Eccentricity Fluctuations in p-p and Elliptic Flow

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We discuss under which conditions elliptic flow becomes measurable in high-multiplicity ($dN_{\rm ch}/dy \geq 50$) p-p collisions, which will soon be collected at the LHC. We observe that fluctuations in the p-p interaction region can result in a sizable spacial eccentricity even for the most central p-p collisions. Under relatively mild assumptions on the nature of such fluctuations and on the eccentricity scaling of elliptic flow, we find that the resulting elliptic flow signal in high-multiplicity p-p collisions at the LHC becomes measurable with standard techniques.

1 Introduction

In a high energy nuclear collision at non-zero impact parameter the interaction region is anisotropic in the transverse plane. Collective phenomena, such as final state interactions of the collision products, lead to a transfer of the spacial anisotropy of the collision geometry into a azimuthal momentum anisotropy of the final particles in the collision. This phenomenon is called elliptic flow and it is one of the most important observables in ultrarelativistic heavy ion collisions. The large values of the elliptic flow parameter $v_2 = \langle \cos 2\phi \rangle$ at RHIC signal very strong collective phenomena of the QCD matter created in these collisions.

The large collectivity observed in nucleus-nucleus collisions leads to the question of wether these effects are also present in other hadronic collisions such as in p-p. There is, however a conceptual difference, since the spacial distribution in the nuclear case is due to the distribution of nucleons inside the nuclei, while in the proton there is only one nucleon. However, protons, as nuclei, are extended objects; the spacial distribution of the individual partonic collisions which lead to particle production are the responsible for the spacial anisotropy.

The transverse plane asymmetry in the collision region is not sufficient for elliptic flow, since final state interactions are needed to transfer this anisotropy into momentum distributions. A typical p-p collision produces too few particles for this transfer to occur . However, high energy p-p collisions at the LHC will produce rare very high multiplicity events, in which the final state multiparticle interactions are important and, thus, certain degree of collective behavior for these class of events is expected.

Finally, we would like to remark that even if an azimuthal asymmetry is imprinted into the momentum distribution in p-p, the experimental determination of v_2 is complicated by the finite number of particles involved in these collisions. The main difficulty is to separate true collective effects on the anisotropy to other sources of multiparticle correlations unrelated to flow. One of the most systematic analysis to perform this task is the cumulant analysis, which uses n_p-particle correlations to determine v_2. The effect of non-flow correlations is reduced as the event multiplicity grows; different n_p-correlations lead to a constraint on the minimum value

Table 1: Constraints on the minimum v_2 value so that it is dominated by flow effects as a function of the multiplicity per unit rapidity and for different n_p-cumulants

	$n_p = 2$	$n_p = 4$
v_2 ($n_{\mathrm{mult}} = 30$)	> 0.18	> 0.09
v_2 ($n_{\mathrm{mult}} = 50$)	> 0.14	> 0.05
v_2 ($n_{\mathrm{mult}} = 80$)	> 0.11	> 0.04

of v_2 such that it is dominated by flow. A table summarizing this constraints is shown in Table 1 [1].

2 Eccentricity fluctuations in p-p collisions

Unlike the nucleus-nucleus case, where the matter distribution in the colliding region is determined by the distribution of nucleons involved in the collision, the spacial distribution of matter in a high energy p-p collision is not known. Since flow is sensitive to this distribution, this type of measurements could be potentially used to constraint models of the proton structure at high energy.

The study of flow in heavy ion collisions shows that v_2 is proportional to the (matter density weighted) eccentricity of the colliding region

$$\epsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} \,. \tag{1}$$

where (x, y) are coordinates in the transverse plane with x along the impact parameter. At a fixed impact parameter, different matter distribution for the proton lead to different values of the eccentricity.

Early attempts to asses flow effects in proton-proton considered the proton as a homogenous and smooth distribution of partons [3,4,5]. In these models, the multiplicity distribution is correlated to the centrality of the collision. Thus, sizable eccentricities occur only at finite impact parameter, where the multiplicity of the event is not maximal. This approach leads to small values for $v_2 \leq 0.03$ which are at odds with the constraints in Table 1.

Even if the proton is homogeneous, it is important to take into account the granularity of the matter distribution: even if assuming that all partons are homogeneously distributed within the proton, only a finite number of partonic collisions occur. Thus, on an event-by-event basis, the distribution of these partons changes; as a consequence, even in the most central collisions, where the eccentricity is naively zero, the eccentricity also changes in an event by event basis. Similar fluctuations in the nucleon distribution within the nuclei are important for the correct description of heavy ion v_2 data [7]. In Fig. 1 a the eccentricity distribution for the most central collision for the model of [5] is shown, which, as argued, extends to large eccentricity values.

A different approach is to assume that the scattering centers are not homogeneously distributed within the proton but that they are concentrated in a finite number N_s of "hot spots" with a typical size r_0. As in the previous case, on an event by event basis the hot spots are distributed within the proton and the different configurations lead to different eccentricity values even in the most central collision, which in this case corresponds to those collisions in which all N_s spots interact. The eccentricity distribution for $N_s = 3$ and different values of r_0 is shown in Fig. 1 b [1]. This somewhat extreme model leads to distributions peaked at very large values of the eccentricity. As the number of spots grows, the eccentricity of these central collisions tends to zero as expected.

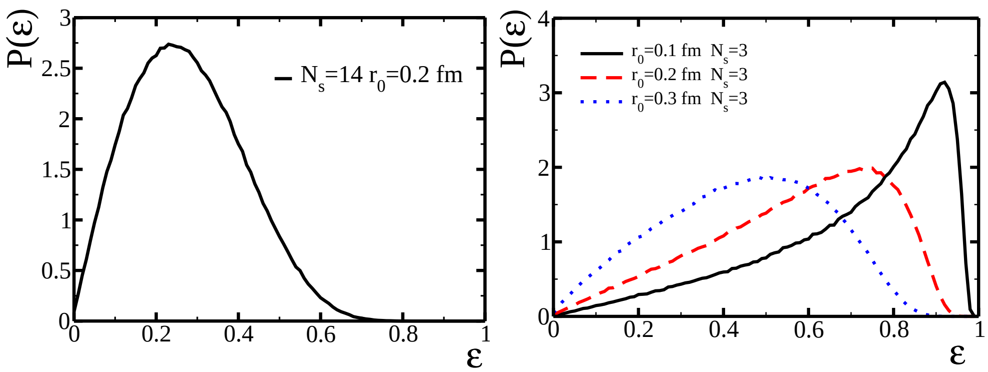

Figure 1: Eccentricity distribution of high multiplicity p-p collisions (central). The left panel shows the distribution for the homogenous model, with $N_s = 14$ scattering. The right panel is for a model in which all the hadronic activity is concentrated in $N_s = 3$ spots of different size r_0.

3 From eccentricity to flow

The eccentricity distributions shown in Fig. 1 are not directly measurable but they are transfered into the momentum distribution via final state interactions. Determining the precise form in which this transfer occurs demands a dynamical calculation along the lines of [4]. However, the flow systematics observed in heavy ion collisions can be used to estimate the final v_2 value. In fact the study of different colliding systems at different energies leads to the conclusion that v_2 is a universal function of the eccentricity of the colliding region and the transverse density of particles[7] . A simple parameterization based on hydrodynamics and which incorporates viscous effects was given in [2]

$$v_2 = \epsilon \left(\frac{v_2}{\epsilon}\right)^{hydro} \frac{1}{1 + \frac{\bar{\lambda}}{K_0} \frac{\langle S \rangle}{\frac{dN}{dy}}} , \tag{2}$$

where $< S >$ is the transverse area of the collision, $(v_2/\epsilon)^{hydro}$ is the hydro limit and $\bar{\lambda}/K_0$ is a fitted parameter.

The relation Eq. (2) is valid in an event by event basis. However, the extraction of v_2 demands average over many events, *i. e.*, the distributions in Fig. 1. In particular, the fourth cumulant v_2, $v_2\{4\}$ is proportional to [6]

$$\epsilon\{4\} \equiv \left(2\left\langle\epsilon^2\right\rangle^2 - \left\langle\epsilon^4\right\rangle\right)^{1/4} . \tag{3}$$

In Fig. 2 we show the predicted (event averaged) $v_2\{4\}$ for the two different models sketched in the previous section for central (high multiplicity) proton-proton collisions as a function of multiplicity. In both plots, the continuous line is the hydrodynamic limit $\bar{\lambda}/K_0 = 0$, the dashed line correspond to the fitted value $\bar{\lambda}/K_0 = 0.58 fm^{-2}$, and the dotted line is twice this value. On Fig. 2 a we show the results for the homogenous model, with $N_s = 14$ scattering centers: the expected values of v_2 is large and it is comparable to the constraints in Table 1; the value extracted is, in particular, twice as large as the model studies in which the smooth approximation is used. In Fig. 2 b we show the expected v_2 values for the more extreme model in which all the hadronic activity is concentrated in $N_s = 3$ hot spots with radius $r_0 = 0.1$, 0.2, 0.3 fm; in all these cases the values of v_2 are very large and they should be easily distinguishable from non-flow correlations.

In both the cases studied, the density fluctuations in the proton lead to a distinct signal on the azimuthal asymmetry of the produced particles, v_2 which is measurable and distinguishable from non-flow correlations.

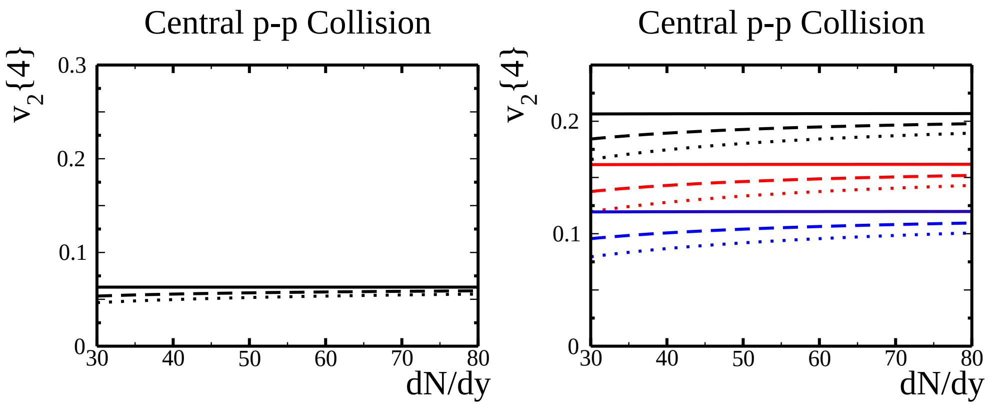

Figure 2: The flow signal $v_2\{4\}$ as a function of multiplicity in most central p-p collisions, for models of $N_s = 14$ (left), $N_s = 3$ (right) interaction regions of radius $r_0 = 0.1$ (top 3 curves), $r_0 = 0.2$ (middle curves), and $r_0 = 0.3$ fm (bottom curves). Signals calculated for $\bar{\lambda}/K_0 = 0$, 5.8 and 11.6 fm^{-2} are displayed by solid, dashed and dotted lines, respectively.

4 Conclusions

In this work we have argued that density fluctuations in the proton wave function play an essential role in determining collective effects in p-p collisions. The effect of these fluctuations is two-fold: on the one hand the eccentricity distribution of the colliding system peaks at sizable values; on the other hand, it allows to have a non vanishing v_2 for those p-p events with highest multiplicity. As shown in Table 1, high multiplicity is an essential requirement for distinguishing flow from non-flow effects on azimuthally asymmetric particle distribution. In both the models studied, the eccentricity fluctuations of the colliding region leads to v_2 values which are measurable for the largest multiplicities achieved in p-p collisions at the LHC.

Determining elliptic flow in p-p is important since it will demonstrate the system independence of the phenomena. Given the small size of the proton, the analysis of collective motion in this smaller system will allow a better constraint of the dissipative and thermalization properties of deconfined QCD matter. Finally, since the elliptic flow signal depends essentially on the density distribution of the proton, this measurement will lead to stringent constraints on models of soft interactions within the proton, as can be concluded from the different values of $v_2\,\{4\}$ in the models we have analyzed.

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