$e^+e^- \rightarrow J/\psi + \eta_c$ at B factory energy in Bethe-Salpeter framework

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BS framework as one way to investigate the virtuality and relative move. of the compositio particles in bound for produc. process

- Experimental results
- Theoretical explanations: consistent & in.
- Introduction on the BS framework
- The calculation procedure
- Disucssion on the investigation

Data and LO Calculations

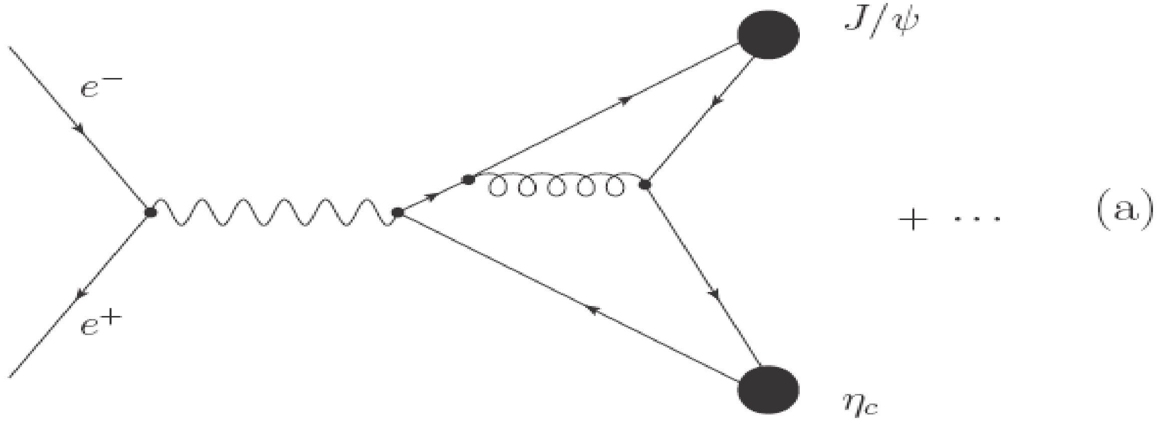
LO=leading order; leading opinion

$$\sigma(e^+ + e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad (\text{Belle})$$

$$2.3 \sim 5.5 \text{ fb}.$$

Braaten & Lee,
Liu, He & Chao
Hagiwara, Kou & Qiao

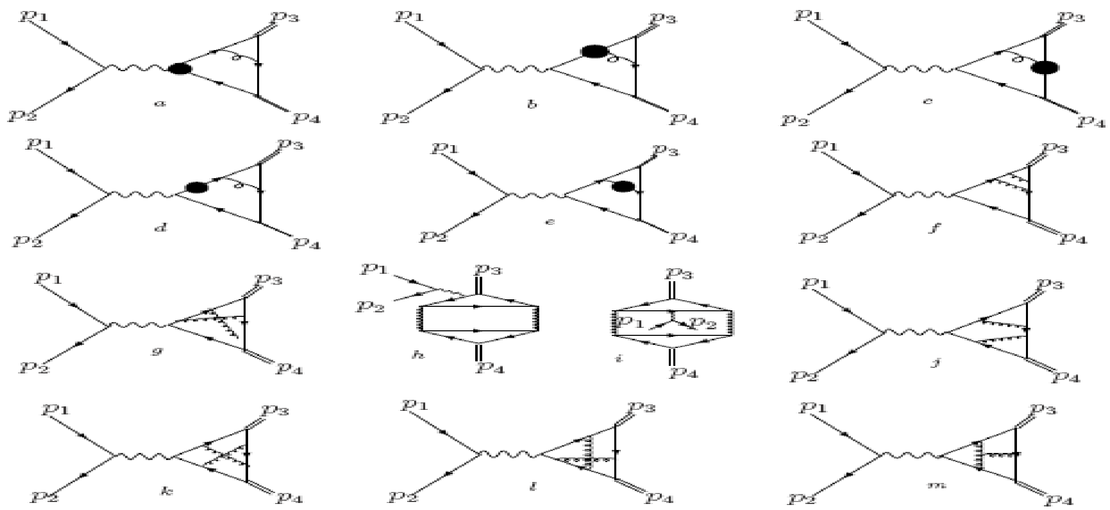
$$\sigma(e^+ + e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad (\text{BaBar}),$$



NRQCD

Higher order (NLO)

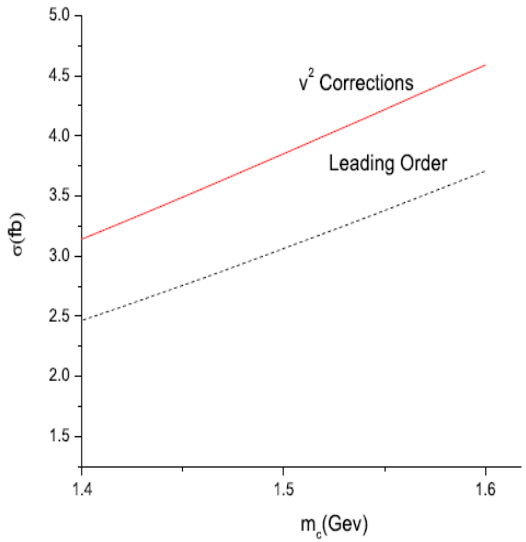
NRQCD

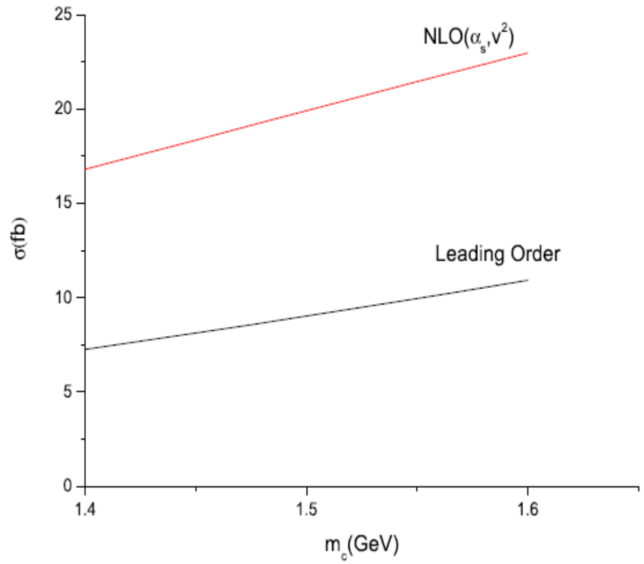


Zhang, Gao & Chao
Gong & Wang*

relativisitc correctior

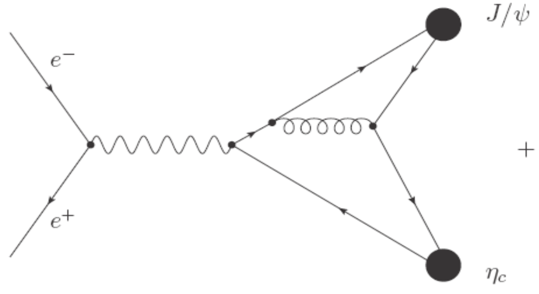
He, Fan & Chao*
Bodwin, Lee & Yu





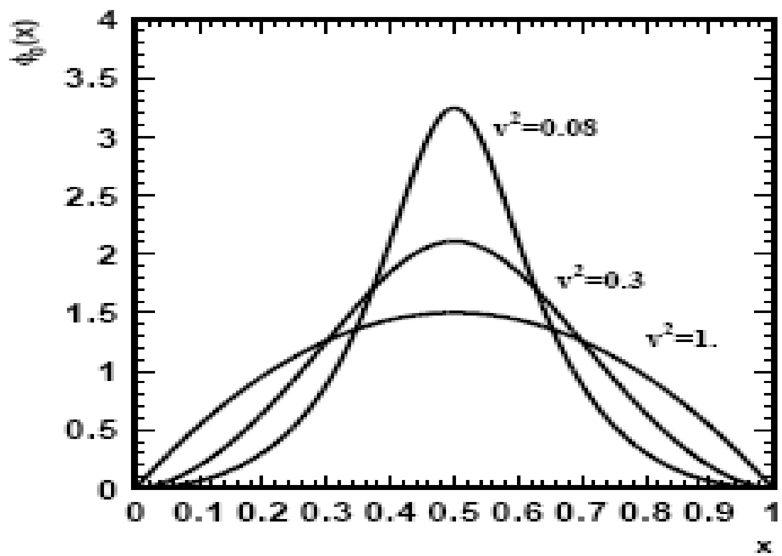
Relativistic approaches
(on shell)

- Lightcone WF (Ma & Si, Bondar & Chernyak*
- QCD sum rule (Sun, Wu, Zuo & Huang)
- BS (Guo, Ke, Li & Wu; Mengesha & Bhatnagar)



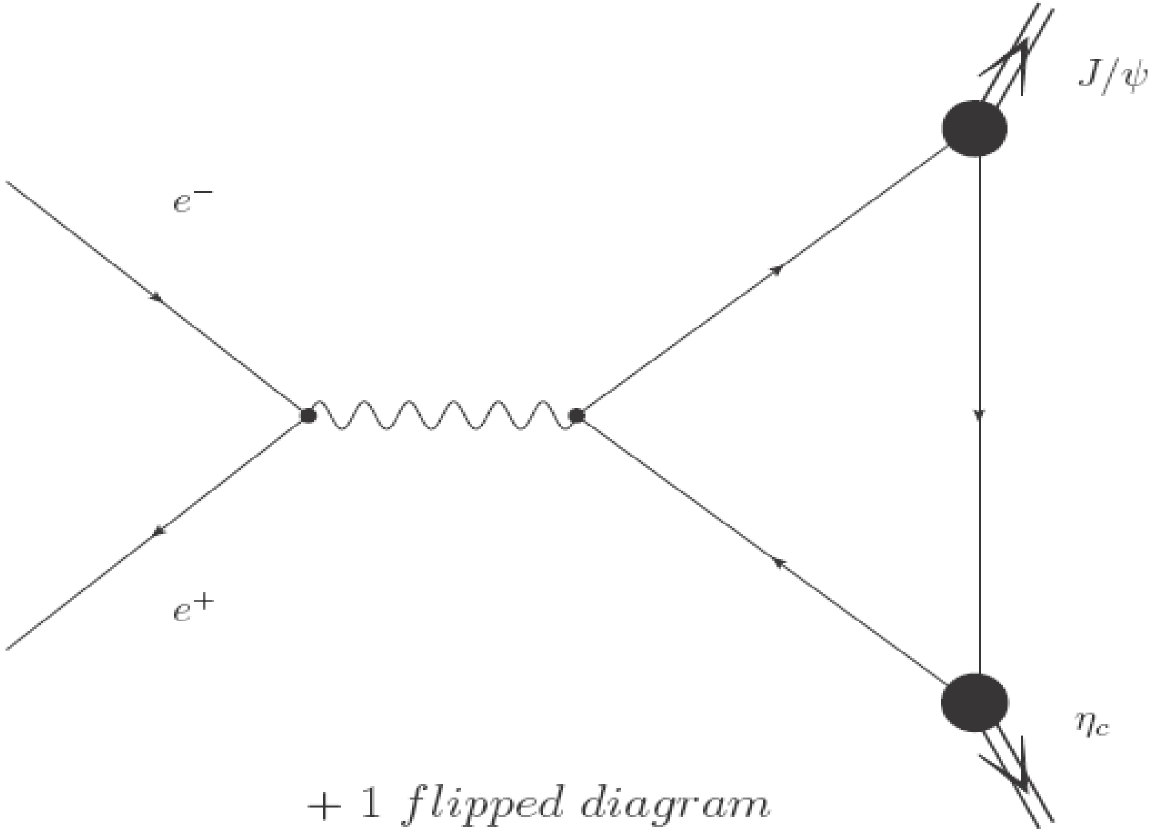
however!

They take the 'NR' approximation to some level (heavy quark limit)



typically with momentum fraction distributions

Both virtuality & relative move. taken into account, can start from the lowest order



$+ 1 \; flipped \; diagram$

Key points for Calculation

- **The vertices c-cbar-bound state**

  the momentum distribution

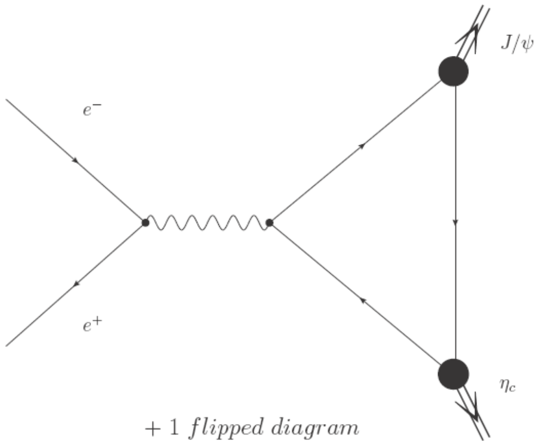
  the Dirac structure

  (both contain relativistic informations)

- **The loop integrals**

  pole of the propagators

  the other 3-dimentions



The coupling vertex and the BS WF

$$\chi(P,q) = \int \frac{d^4x}{(2\pi)^4} e^{-iqx} \frac{1}{\sqrt{3}} \delta_{ij} < 0 \mid T\psi^i(\frac{x}{2})\bar{\psi}^j(-\frac{x}{2}) \underline{\mid B >}$$

$$\underline{= S_F(p_1)\Gamma(\hat{q})S_F(-p_2)}$$

by definition

$$\mathscr{L}_I = \bar{\psi}^i(x)\Theta_{ij}(i\frac{\partial}{\partial x})\delta_{ij}\psi^j(x)\underline{B(x)}$$

Reduction Formula

$$p_1 = \frac{P}{2} + q,$$
$$p_2 = \frac{P}{2} - q,$$
$$P = p_1 + p_2,$$
$$q = \frac{1}{2}(p_1 - p_2)$$

Some more details in CIA approx.

$$\hat{q}_\mu = q_\mu - \frac{q.P}{P^2} \quad \phi(\hat{q}) \sim e^{-\hat{q}^2/2\beta^2}$$

$$\Gamma(\hat{q}) = D'(\hat{q})\phi(\hat{q}) = \tilde{\Gamma} N D(\hat{q})\phi(\hat{q})$$

First analyzed by Llewellyn-Smith:

$$\tilde{\Gamma}_J = i\not{\varepsilon} A_0 + \not{\varepsilon}\not{P}\frac{A_1}{M} + (q\varepsilon - \not{\varepsilon}\not{q})\frac{A_2}{M} + i(\not{\varepsilon}\not{P}\not{q} - \not{\varepsilon}\not{q}\not{P} + 2i(q\varepsilon)\not{P})\frac{A_3}{M^2} + (q\varepsilon)\frac{A_4}{M} + iq\varepsilon\not{P}\frac{A_5}{M^2} - iq\varepsilon\not{q}\frac{A_6}{M^2} + (q\varepsilon)(\not{P}\not{q} - \not{q}\not{P})\frac{A_7}{M^3}$$

$$\tilde{\Gamma}_y = \gamma_5 B_0 - i\gamma_5\not{P}\frac{B_1}{M} - i\gamma_5\not{q}\frac{B_2}{M} - \gamma_5(\not{P}\not{q} - \not{q}\not{P})\frac{B_3}{M^2}$$

see, Bhatnagr & LSY

Successful in describing decays

	f_P^0	f_P^1	f_P^2	f_P^3	$\|f_P^{LO}\|$	$\|f_P^{NLO}\|$	$f_P^{LO}(\%)$	$f_P^{NLO}(\%)$	$\boldsymbol{f_P = f_P^{LO} + f_P^{NLO}}$
π	0.110	-0.154	0.000	0.175	0.044	0.175	25%	75%	**0.130**
K	0.202	-0.104	0.025	0.039	0.098	0.064	60%	40%	**0.164**
D	0.271	-0.097	0.010	0.009	0.174	0.019	90%	10%	**0.194**
D_S	0.426	-0.156	0.013	0.013	0.270	0.026	91%	9%	**0.296**
B	0.345	-0.125	0.005	0.003	0.220	0.008	96%	4%	**0.228**

	f_π	f_K	f_D	f_{D_s}	f_B
BSE (3.5% average error) present paper	**0.130**	**0.164**	**0.194**	**0.296**	**0.228**
BSE [5]				0.248	
SDE [6]		0.164			
Lattice [15]			0.208±0.004	0.241±0.003	
QCD-SR [16]			0.20±0.02	0.23±0.02	
Exp. Results [13]	0.1300±0.0001	0.159±0.001	0.22±0.02	0.29±0.03	
Babar+Belle Collaboration [14]					0.24±0.04

see Bhatnagar, LSY & Mahecha

$$\sigma = \int \frac{|\mathbf{M}|^2}{F} dR_2$$
$$= \int \frac{|\mathbf{M}|^2}{2s} \frac{k}{32\pi^2 E} 4E^2 (2\pi)^6 sin\theta d\theta d\varphi$$

$$\mathrm{M} = 2 l_\mu^{(e)} \frac{ie^2 e_Q}{q^2} \frac{1}{-(2\pi)^2} N_J N_y \int \frac{d^4p}{(2\pi)^4} \frac{D(\hat{q}_J) D(\hat{q}_y) \phi(\hat{q}_J) \phi(\hat{q}_y)}{((p-P_y)^2 - m^2)(p^2 - m^2)((p+P_J)^2 - m^2)}$$
$$\times Tr[\gamma^\mu (\not{p} - \not{P}_y + m) \Gamma_y (\not{p} + m) \Gamma_J (\not{p} + \not{P}_J + m)] \qquad (3.20)$$

$$N_J^{-2}(\beta) = \int_0^\infty d\hat{q} 16\pi^3 e^{-\frac{\hat{q}^2}{\beta^2}} \hat{q}^2 \left(m^2 + \hat{q}^2\right) \left(-M^2 + 4\left(m^2 + \hat{q}^2\right)\right)^2 A$$
$$N_y^{-2}(\beta) = \int_0^\infty d\hat{q} 16\pi^3 e^{-\frac{\hat{q}^2}{\beta^2}} \hat{q}^2 \left(m^2 + \hat{q}^2\right) \left(-M^2 + 4\left(m^2 + \hat{q}^2\right)\right)^2 B$$

$$A = \frac{2\pi}{3\left(m^2+\hat{q}^2\right)^{1.8}} + \frac{4m^2\pi}{3M^2\left(m^2+\hat{q}^2\right)^{1.8}} - \frac{2\pi}{3M^2\left(m^2+\hat{q}^2\right)^{0.8}}$$
$$+ \frac{4m^2\pi\left(M^2-12\left(m^2+\hat{q}^2\right)\right)}{3\left(m^2+\hat{q}^2\right)^{1.8}\left(M^2-4\left(m^2+\hat{q}^2\right)\right)^2} + \frac{M^2\pi\left(M^2-12\left(m^2+\hat{q}^2\right)\right)}{3\left(m^2+\hat{q}^2\right)^{1.8}\left(M^2-4\left(m^2+\hat{q}^2\right)\right)^2}$$
$$+ \frac{4\pi\left(-\frac{M^2}{2}+\left(m^2+\hat{q}^2\right)\right)}{3M^2\left(m^2+\hat{q}^2\right)^{1.8}} + \frac{4\pi}{\left(m^2+\hat{q}^2\right)^{0.8}\left(-M^2+4\left(m^2+\hat{q}^2\right)\right)}$$
$$- \frac{16m^2\pi}{3M^2\left(m^2+\hat{q}^2\right)^{0.8}\left(-M^2+4\left(m^2+\hat{q}^2\right)\right)} \qquad (3.24)$$

$$B = -\frac{1}{M^2}\left(\frac{\pi(-2m^2M^2-\frac{M^4}{2}-6M^2\hat{q}^2)(-M^2(m^2+\hat{q}^2)^{0.8}+12(m^2+\hat{q}^2)^{1.8})}{2(m^2+\hat{q}^2)^2\cdot(M^2-4(m^2+\hat{q}^2))^2}\right.$$
$$+ 2\pi(4m^2M^2-M^4-4M^2\hat{q}^2)(-\frac{M^2}{8(m^2+\hat{q}^2)^{1.8}(M^2-4(m^2+\hat{q}^2))^2}$$
$$+ \frac{5}{2(m^2+\hat{q}^2)^{0.8}(M^2-4(m^2+\hat{q}^2))^2}) + 8M^4\pi(-\frac{0.25}{M^8(M^2-2M(m^2+\hat{q}^2)^{0.8})^2}$$
$$+ \frac{M^2+2M(m^2+\hat{q}^2)^{0.8}}{8M^8(m^2+\hat{q}^2)^{0.8}} + \frac{M^4-2M^2(m^2+\hat{q}^2)}{16M^6(m^2+\hat{q}^2)^{1.8}})$$
$$+ \frac{1}{4}\pi(\frac{2(M^2+2M(m^2+\hat{q}^2)^{0.8})^2}{M^2(m^2+\hat{q}^2)^{0.8}(M+2(m^2+\hat{q}^2)^{0.8})^2}$$
$$+ \frac{2M(-m+M)(M^2-2M(m^2+\hat{q}^2)^{0.8})^2}{(m^2+\hat{q}^2)(-(m^2+\hat{q}^2)+(M-(m^2+\hat{q}^2)^{0.8})^2)^2}$$
$$- \frac{4M^2(M^2-2M(m^2+\hat{q}^2)^{0.8})}{(m^2+\hat{q}^2)(-(m^2+\hat{q}^2)+(M-(m^2+\hat{q}^2)^{0.8})^2)}$$
$$\left.- \frac{M(M^2-2M(m^2+\hat{q}^2)^{0.8})^2}{(m^2+\hat{q}^2)^{1.8}(-(m^2+\hat{q}^2)+(M-(m^2+\hat{q}^2)^{0.8})^2)}\right)) \qquad (3.25)$$

$$| \mathrm{M} |^2 = 4L_{\mu\nu}^{(e)} L_{(h)}^{\mu\nu} \frac{e^4 e_Q^2}{(2\pi)^4 q^4} N_J^2 N_y^2 \mid I(\beta) \mid^2$$

$$I(\beta) = \int \frac{d^4p}{(2\pi)^4} \frac{D(\hat{q}_J)D(\hat{q}_y)\phi(\hat{q}_J,\beta)\phi(\hat{q}_y,\beta)}{((p-P_y)^2-m^2)(p^2-m^2)((p+P_J)^2-m^2)}$$

$$I(\beta) = \frac{1}{2(2\pi)^3} \int_0^{+\infty} d\vec{p}_\perp^{\,2} \int_{-\infty}^{+\infty} dp^- \int_{-\infty}^{+\infty} dp^+ F(p^+,p^-,\vec{p}_\perp^{\,2},\beta) \frac{1}{2p^+p^- - \vec{p}_\perp^{\,2} - m^2}$$
$$\times \frac{1}{2(p^+ + P_J^+)(p^- + P_J^-) - \vec{p}_\perp^{\,2} - m^2} \frac{1}{2(p^+ - P_y^+)(p^- - P_y^-) - \vec{p}_\perp^{\,2} - m^2}$$

$$I(\beta) = I_1(\beta) + I_2(\beta)$$

$$I_1(\beta) = \frac{1}{2(2\pi)^3} \int_0^{+\infty} d\vec{p}_\perp^{\,2} \int_0^{P_y^-} dp^- \int_{-\infty}^{+\infty} dp^+$$

$$I_2(\beta) = \frac{1}{2(2\pi)^3} \int_0^{+\infty} d\vec{p}_\perp^{\,2} \int_{-P_J^-}^{0} dp^- \int_{-\infty}^{+\infty} dp^+$$

β	I^2	N^2	I^2/N^2	$\sigma(fb)$
$1/2$	3.39×10^{-14}	1.29×10^{6}	2.63×10^{-20}	1.4×10^{-6}
$1/\sqrt{2}$	7.90×10^{-9}	1.47×10^{7}	5.37×10^{-16}	2.9×10^{-2}
1	1.36×10^{-6}	1.52×10^{8}	8.95×10^{-15}	4.9×10^{-1}
1.15	8.00×10^{-6}	2.83×10^{8}	2.83×10^{-14}	1.5
1.3	9.80×10^{-6}	8.06×10^{7}	1.22×10^{-13}	6.6
$\sqrt{2}$	9.80×10^{-5}	1.08×10^{9}	9.09×10^{-14}	5.0
1.5	1.00×10^{-4}	3.36×10^{9}	2.97×10^{-14}	1.6
1.6	3.59×10^{-4}	8.85×10^{9}	4.05×10^{-14}	2.2
1.7	1.08×10^{-3}	1.89×10^{10}	5.73×10^{-14}	3.1
1.8	2.95×10^{-3}	3.40×10^{10}	8.69×10^{-14}	4.7
1.9	7.32×10^{-3}	4.96×10^{10}	1.48×10^{-13}	8.1
2	1.05×10^{-2}	4.72×10^{10}	2.23×10^{-13}	12.2
2.1	3.41×10^{-2}	2.50×10^{10}	1.37×10^{-12}	74.6
2.25	8.59×10^{-2}	5.73×10^{11}	1.50×10^{-13}	8.2
2.5	1.81×10^{-1}	6.59×10^{12}	2.75×10^{-14}	1.5
3	1.41	2.08×10^{14}	6.81×10^{-15}	0.4
4	26.35	3.07×10^{16}	8.57×10^{-16}	0.05

Conclusion

- Taking into account the virtual quark state in the bound state in a full relativistic framework
- The lowest order contributing sounding cross section
- A consistent framework to higher precision (higher order) and for global analysis