Unintegrated sea quark at small x and forward Z production

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Based on results obtained with F. Hautmann & H. Jung

Recontres de Moriond 2012 – QCD & High Energy Interactions

BFKL evolution & High Energy Factorization

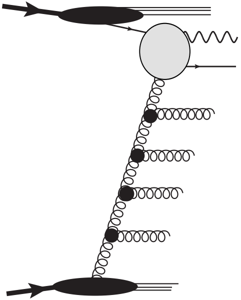
- hadronic scattering processes with a hard scale $Q^2 \gg \Lambda^2_{\text{QCD}}$ described within perturbative QCD
- limit of high c.o.m. eneries $s \gg Q^2$ ➡ enter multi scale regime ➡ $\alpha_s(Q^2) \ln s/Q^2 \sim 1$ ➡ require resummation

BFKL evolution:
- LL [Fadin, Kuraev, Lipatov (1977)], [Balitsky, Lipatov (1978)]
- NLL [Fadin, Lipatov (1998)], [Ciafaloni, Gamici (1998)]

- Predicts rise of cross-sections with s
- Derived from QCD & QFT
- Hints, but no clear experimental evidence till nowadays

Follow four paths
- study forward & forward-backward observables ➡ probe directly BFKL evolution
- (TMD) pdfs at small x ➡ x dependence described by BFKL
- Design Monte Carlo tools designed for small x phenomenology
- Develop further the theoretical formulation of high energy factorization

TMD parton distributions at small x

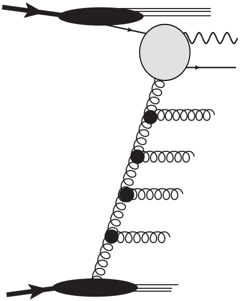


- collinear factorization (inclusive pdfs) well established for processes with a single large mass scale M_Z^2
- processes with multiple scale ➡ large perturbative logarithms ➡ need generalized factorization formula
- leads to transverse momentum dependent (TMD) or unintegrated parton distribution functions

general definition not yet available [J. Collins, Foundations of perturbative QCD, CUP (2011)]
➡ restrict to certain regions of phase space.

- **Here:** Use definition of TMD pdfs in small x limit
- Specific process: forward Z (DY) production, $s \gg M_Z^2 \gg \Lambda^2_{\text{QCD}}$

TMD factorization from high energy factorization



high energy factorization & BFKL evolution:

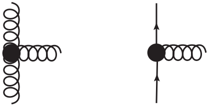
- obtain cross sections as convolutions in transverse momentum space
  ➡ natural basis for definition of TMD distributions
- dominated by gluon → TMD gluon distribution
- Note: multiple gluon exchanges only suppressed only by logarithms, not powers of s
- matching to collinear factorization $\equiv k_T$-factorization
  [Catani, Ciafaloni, Hautmann (1991)],[Catani, Hautmann (1994)]

- inclusive analysis (small x resummed F_2 structure function etc.)
- exclusive final states ➡ sensitive to soft radition
  - suitable generalization ➡ CCFM evolution : resum both soft and small x logarithms + takes into account associated coherence effects
  - TMD pdf required for $z \to 0$ coherence [Ciafaloni (1988), (1998)])
  - for inclusive observables: interpolation between DGLAP and BFKL
  - realized in Monte Carlo event generator CASCADE [Jung, Salam (2001)]; [Jung et.al. (2010)]

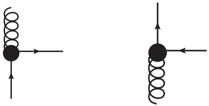
CCFM evolution resums

- soft logarithms ('$z \to 1$')
- high energy logarithms ('$z \to 0$')

➡ emssions **gluonic** (LL)



gluon and valence quark naturally defined



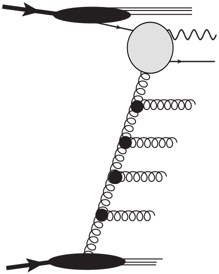
no non-diagonal transitions

Consequences for Cascade parton shower

- only gluonic emissions, no quarks
  ➡ jets purely gluonic

Consequences for the hard process:

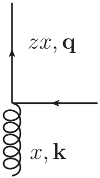
- quark $\equiv$ valence quark, seaquark through $2 \to 2$, $2 \to 3$ tree-level matrix elements
- contain soft & collinear divergence
  ➡ difficult to disentangle and isolate



Goal of this study: Construct sea quark density

At first: quark from last splitting, on top of small x gluon

TMD sea quark distribution at small x



- attempts so far: use LO DGLAP splitting function $P_{qg}(z)$ on top of TMD gluon distribution [Martin, Ryskin, Watt (2003, 2010)]; [Gawron, Kwiecinski, Broniowski (2003)]; [Hoeche, Krauss, Teubner (2008)]
- correct collinear limit, but miss corrections $\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2}$
  ➡ quark transverse momentum q_T: a 1-loop effect

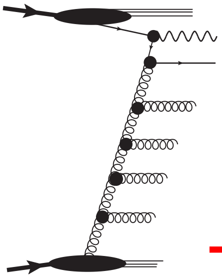
Here : use TMD splitting function [Catani, Hautmann, (1994)]

$$P_{qg}^{\mathrm{CH}} = T_R \left(\frac{(\boldsymbol{q} - z\boldsymbol{k})^2}{(\boldsymbol{q} - z\boldsymbol{k})^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[P_{qg}(z) + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{(\boldsymbol{q} - z\boldsymbol{k})^2} \right]$$

- from matching of high energy resummation on collinear factorization
  ➡ small x enhanced collinear logarithms included to all orders
- corrections $\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2}$ to all orders included ➡ quark q_T follows from multiple small x enhanced branchings
- universal despite of off-shellness [Catani, Hautmann (1994)]; [Ciafaloni, Colferai (2005)]

Definition of q_T-dependent sea-quark density:

$$\mathcal{Q}^{\text{sea}}(x, \mathbf{\Delta}^2, \mu^2) := \int_x^1 dz \int_0^{\mu^2/z} d\boldsymbol{k}^2 \frac{P_{qg}^{\text{CH}}\left(z, \frac{\boldsymbol{k}^2}{\mathbf{\Delta}^2}\right)}{\mathbf{\Delta}^2} \mathcal{G}^{\text{gluon}}\left(\frac{x}{z}, \boldsymbol{k}^2, \mu^2, \right)$$

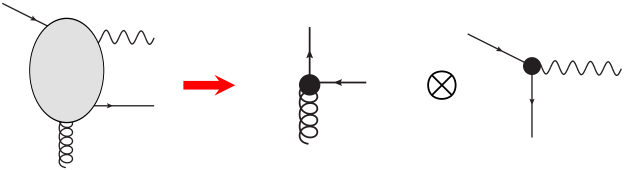


- shifted transverse momentum $\mathbf{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$
- μ^2: factorization scale
- $\mathcal{G}^{\text{gluon}}$: TMD gluon distribution, determined from BFKL/CCFM evolution, contains LL small x resummation + matched to collinear factorization
- upper bound fixed up to NLL correction, here: CCFM prescription = angular ordering

→ TMD sea quark distribution:
- correct collinear limit
- small x resummation,
- $\mathbf{\Delta}^2$ from multiple branchings

Determination of off-shell coefficients

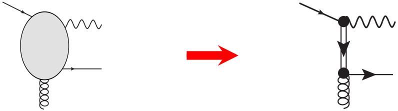
- Consider specific process: forward Z production (interesting by itself, probes proton at very small x at the LHC)
- gluon incuded $2 \to 2$ process $qg^* \to Zq$ known [Marzani, Ball (2008)]



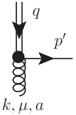
- forward Z: incoming quark valence like ➡ simplifies treatment
- possible to extract $qq^* \to Z$ from matching procedure
- Here: develop formalism to calculate this coefficient ➡ can generalize to off-shell case $q^*q^* \to Z$
- Strategy: mimic gluonic case ➡ use factorization of $qg^* \to Zq$ in high energy limit

High energy limit for quark exchange: ‘reggeized quark’ formalism

- use formulations by [Bodgan, Fadin, (2005)],[Lipatov, Vyazosky (2001)]

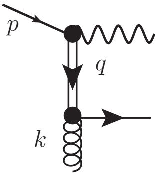


- achieve gauge invariant off-shell factorization through effective vertices $\equiv$ re-organization of QCD diagrams



$$= igt^a \left(\gamma^\mu - \not{q} \frac{(n^+)^\mu}{k^+} \right) \qquad \text{etc.}$$

- agree for on-shell quarks with QCD vertex, extra term provides gauge invariance $\rightarrow$ from expansion of Wilson line
- Formalism used in phenomenological studies [Saleev (2008)]; [Kniehl, Saleev, Shipilova (2008)] in combination with KMRW-u pdfs (initial parton on-shell $\boldsymbol{k}^2 \rightarrow 0$)



Reggeized quark formalism gives high energy expansion:

- imposes limit $\bar{z} = \frac{q^+}{p^+} \to 0$, $z = \frac{q^-}{k^-} \to 0$
- rough approximation at finite energies: splitting function a constant
- $qq^* \to Z$ coefficient with correct collinear limit & can be directly used

- generalize formalism to finite $z, \bar{z}$ ➡ possible in gauge invariant way (at level of current conservation)
- finite z: TMD splitting function of Catani&Hautmann
- finite $\bar{z}$: not dictated by correct collinear limit ➡ 2 possible versions for off-shell coefficient $qq^* \to Z$

$$\hat{\sigma}_{q\bar{q}^* \to Z}(\nu, \boldsymbol{q}^2) = \underbrace{\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(z\nu - M_Z^2 - t), \quad \nu = x_1 x_2 s$$

- $\bar{z} = 0$ leads to $t = \boldsymbol{\Delta}^2$, $\boldsymbol{\Delta} = (\boldsymbol{q} - z\boldsymbol{k})$ (up to a subleading correction); finite $\bar{z}$ gives $t = -q^2 = \frac{\boldsymbol{\Delta}^2}{1-z} + z\boldsymbol{k}^2$ (agrees with exact kinematics)
- both choices coincide in high energy ($z \to 0$) and collinear ($\boldsymbol{k}^2 \to 0$) limit.

- depending on choice, $qg^* \to qZ$ factorized as convolution w.r.t $\boldsymbol{q}^2$ or $|q^2|$
  ➡ different virtuality of the off-shell quark

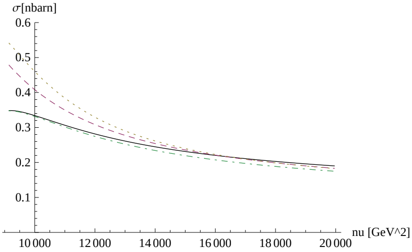
$$\sigma^{q_T-\text{fact.}}_{qg^*\to Zq} = \int_0^1 dz \int_0^{(1-z)(\mu^2 - z\boldsymbol{k}^2)} \frac{d\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2}\,\hat{\sigma}_{qq^*\to Z}(z\nu, \boldsymbol{\Delta}^2)\cdot\frac{\alpha_s}{2\pi}P_{qg}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right)$$

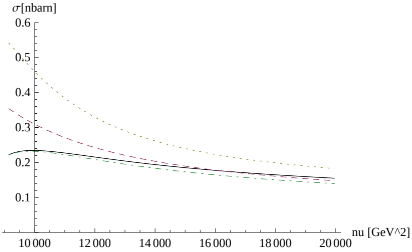
$$\sigma^{q-\text{fact.}}_{qg^*\to Zq} = \int_0^1 dz \int_{z\boldsymbol{k}^2}^{\mu^2} \frac{d|q^2|}{|q^2| - z\boldsymbol{k}^2}\,\hat{\sigma}_{qq^*\to Z}(z\nu, |q^2|) \cdot\frac{\alpha_s}{2\pi}P_{qg}\left(z, \frac{\boldsymbol{k}^2}{(1-z)(|q^2| - z\boldsymbol{k}^2)}\right)$$

- Constraint $\mu^2 > q^2$ from collinear factorization [Catani, Hautmann (1994)]
- Interesting result for u-pdfs on pure DGLAP basis [Martin, Ryskin, Watt (2010)]
  ➡ Use of $|q^2|$ instead of $\boldsymbol{q}^2$ allows to reproduce NLO DGLAP corrections with LO splitting kernel (through NLO kinematic alone)

Numerical comparsion of factorized expressions

- Comparsion of q_T (dashed) and q (dashed-dotted) factorized expression against pure LO DGLAP splitting function (dotted) and exact $qg^* \to Zq$ cross-section (black)
- expression with LO DGLAP splitting function divergent even for finite gluon $\boldsymbol{k}^2$: introduce cut-off & match to q_T-factorized expression at $\nu = 2M_Z^2$ and $\boldsymbol{k}^2 = 1\text{GeV}^2$.

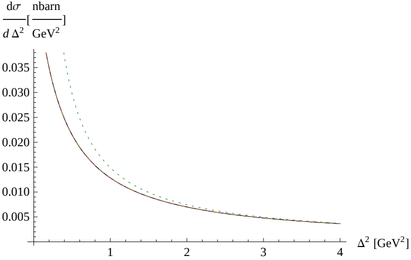


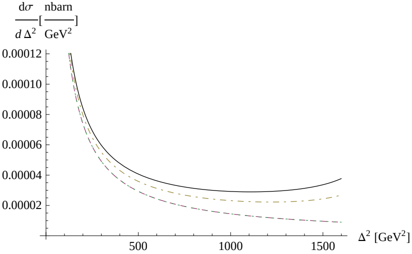


- ν -dependence of total cross-section for $\boldsymbol{k}^2 = 1\text{GeV}^2$ and $\boldsymbol{k}^2 = 10\text{GeV}^2$

Numerical comparision of collinear behavior

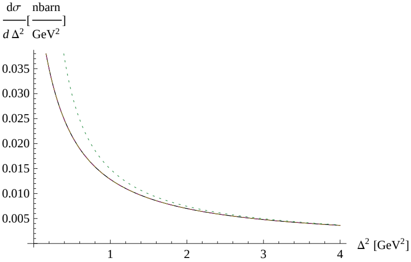
- Collinear logarithm arises (for $M_Z^2 \gg q^2, \boldsymbol{q}^2$) from integral over $\boldsymbol{\Delta}^2 = (\boldsymbol{q} - z\boldsymbol{k})^2$
  ➡ study $\frac{d\sigma_{qg^* \to qZ}}{d\boldsymbol{\Delta}^2}$ to investage collinear behavior
- compare again q_T (dashed) and q (dashed-dotted) factorized expression against pure LO DGLAP splitting function (dotted) and exact $qg^* \to Zq$ cross-section (black)
- plots shown for $\nu = 2.5 M_Z^2$ and $\boldsymbol{k}^2 = 2\text{GeV}^2$

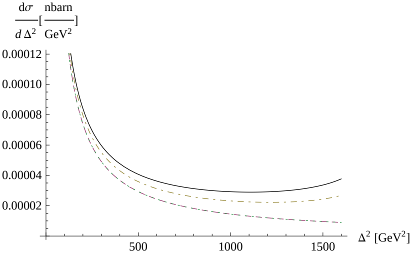




Numerical comparision of collinear behavior

- LO DGLAP splitting provides good description in region $M_z^2 \gg \mathbf{\Delta}^2 \gg \boldsymbol{k}^2$
- q_T factorized expression can describes $M_z^2 \gg \mathbf{\Delta}^2 \sim \boldsymbol{k}^2$ but fails for large $\mathbf{\Delta}^2$
- q factorized expression equally good for small $\mathbf{\Delta}^2$; catches in addition some of the effects at large $\mathbf{\Delta}^2 \leftrightarrow$, naturally misses effects of subleading s-channel contributions contained in the unfactorized cross-section





external parameters: $\nu = 2.5 M_Z^2$ and $\boldsymbol{k}^2 = 2\text{GeV}^2$

Conclusions and Outlook:

- Current small-x parton showers include only gluon & valence quarks at TMD level
- Here: go beyond this approximation by including TMD sea-quark
- The presented method includes finite-k_T terms in the gluon - quark splitting P_{qg}, which control resummation of $\alpha_s(\alpha_s \ln 1/x)^n$ corrections to flavor-singlet observables
- We obtained an off-shell (but gauge-invariant) hard matrix element for coupling Z production to the TMD sea-quark using the "reggeized quark" formalism [Bodgan, Fadin (2005)],[Lipatov, Vyazosky (2001)]

Current results are based on:
– 1 quark shower interaction only
– 1 off-shell quark only

➡ this needs extending; nevertheless, it is starting point to include systematically quark-initiated processes in small-x showers

➡ hopefully achieve more general defs of TMD pdfs: e.g., match on to SCET definitions [Stewart, Tackmann, Waalewijn (2009)]; [Garcia-Echevarria, Idilbi, Scimemi (2011)]; [Becher, Neubert (2011)]; [Mantry, Petriello (2011)] and TMD evolution and fits [Aybat, Rogers, 2011]