirmed in experiment yet. Below the open charm threshold, all expected charmonia have been identified in recent years, but experimental measurements of the physical natures of P-wave spin-singlet charmonium, $h_c(^1P_1)$, are quite limited, partly due to the low production rate in leptonic collision and complicated background in hadronic collision. In hadron-hadron collision, the $^1P_1$ state can be formed directly in many ways. The goal of this work is to analyze the h_c production at hadron colliders, e.g. the Large Hardon Collider(LHC).

The rest of this paper is organized as follows: in section 2, h_c production rate at LHC is calculated; in section 3, we give out the analytical expression for the fragmentation function of gluon splitting into h_c; in section 4, a brief conclusion of our results is given.

2 h_c production at the LHC

To obtain more knowledge of the nature of h_c, a key point for experimentalists is to obtain enough h_c data. The newly run collider LHC may supply a good opportunity to study quarkonium physics, including h_c. With a luminosity of about $10^{32} \sim 10^{34} cm^{-2}s^{-1}$ and a center of mass

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energy of $10 \sim 14$ TeV, the LHC will produce copious charmonium data, which in principle will enable the measurement of h_c state precisely. In the following we evaluate the h_c production rate at the LHC .

The differential cross section for h_c hadroproduction is formulated in a standard way,

$$\frac{d\sigma}{dp_T}(pp \to h_c + X) = \sum_{a,b} \int dx_a dy f_{a/p}(x_a) f_{b/p}(x_b) \frac{4p_T x_a x_b}{2x_a - \bar{x}_T e^y} \frac{d\hat{\sigma}}{dt}(a + b \to h_c + X) \ , \tag{1}$$

where $f_{a/p}$ and $f_{b/p}$ denote the parton densities; s, t, and u are Mandelstam variables at the parton level; y stands for the rapidity of produced h_c; $\bar{x}_T \equiv \frac{2m_T}{\sqrt{S}}$ with $m_T = \sqrt{M^2 + p_T^2}$; and the capital $\sqrt{S}$ and M denote the total energy of incident beam and the mass of h_c, respectively. To leading order and with moderate transverse momentum, the dominant partonic sub-processes for h_c hadroproduction evidently include

$$\begin{aligned} g\ +\ g &\ \to\ h_c(^1S_0^{[8]})\ +\ g\ , & (2)\\ g\ +\ q(\bar{q}) &\ \to\ h_c(^1S_0^{[8]})\ +\ q(\bar{q})\ , & (3)\\ q\ +\ \bar{q} &\ \to\ h_c(^1S_0^{[8]})\ +\ g\ , & (4)\\ g\ +\ g &\ \to\ h_c(^1P_1^{[1]})\ +\ g\ , & (5)\\ g\ +\ c(\bar{c}) &\ \to\ h_c(^1P_1^{[1]})\ +\ c(\bar{c})\ , & (6) \end{aligned}$$

where the first three represent the h_c production processes in the color-octet model(COM)[1,2], while the last two are through color-singlet model(CSM)[3,4,5,6,7].

In our numerical evaluation, the input parameters are taken as follows: $\sqrt{S} = 14$TeV, $m_c = M/2 = 1.78$GeV, the value of the color-singlet matrix element $\langle 0|\mathcal{O}_1^{h_c}(^1P_1)|0\rangle = 0.32$ GeV^5 [9], the value of the color-octet matrix element $\langle 0|\mathcal{O}_8^{h_c}(^1S_0)|0\rangle = 9.8 \times 10^{-3}$ GeV^3 [8], and the pseudorapidity cut $|\eta(h_c)| < 2.2$ is enforced according to the LHC experimental environment, and the CTEQ5L [10] parton distribution function is employed. The numerical results of the integrated cross section for different p_T lower bounds are given in Figure 1. From the figure, it can be found that the contribution from COM is about two orders of magnitude larger than that from CSM in almost every transverse momentum region. Among the three color-octet processes, the contribution from process (2) dominates over the other two. Of the two color-singlet processes, the yield from process (6) overshoots that from process (5) in the large transverse momentum region, in spite of the suppression of the extrinsic charm distribution. Because of the big gap between the yields from the color-singlet and color-octet, one result of this calculation is that the experimental measurement may tell whether the color-octet estimate of h_c production is reliable or not. The calculation details and the possibility of detecting h_c at the LHC can be found elsewhere [11].

3 The gluon fragmentation to h_c

It is well-known that the fragmentation mechanism dominates the heavy quarkonium hadroproduction at large transverse momentum [12,13]. It is hence important to obtain the corresponding fragmentation function in order to properly estimate the production rate of a specific charmonium state. Fortunately it was found that these fragmentation functions for heavy quarkonium production are analytically calculable by virtue of perturbative QCD, with limited universal(phenomenological) parameters.

In the framework of non-relativistic QCD, the fragmentation function of a virtual gluon splitting into heavy quarkonium H reads

$$D_{g\to H}(z,\mu) = \sum_n d_n(z,\mu)\langle 0|\mathcal{O}_n^H|0\rangle \ . \tag{7}$$

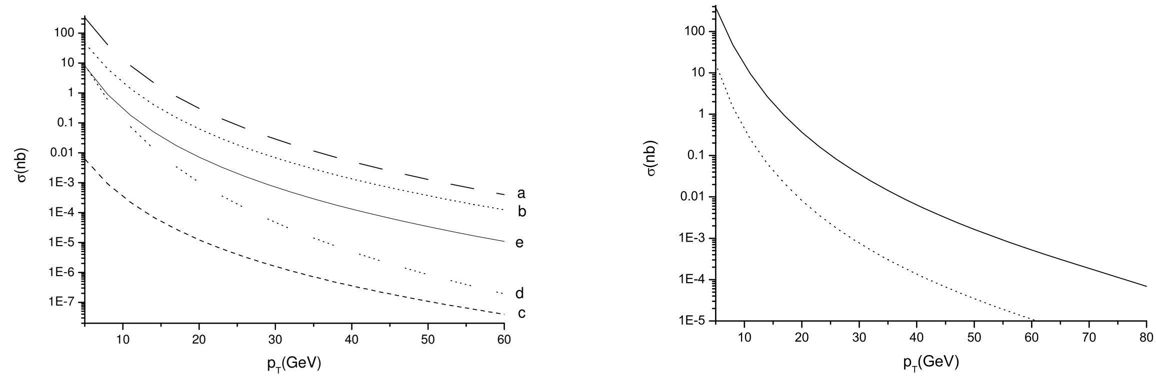

Figure 1: The h_c production rates as a function of the transverse momentum lower bound p_T in pp collision at the center-of-mass energy $\sqrt{S} = 14$ TeV. The left diagram demonstrates the integrated cross-sections of h_c production via processes (2) to (6) shown as lines a to e, respectively. The solid line in the right diagram represents the yield from the color-octet scheme, and the dashed line represents the yield from the color-singlet scheme.

Here, n is the color-spin-orbital quantum number of the heavy quark pair with null relative momentum. The short-distance coefficient $d_n(z,\mu)$, describing the production of heavy quark pair with appropriate quantum number n, is pQCD calculable. $\mathcal{O}_n^H$ are local four-fermion operators in NRQCD, and their vacuum expectation values are proportional to the probabilities of heavy quark pairs with quantum number n hadronizing into quarkonium states. According to NRQCD, in (7) the short distance sector $d_n(z,\mu)$ can be computed order-by-order in strong coupling $\alpha_s(2m_c)$, and the long distance sector, the matrix elements, can be expanded in series of the typical relative velocity v of heavy quarks inside heavy quarkonium [14,15]. In principle the fragmentation function is calculable to any order in α_s and v as desired; in practice, normally the NLO results are enough for phenomenological aim. It should be mentioned that in recently there are discussions about whether the definition of fragmentation function beyond NLO in strong coupling expansion is complete or not in the framework of NRQCD [16,17], whereas it has no influence on our study in this work.

For process of gluon splitting into h_c, there are two classes of processes at leading order of α_s and v. One is the color-singlet process, in which $c\bar{c}$ pair is in spin-singlet, color-singlet and P-wave state (denoted by ${}^1P_1^{(1)}$); the other is color-octet process, in which the quark pair is in spin-singlet, color-octet and S-wave state(${}^1S_0^{(8)}$). The full fragmentation function for $g \to h_c$ then composes of color-singlet and color-octet terms, like

$$D_{g\to h_c}(z,\mu) = d_1(z,\Lambda)\langle 0|\mathcal{O}_1^{h_c}({}^1P_1)|0\rangle + d_8(z)\langle 0|\mathcal{O}_8^{h_c}({}^1S_0)|0\rangle(\Lambda)\,, \tag{8}$$

The factorization scale Λ is introduced to separate the effect at short distance of order $1/m_c$ from the one at long distance in order of heavy quarkonium radius $1/(m_c v)$. $\langle 0|\mathcal{O}_1^{h_c}({}^1P_1)|0\rangle$ and $\langle 0|\mathcal{O}_8^{h_c}({}^1S_0)|0\rangle$ are matrix elements of NRQCD operators, scaling as $m_c^5 v^5$ and $m_c^3 v^5$ according to velocity-scaling rules in NRQCD. Their dependence on Λ can be obtained by renormalization group equations. To leading order of α_s they are [15],

$$\begin{aligned}
\Lambda\frac{d}{d\Lambda}\langle 0|\mathcal{O}_1^{h_c}({}^1P_1)|0\rangle &= 0 \\
\Lambda\frac{d}{d\Lambda}\langle 0|\mathcal{O}_8^{h_c}({}^1S_0)|0\rangle &= \frac{4C_F\alpha_s}{3N_c\pi m_c^2}\langle 0|\mathcal{O}_1^{h_c}({}^1P_1)|0\rangle.
\end{aligned} \tag{9}$$

The calculation for $d_1(z,\Lambda)$ and $d_8(z)$ is straightforward in perturbative QCD, the results are

$$d_1(z,\Lambda) = f(z) + \frac{5\alpha_s^3(\mu)}{162\pi m_c^5}\Big\{ -\big[-2z^2+3z+2(1-z)\ln(1-z)\big]\ln\frac{\Lambda}{m_c} + w(z)\Big\}, \tag{10}$$

$$d_8(z) = \frac{5\alpha_s^2(\mu)}{96 m_c^3}[-2z^2+3z+2(1-z)\ln(1-z)] . \tag{11}$$

The functions $f(z)$ and $w(z)$ are Λ-independent, their analytical expressions are shown in Appendix. With the Eq.9, one can easily check the fragmentation function (8) are scale-independent,

$$\frac{d}{d\Lambda} D_{g\to h_c}(z,\Lambda) = 0 . \tag{12}$$

Integrating the fragmentation function $D_{g\to h_c}(z, 2m_c)$ over the momentum fraction z, we get the fragmentation probability,

$$\int_0^1 dz\, D_{g\to h_c}(z, 2m_c) = \frac{5\alpha_s^2(2m_c)}{96 m_c^3}\Big[\frac{(-54.5)\alpha_s(2m_c)}{216\pi m_c^2}\langle \mathcal{O}_1^{h_c}\rangle + (0.33)\langle \mathcal{O}_8^{h_c}\rangle(m_c)\Big] . \tag{13}$$

To make the total probability positive, the lower bound for the magnitude of color-octet matrix element at factorization scale $\Lambda = m_c$ is set to be

$$\langle \mathcal{O}_8^{h_c}\rangle(m_c) > \frac{3\alpha_s(2m_c)}{4\pi m_c^2}\langle \mathcal{O}_1^{h_c}\rangle . \tag{14}$$

For numerical evaluation, the input parameters are taken as follows: $m_c = m_{h_c}/2 = 1.78$GeV, $\alpha_s(2m_c) = 0.26$, $\langle 0|\mathcal{O}_1^{h_c}(^1P_1)|0\rangle = 0.32 GeV^5$ [9], which enables $\langle 0|\mathcal{O}_8^{h_c}(^1S_0)|0\rangle(m_c) > 6.3 \times 10^{-3} GeV^3$, $\langle 0|\mathcal{O}_8^{h_c}(^1S_0)|0\rangle(m_c) = 9.8 \times 10^{-3} GeV^3$ [8,11]. With above inputs the fragmentation probability in Eq.(13) is found to be about 7.1×10^{-7}. Compared with the probabilities of gluon fragmenting into χ_{c0}, χ_{c1}, and χ_{c2}, which are 0.4×10^{-4}, 1.8×10^{-4} and 2.4×10^{-4} [13], the value of gluon fragmentation to h_c is smaller by two to three orders, and it is even less than the color-singlet probability of process $g^* \to J/\psi gg$ by an order [14]. The z dependence of the fragmentation function at $\mu = 2m_c$ and $\Lambda = m_c$ is shown in Figure 2.

4 Conclusions

In conclusion, we have evaluated the h_c direct production rate at the LHC, where the h_c indirect yields are much less than the direct ones according to a similar analysis for h_c production at HERA-b[18]. Our calculation is performed to leading order of the strong coupling constant α_s and to second order in the relative velocity v^2 expansion. Both color-singlet and -octet production schemes are taken into account in this work. We find that there will be enough h_c yields at the LHC for a precise measurement on the nature of this P-wave spin singlet. Although as usual the high order corrections may induce some uncertainties in the calculation, as an order-of-magnitude estimate our results should hold. Due to the large discrepancy between predictions from the color-singlet and color-octet schemes, the experimental measurement of the h_c production rate at the LHC may tell to what degree the color-octet mechanism plays a role in charmonium production as well.

At the region of large transverse momentum, the dominant mechanism for heavy quarkonium hadronproduction is fragmentation, hence the fragmentation function is important for h_c production. We have computed in this work the fragmentation function of gluon to P-wave spin singlet quarkonium h_c. While both color-singlet and -octet processes are taken into account, the analytic expression is independent of factorization scale. It is found that the fragmentation probability of a high energy virtual gluon splitting into h_c is about 7.1×10^{-7}. Finally, it is worthy to note that result in this work can be readily applied to the study of h_b physics.

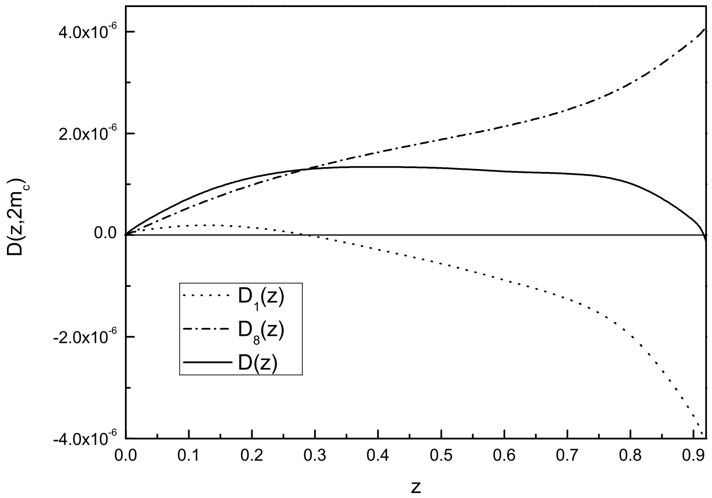

Figure 2: The fragmentation function $D_{g\to h_c}(z,\mu)$ as a function of z for $\mu = 2m_c$ and $\Lambda = m_c$, the dotted line is the contribution from color-singlet process, the dash-dotted line is the contribution from color-octet process and the solid line is the total result.

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Appendix

$$\begin{aligned} w(z) &= 2(1-z)\ln^2(1-z) + [-2-3(-2+z)z+3(-1+z)\ln z]\ln(1-z) \\ &+\frac{1}{6}[2\pi^2(1-z)+3z(-7+8z)+3z(-3+2z)\ln z] - 2(1-z)Li_2(1-z) - (1-z)Li_2(z)\,, \end{aligned} \tag{15}$$

and

$$\begin{aligned} f(z) &= \frac{5\alpha_s^3(\mu)}{20736\pi m_c^5}\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2} dy\frac{1}{(1-y)^4(y^2-r)^2}\Bigg\{\frac{(1+r^2)-4yz+2z^2}{(1-r)^2}h(y,r) \\ &+\frac{1}{(y-r)^5}\sum_{i=0}^{2} z^i\left((r-y)f_i(y,r)+\frac{g_i(y,r)}{\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right)\Bigg\} \end{aligned} \tag{16}$$

with functions $w(z)$, $h(y,r)$, $f_i(y,r)$ and $g_i(y,r)$ read

$$\begin{aligned} h(y,r) &= (1+11r-5r^2+r^3)(1-20r+6r^2-4r^3+r^4)+(2-20r+606r^2-280r^3+94r^4-20r^5+2r^6)y \\ &+(4+468r-792r^2+232r^3-44r^4+4r^5)y^2+(8-1120r+560r^2-96r^3+8r^4)y^3 \\ &+(-240+1072r-208r^2+16r^3)y^4+(544-448r+32r^2)y^5+(-448+64r)y^6+128y^7\,, \quad (17) \\ f_0(y,r) &= r^3(36-15r+25r^2-57r^3+59r^4-97r^5+23r^6-7r^7+r^8) \\ &+(-164r^3+54r^4+80r^5-34r^6+344r^7-2r^8+12r^9-2r^{10})y \\ &+(-82r^2+274r^3-374r^4-46r^5-746r^6+26r^7-14r^8+2r^9)y^2 \\ &+(-4r+488r^2+88r^3+652r^4+824r^5-564r^6-108r^7)y^3 \\ &+(1+33r-1277r^2-637r^3-877r^4+1947r^5+553r^6+r^7)y^4 \\ &+(2-160r+1734r^2+488r^3-2794r^4-1032r^5+66r^6)y^5 \end{aligned}$$

$$
\begin{aligned}
&+(-28+356r-1144r^2+2232r^3+644r^4-396r^5)y^6+(136-352r-352r^2+400r^3+968r^4)y^7\\
&+(-336+80r-1136r^2-1264r^3)y^8+(448+736r+992r^2)y^9+(-384-448r)y^{10}+128y^{11}\,, \qquad (18)
\end{aligned}
$$

$$
f_1(y,r) = -2yf_2(y,r)\,, \qquad (19)
$$

$$
\begin{aligned}
f_2(y,r) = \; & 2\left[r^2(6+8r-47r^2+33r^3-24r^4+46r^5-7r^6+r^7)\right.\\
&+(-16r^2+96r^3+56r^4+2r^5-206r^6-10r^7-2r^8)y\\
&+(-16r-148r^2-248r^3-74r^4+502r^5+110r^6+2r^7)y^2\\
&+(124r+470r^2+286r^3-614r^4-234r^5+16r^6)y^3\\
&+(-7-285r-592r^2+320r^3+151r^4-99r^5)y^4+(34+336r+204r^2+128r^3+242r^4)y^5\\
&\left.+(-84-220r-340r^2-316r^3)y^6+(112+232r+248r^2)y^7+(-96-112r)y^8+32y^9\right]\,, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
g_0(y,r) = \; & 2\left[r^4(-9+r-8r^2-8r^3+r^4-r^5-8r^6)+(46r^4+10r^5+71r^6+20r^7+4r^8+66r^9+7r^{10})y\right.\\
&+(28r^3-111r^4-94r^5-148r^6-34r^7-213r^8-20r^9)y^2\\
&+(-148r^3+193r^4+146r^5+150r^6+356r^7-111r^8-26r^9)y^3\\
&+(-28r^2+297r^3-232r^4-120r^5-330r^6+695r^7+182r^8)y^4\\
&+(192r^2-166r^3+452r^4+152r^5-1604r^6-498r^7+16r^8)y^5\\
&+(-608r^2-590r^3-662r^4+1930r^5+614r^6-124r^7)y^6\\
&+(1160r^2+1152r^3-896r^4-112r^5+408r^6)y^7\\
&+(8r-1152r^2-216r^3-696r^4-744r^5)y^8+(-104r+360r^2+864r^3+816r^4)y^9\\
&\left.+(8+216r-376r^2-520r^3)y^{10}+(-16-48r+160r^2)y^{11}\right]\,, \qquad (21)
\end{aligned}
$$

$$
g_1(y,r) = -2yg_2(y,r)\,, \qquad (22)
$$

$$
\begin{aligned}
g_2(y,r) = \; & 2\left[r^3(-3-5r+8r^2+3r^4+13r^5+16r^6)-(-10r^3-14r^4+11r^5+4r^6+84r^7+138r^8+11r^9)y\right.\\
&-(-10r^2+11r^3+114r^4+60r^5-196r^6-495r^7-76r^8)y^2\\
&-(16r^2-183r^3-492r^4+86r^5+940r^6+201r^7-8r^8)y^3\\
&-(16r+175r^2+796r^3+640r^4-914r^5-215r^6+62r^7)y^4\\
&-(-96r-688r^2-1172r^3+108r^4-76r^5-204r^6)y^5\\
&-(218r+922r^2+562r^3+502r^4+372r^5)y^6+(200r+472r^2+568r^3+408r^4)y^7\\
&-(-4+36r+268r^2+260r^3)y^8-(8-8r-80r^2)y^9\,. \qquad (23)
\end{aligned}
$$

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