Optimizing Higgs Identification at the LHC

Seung J. Lee

Rencontres de Moriond

L. Almeida, SL, G. Perez, G. Sterman, & I. Sung

PRD 82, 054034 (2010)

L. Almeida, O. Erdogan, J. Juknevich, SL, G. Perez, & G. Sterman

arXiv:1112.1957

Moriond QCD and High Energy Interactions, Mach 10
Outline

♦ Introduction (motivation)
♦ Template Overlap Method
♦ LO Template for Higgs
♦ NLO template (+color flow) for Higgs
♦ Summary
Looking at boosted massive objects, generic motivations

TeV scale New Physics => boosted electroweak+top particles.

Observing signal => identify collimated $W/Z/h/t$, $\Delta \theta_{ij} \sim m_J / E_J$.

Massive particles easier to identify when boosted.

Combinatorial background is removed, less soft junk collected & often backgrounds fall faster than signal with energy.

For instance $h + V, t, \chi^0$. 
The challenge of highly boosted Massive Jets

♦ Fine tuning solution => New states decay quickly to massive SM particles

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♦ Since $M_{t,h} \ll M_X$ the outgoing SM particles are ultra-relativistic, their decay products are collimated
The challenge of highly boosted Massive Jets

✦ Fine tuning solution => New states decay quickly to massive SM particles

✦ Since $M_{t,h} \ll M_X$ the outgoing SM particles are ultra-relativistic, their decay products are collimated

✦ The concept of boosted massive jet emerges

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Higgs hunting

Search for Higgs boson in $W/Z+H$, $H \rightarrow b\bar{b}$.

Butterworth, Davison, Rubin & Salam (08).

Example improvement from boosted regime

[Introduction]

Search for main decay of light Higgs boson in $W/Z+H$, $H \rightarrow b\bar{b}$.

ATLAS TDR (unboosted)

(boosted)

restricting search to $p_{TH} > 200$ GeV

using the method from Butterworth, Davison, Rubin & GPS '08

Gavin Salam (CERN/Princeton/Paris)
Higgs hunting

Search for Higgs boson in \( W/Z + H, H \rightarrow bb \).

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Search for main decay of light Higgs boson in \( W/Z + H \), \( H \rightarrow b\bar{b} \).

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(uncorrected)

(uncorrected)

restricting search to \( p_{T H} > 200 \text{ GeV} \)

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Gavin Salam (CERN/Princeton/Paris)

Boosted Theory

LHC4TSP, 2011-08-30 4 / 19

Less competitive than \( h \rightarrow \gamma \gamma \) but important. (can be improved?)
Higgs hunting
Need to understand the energy flow inside jet shapes or jet substructure
Need to understand the energy flow inside jet jet shapes or jet substructure

i) Jet Shapes

ii) Template Overlap Method
   1) LO for Higgs
   2) NLO Higgs (+color flow)
Why jets? What else?

✦ QCD amplitudes have soft-collinear singularity

✦ Observable: IR safe, smooth function of $E$ flow

Sterman & Weinberg, PRL (77)

✦ Jet is a very inclusive object, defined via direction + $p_T$ (+ mass)

✦ Even $R=0.4$ contains $O(50)$ had-cells => huge amount of info’ is lost
Beyond mass, higher moments, angularity (2 body)

Given jet mass & momenta, only one additional independent, variable to describe energy flow:

$$\tau_{-2} \sim \frac{1}{m} \sum_{i \in J} E_i \theta_i^4$$

QCD: $\propto \frac{1}{\tau_{-2}}$

$$h: \propto \frac{1}{\tau_{-2}^2}$$

Berger, Kucs, Sterman, PRD (03);
Almeida, SL, Perez, Sterman, Sung & Virzi, PRD (09).
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\begin{equation}
\theta_i^4
\end{equation}

\begin{equation}
-2
\end{equation}

after fixing mass - signal & background dist' are similar in shape!

The angularity distribution for QCD (red-dashed curve) and longitudinal Z (black-solid curve) jets obtained from MADGRAPH. Both distributions are normalized to the same area.

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Beyond mass, higher moments, angularity (2 body)

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\[ \tau_{-2} \sim \frac{1}{m} \sum_{i \in J} E_i \theta_i^4 \quad \rightarrow \quad \text{QCD: } \propto \frac{1}{\tau_{-2}} \]

\[ h: \propto \frac{1}{\tau_{-2}^2} \]

If mass is due to 2-body => sharp prediction (kinematics):

\[ \theta_{\text{min}} \sim \frac{m_J}{p_J} \Rightarrow \tau_{-2}^{\text{min}} \approx \left( \frac{m_J}{p_J} \right)^3 \]

\[ \theta_{\text{max}} \sim R \Rightarrow \tau_{-2}^{\text{max}} \approx R^2 \frac{m_J}{p_J} \]

Berger, Kucs, Sterman, PRD (03); Almeida, SL, Perez, Sterman, Sung & Virzi, PRD (09).

Almeida, SL, Perez, Sterman & Sung, PRD (10).
Beyond mass, higher moments, angularity (2 body)

Given jet mass & momenta, only one additional independent, variable to describe energy flow:

\[ \tau_2 \sim t \]

\[ \tau_{\min} \sim m_J \]

\[ \tau_{\max} \sim R \]

\[ \tau_2 \sim m_J^2 \]

\[ \tau_2 \sim R^2 m_J^2 \]

Angularity for jets with mass \( \in (90, 120) \) GeV/c^2, \( p_T > 400 \) GeV/c, \( 0.1 < |\eta| < 0.7 \), cone \( R=0.7 \). Black crosses are the data, red dashed is QCD MC, \( \tau_{\min} \) and \( \tau_{\max} \) predictions are also shown. The inset plot compares the results with Midpoint/SC and Anti-\( k_T \).
Planar Flow

♦ Top-jet is 3 body vs. massive QCD jet $\leftrightarrow$ 2-body (our result)

QCD massive jet

top jet
Planar Flow

♦ Top-jet is 3 body vs. massive QCD jet <=> 2-body (our result)
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QCD massive jet

top jet
Planar flow

Top-jet is 3 body vs. massive QCD jet \(\leftrightarrow\) 2-body (our result)

Planar flow, \(P_f\), measures the energy ratio between two primary axes of cone surface:

(i) “moment of inertia”:

\[
I_{E}^{k,l} = \frac{1}{m_J} \sum_{i \in R} E_i \frac{p_{i,k}}{E_i} \frac{p_{i,l}}{E_i},
\]

(ii) Planar flow:

\[
P_f = 4 \frac{\det(I_E)}{\text{tr}(I_E)^2} = \frac{4\lambda_1\lambda_2}{(\lambda_1 + \lambda_2)^2}
\]

leading order QCD, \(P_f=0\)

top jet, \(P_f=1\)

Thaler & Wang, JHEP (08);
Almeida, SL, Perez, Sterman, Sung & Virzi, PRD (09).
Planar flow

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Planar flow

![Planar Flow Plot]

**Planar Flow Distribution at the Tevatron** assumes the range of $f_{bb} [g_{bb} \text{yeV}]$. An excess of $f_{bb} [g_{bb} \text{yeV}]$ is observed. If one were to interpret this excess as an actual moment of inertia, it can be expanded as a sum of three basis matrices.

The solid red line corresponds to a straightforward model, and the black circles with error bars describe the CDF data. The prediction is observed where $E \geq 90 \text{ GeV}$ and a cone size of $w_{tv}$ and a cone size of $m_{t\text{kin}}$.

Acknowledgments:

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**Mathematical Expressions**

Planar flow $\tau$ can be defined as:

$$\tau = \frac{\vec{p}_T \cdot \vec{E}_{\text{jet}}}{\sqrt{\vec{p}_T^2 + \vec{E}_{\text{jet}}^2}}$$

where $\vec{p}_T$ is the component of its transverse momentum for a given jet $a_j$. We shall return to this point in the following section.

Given a two-cone dimensional Lorentz group becomes clear in the following section.

For a given jet $a_j$, we find a prediction is observed. The corresponding search relies on a constraint $\delta \tau$ that $\sim \sqrt{\frac{1}{2} I_{ij}}$.

**Arbitrary Units / bin of 0.1**

$P_{\text{CDF data}}$, $QCD+\text{top jet (SM)}$, $QCD+\text{top jet (FT)}$.

**General**

The relative correction to angularity is small for a given jet $a_j$. While the precision observables are the corresponding Pauli matrices $\lambda_i$, $\eta$, $\mu$ of $f_{bb} [g_{bb} \text{yeV}]$ below the observed excess of $e_{bb} [g_{bb} \text{yeV}]$.

**Conclusion**

We find that our model yields a contribution to the distribution of boosted massive jets. This analysis looks for an excess of $e_{bb} [g_{bb} \text{yeV}]$ for a given jet $a_j$.
Background rejection, basic approaches

Filtering, pruning, trimming. (simple to implement, very successful)

Seymour (93); Butterworth, Cox, Forshaw (02); Butterworth, Davison, Rubin & Salam (08); Krohn, Thaler & Wang (10); Ellis, Vermilion & Walsh (09). Soper & Spannowsky (10,11)

Moments. (easy to get LO PQCD, weak jet finder dependence, etc)

Recently: Almeida, SL, Perez, Sterman, Sung & Virzi; Thaler & Wang (08), etc.

Template Overlap. (easy to get LO PQCD, weak jet finder dep’ & beyond, fits the spiky nature of signals)

Almeida, SL, Perez, Sterman & Sung (10);
Almeida, Erdogan, Juknevich, SL, Perez, Sterman (11);
Template Overlap Method

♦ Template overlaps: functional measures that quantify how well the energy flow of a physical jet matches the flow of a boosted partonic decay

\[ |j> = \text{set of particles or calorimeter towers that make up a jet. e.g.} \]

\[ |j> = |t>,|g>, \text{etc, where:} \]

\[ |t> = \text{top distribution} \]
\[ |g> = \text{massless QCD distribution} \]

We need a probe distribution, \(|f>\), such that “template”

\[ R = \left( \frac{\langle f|t>}{\langle f|g>} \right) \] is maximized.

Lunch table discussion with Juan Maldacena

Sunday, March 11, 2012
Example: The Golden Triangle

\[ E(\hat{p}_x, \hat{p}_y) \]

Plane \( \perp \) to Jet Axis:

Our templates will be sets of partonic momenta \( t = \{p_1, p_2, ..., p_n\} \)

\[ \sum_{i=1}^{n} p_i = P, \quad P^2 = M^2 \]
Template Overlap Method

♦ General overlap functional:

\[ Ov(j, f) = \langle j | f \rangle = \mathcal{F} \left[ \frac{dE(j)}{d\Omega}, \frac{dE(f)}{d\Omega} \right] \]

♦ Define “template overlap” as the maximum functional overlap of j to a state f[j]:

\[ Ov(j, f) = \max \{ f \} \mathcal{F}(j, f) \]

♦ Can match arbitrary final states j to partonic partners f[j] at any given order.
Two-particle Templates and Higgs Decay

♦ Construct template: two particle phase space for Higgs decay (easy)  
\[ |f\rangle = |h^{(LO)}\rangle = |p_1, p_2\rangle \]

♦ Higgs: at fixed \( z = m_J/P_0 \ll 1 \), \( \Theta_s \) distribution is peaked around \( \Theta_s \) in its minimum value  
\[ \Rightarrow \text{decays “democratic” (sharing energy evenly)} \]
\[ \frac{dJ^h}{d\Theta_s} \propto \frac{1}{\Theta_s^3} \]

♦ Lowest-order QCD events is also peaked, but much less so  
\[ \frac{dJ^{QCD}}{d\Theta_s} \propto \frac{1}{\Theta_s} \]
Two-particle Templates and Higgs Decay

♦ jet mass window $110 \text{ GeV} < m_J < 130 \text{ GeV}$, cone size $R = 0.4$ ($D = 0.4$ for anti-\(kT\) jet), jet energy $950 \text{ GeV} < E_j < 1050 \text{ GeV}$.

♦ Template Overlap with data discretization

$$Ov(j, f) = \max_{\tau_n^{(R)}} \exp \left[-\sum_{a=1}^{2} \frac{1}{2\sigma_a^2} \left( \sum_{k=i_a-1}^{i_a+1} \sum_{l=j_a-1}^{j_a+1} E(k, l) - E(i_a, j_a)^{(f)} \right)^2 \right]$$
Two-particle Templates and Higgs Decay

♦ Template Overlap with data discretization

\[
Ov(j, f) = \max_{\tau_n^{(R)}} \exp \left[ - \sum_{a=1}^{2} \frac{1}{2\sigma_a^2} \left( \sum_{k=i_a-1}^{i_a+1} \sum_{l=j_a-1}^{j_a+1} E(k, l) - E(i_a, j_a)^{(f)} \right)^2 \right]
\]
Two-particle Templates and Higgs Decay

\[ \frac{d\sigma}{\sigma} = \frac{Ov(M_J=m_H, P_0=1000 \text{ GeV})}{Ov} \]

- Higgs
- QCD jet

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Two-particle Templates and Higgs Decay

![Graph showing the relationship between h-Jet Efficiency and Fake Rate for different values of $O_\nu$.](image)

- $O_\nu > 0.0$
- $O_\nu > 0.6$
- $O_\nu > 0.7$
- $O_\nu > 0.8$
The templates can be systematically improved by including the effects of gluon emissions, which contain color flow information.
Two-particle Templates and Higgs Decay

♦ The templates can be systematically improved by including the effects of gluon emissions, which contain color flow information

♦ The effects of higher-order effects can be partly captured by using Planar flow

(expect soft radiation from the boosted color singlet Higgs to be concentrated between the b and b̅ decay products, in contrast to QCD light jet)
Two-particle Templates and Higgs Decay

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Two-particle Templates and Higgs Decay

♦ Combined with angularity or $\Theta_s$ : can improved rejection power ($\Theta_s$ and angularities are related)
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Compared to angularities, $\Theta_s$ is a parameter for two-body template states, which already provides useful information on physical states, as well as a clear picture of their energy flow.
Two-particle Templates and Higgs Decay

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Two-particle Templates and Higgs Decay

$\theta_s \leq 0.2$

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$\rho_{\nu} > 0.0$

$\rho_{\nu} > 0.6$

$\rho_{\nu} > 0.7$
Two-particle Templates and Higgs Decay

Rejection Power: combining jet mass cut (fake rate: 4.5%, efficiency: 79%) efficiency of 9.3%, a fake rate of 0.084%

(rejection power 1: 110)

after mass cut, without optimization

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**NLO** => **Soft radiation (+color flow???)**

I. Sung (09)
J. Gallicchio and M. Schwartz (10),
K. Black, J. Gallicchio, J. Huith, M. Kagan, M. Schwartz, B. Tweedie (10)

**NLO template:**

\[ x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i \cdot q}{s} \]
\[ 0 < x_i < 1. \]
NLO Templates and Higgs Decay

L. Almeida, O. Erdogan, J. Juknevich, SL, G. Perez, & G. Sterman (11)

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♦ NLO template

Sunday, March 11, 2012
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**NLO templates**

Since Higgs is a color singlet we can provide a precise NLO calculation in the rest frame.
Higgs NLO template, cont’

\[ |f\rangle = |h\rangle^{(NLO)} = |p_1, p_2, p_3\rangle \]

- Three Euler angles \(\psi, \theta, \phi\)
- Two energy fractions \(x_1, x_2\)

\[ p^\mu_a(x_1, x_2, \psi, \theta, \phi) = L_z(\gamma) R_z(\psi) R_x(\theta) R_z(\phi) p^\mu_a|p_j^z=0(x_1, x_2) \]
Showering smears the Higgs distribution, although Higgs events are concentrated at large peak overlaps than QCD.
Density plots of 2-body overlap vs. 3-body overlap for boosted Higgs and QCD jets with $R = 0.7$ and same number of events (20000).
Max template $O_v =>$ access to partonic information.
Can do better than that ... 

Max template $O_{\nu}$ $\Rightarrow$ access to partonic information.

However, templates are purely 3-prong kinematics $\Rightarrow$ If S & B were genuinely only 3body then both would always yield large overlaps $\Rightarrow$ no separation. 😞

$h\rightarrow g$
Distributions of some of 5 variables differ!

Can use angular ordering:

\[ V_{\text{dip}} \approx \frac{R}{r^2} \]

\[ r_\theta = \min\{\theta_{13}/\theta_{12}, \theta_{23}/\theta_{12}\} \]

\[ \bar{\theta} = \sum_i \sin \theta_{iJ} \]

\[ 1 - \cos \theta_{iJ} = \frac{z x_i m_J}{2 E_i} \]
Distributions of some of 5 variables differ!

♦ Can use angular ordering:

![Diagram showing angular ordering](image1)

♦ Can use monopole vs. dipole (soft gluon):

![Diagram showing monopole and dipole](image2)

\[ V_{\text{dip}} \approx \frac{R}{r^2} \]

\[ r_\theta = \min\{\theta_{13}/\theta_{12}, \theta_{23}/\theta_{12}\} \]
Distributions of some of 5 variables differ!

Can use angular ordering:

**Color Flow**: Radiation from a colour dipole prefers to radiate among the color connected partners.

On the other hand, radiation from a coloured object will be color connected to other parts of the event leading to additional radiation in-between jets or a jet and beam.

\[ V_{\text{dip}} \approx R/r^2 \]
Fake vs. efficiency 2-body vs. 3-body

Varying 2-body $\text{max}(O_v)$ value (including mass cut)

![Graph showing efficiency and fake rates for different conditions.]

<table>
<thead>
<tr>
<th>MC</th>
<th>Jet mass cut only</th>
<th>Mass cut + $O_v$ + $\bar{\eta}$ + $P_f$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Higgs-jet efficiency [%]</td>
<td>fake rate [%]</td>
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</table>

Table 2: Efficiencies and fake rates for jets with $R = 0.7$ (using anti-$k_T$: $D = 0.7$), $950 \text{ GeV} \leq P_0 \leq 1050 \text{ GeV}$, $110 \text{ GeV} \leq m_J \leq 130 \text{ GeV}$ and $m_H = 120 \text{ GeV}$. 

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Fake vs. efficiency 2-body vs. 3-body

Varying 2-body \( \max(Ov) \) value (including mass cut)

**Naive rejection power** (eff'/fake rate) -

- Pythia8 & MG/ME: better than 1 in 200
  - without optimization and without b-tagging!

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Work in progress (PT >200 GeV with pileup)

Mihailo Backovic, Jose Juknevich, Gilad Perez, Jan Winter, Weizmann Group

Rejection Rate

For 10% efficiency rejection Factor of ~ 5.

Total rejection factor:

- mass cut
- $b$-tagging
- template+pf

= 3000

Need double $b$-tag
Work in progress (PT >200 GeV with pileup)

Mihailo Backovic, Jose Juknevich, Gilad Perez, Jan Winter, Weizmann Group

Rejection Rate

For 10% efficiency rejection
Factor of ~ 5.

Total rejection factor:

\[ \text{mass cut} \times 15 \times \text{b-tagging} \times 40 \times \text{template+pf} \times 5 = 3000 \]
Summary

♦ Template overlaps: new class of infrared safe jet observables, based on functional comparison of the energy flow in data with the flow in selected templates of partonic states.

♦ Allows for systematic improvement  
  e.g. by incorporating the effect of gluon emission,  
  by weighting according to the lowest order matrix elements.

♦ Can use our knowledge of the signal to design a custom analysis for each resonance.

♦ Template Overlap Method works well for ~TeV Higgs jet

♦ Work in progress (P_T >200 GeV with pileup)
An Emergency Tire beyond the SM

Thank You

Sunday, March 11, 2012
Backups
NLO Planar Flow for Higgs Decay

♦ NLO Planar flow for Higgs:

\[
\frac{1}{\sigma_0} \frac{d\sigma^{NLO}}{dPf} = \frac{\alpha_s C_F}{8\pi^2} \int dx_1 dx_2 d(\cos \theta) d\phi \frac{(1 - x_1 - x_2)^2 + 1}{(1 - x_1)(1 - x_2)} \times \delta \left( Pf - \frac{E_j^3}{E_1 E_2 E_3} S \cos^2 \theta \right)
\]

\( m_H/E_J \ll 1 \)

♦ \( \theta \approx 0 \) approximaition:

\[
\frac{1}{\sigma_0} \frac{d\sigma^{(3)}}{dPf} = \frac{\alpha_s C_F}{2\pi} \int_{x^+_2}^{x^+_2} dx \frac{8x \left[ Pf((x - 3)x^2 + 4x) + 8x((x - 1)x + 1) - 8 \right]}{Pf(Pf + 8)^2 \left( \frac{8}{Pf + 8} - x \right) \sqrt{\left( \frac{8}{Pf + 8} - x \right)(x^+_2 - x)(x^-_2 - x)x}}
\]

\[ x^\pm_2 = \frac{2(Pf + 2) - 4\sqrt{1 - Pf}}{Pf + 8} \]
NLO Planar Flow for Higgs Decay

\[ \frac{1}{\sigma} \frac{d \sigma}{d P_f} \]

Full Numerical Distribution

\[ \theta = 0 \text{ Approximation} \]

\[ P_f(M_J = M_{Higgs}, P_0 = 1000 \text{ GeV}) \]
NLO Planar Flow for Higgs Decay

The $\theta \approx 0$ approximation needs resummation. But, the tail region is already well described.
2-body jet's kinematics, $Z/W/h$

$$\tilde{\tau}_a(R, m_J) = \frac{1}{m_J} \sum_{i \in \text{jet}} \omega_i \sin^a \left( \frac{\pi \theta_i}{2R} \right) \left[ 1 - \cos \left( \frac{\pi \theta_i}{2R} \right) \right]^{1-a} \sim \frac{1}{m_J} \frac{1}{2^{1-a}} \sum_{i \in \text{jet}} \omega_i \left( \frac{\pi \theta_i}{2R} \right)^{2-a}$$

$a \leq 2$ for IR safety

Angularities distinguish between Higgs and QCD jets:

$$\frac{dJ^h}{d\tilde{\tau}_a} \propto \frac{1}{|a| (\tilde{\tau}_a)^{1-\frac{2}{a}}}$$

V.S.

$$\frac{dJ^{QCD}}{d\tilde{\tau}_a} \propto \frac{1}{|a| \tilde{\tau}_a}$$
Constructing a functional

- A natural measure: weighted difference of their energy flows integrated over a region (simple example: Gaussian)

\[ O_{ij}(F)(j, f) = \max_{\tau_n(R)} \exp \left[ -\frac{1}{2\sigma^2} \left( \int d\Omega \left[ \frac{dE(j)}{d\Omega} - \frac{dE(f)}{d\Omega} \right] F(\Omega, f) \right)^2 \right] \]

n-particle phase space:

IR safety: \( F \) should be a sufficiently smooth function of the angles for any template state \( f \):

-we may choose \( F \) to be a normalized step function around the directions of the template momenta \( p_i \)

- For a given template, with direction of particle \( a \), \( n_a \) and its energy \( E_a^{(f)} \):

\[ O_{ij}(j, p_1 \ldots p_n) = \max_{\tau_n(R)} \exp \left[ -\sum_{a=1}^{n} \frac{1}{2\sigma^2_a} \left( \int d^2\hat{n} \frac{dE(j)}{d^2\hat{n}} \theta(\hat{n}, \hat{n}_a^{(f)}) - E_a^{(f)} \right)^2 \right] \]
Scatter plots of planar flow $P_f$ vs. template overlap $Ov$ for Higgs jets and QCD jets from PYTHIA, for $R = 0.7$, $950 \text{ GeV} \leq P_0 \leq 1050 \text{ GeV}$, $110 \text{ GeV} \leq m_J \leq 130 \text{ GeV}$ using three-body templates.