Drell-Yan lepton pair production: transverse-momentum resummation with leptonic variables dependence

Giancarlo Ferrera
giancarlo.ferrera@mi.infn.it

Università di Milano
Outline

1. Drell-Yan transverse-momentum ($q_T$) distribution
2. DY $q_T$ resummation at full NNLL+NLO
3. DY $q_T$ resummation with leptonic variables dependence
4. Conclusions
Motivations

The Drell-Yan process [Drell,Yan(’70)] is a benchmark process in hadron collider physics. Its study is well motivated:

- Large production rates and clean experimental signatures.
- Constraint for fits of PDFs.
- $q_T$ spectrum: important for $M_W$ measurement and Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.

The above reasons and precise experimental data demands for accurate theoretical predictions $\Rightarrow$ computation of higher-order QCD corrections.
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The above reasons and precise experimental data demands for accurate theoretical predictions $\Rightarrow$ computation of higher-order QCD corrections.
The Drell-Yan $q_T$ distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M) + X \rightarrow \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_\ell$

QCD factorization:

$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a, b} \int_0^1 dx_1 \int_0^1 dx_2 \ f_a/h_1(x_1, \mu_F^2) \ f_b/h_2(x_2, \mu_F^2) \ \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R, \mu_F^2).$$

The standard fixed-order QCD perturbative expansions gives:

$$\int_0^{Q_T^2} dq_T^2 \ \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim 1 + \alpha_S \left[ c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$

$$+ \alpha_S^2 \left[ c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + O(\alpha_S^3)$$

Fixed order calculation reliable only for $q_T \sim M_V$

For $q_T \rightarrow 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections.
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For \( q_T \to 0, \alpha_S^n \log^m(M^2/q_T^2) \gg 1: \text{need for resummation of logarithmic corrections.} \)
State of the art: transverse-momentum \((q_T)\) resummation

- The method to perform the resummation of the large logarithms of \(q_T\) is known
  
  [Dokshitzer, Diakonov, Troian ('78)], [Parisi, Petronzio ('79)],
  [Kodaira, Trentadue ('82)], [Altarelli et al. ('84)],
  [Collins, Soper, Sterman ('85)], [Catani, de Florian, Grazzini ('01)]
  [Catani, Grazzini ('10)]

- Various phenomenological studies of the vector boson transverse momentum distribution exist
  
  [Balasz, Qiu, Yuan ('95)], [ResBos: Balasz, Yuan, Nadolsky et al.], [Ellis et al. ('97)], [Kulesza et al. ('02)]

- Recently various results for transverse momentum resummation in the framework of Effective Theories appeared
  
  [Gao, Li, Liu ('05)], [Idilbi, Ji, Yuan ('05)], [Mantry, Petriello ('10)],
  [Becher, Neubert ('10)], [García, Idibli, Scimemi ('11)].
Transverse momentum resummation in pQCD

\[ \frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{\text{res}}}{dq_T^2} + \frac{d\hat{\sigma}^{\text{fin}}}{dq_T^2}; \]

\[ \int_0^{Q_T^2} dq_T^2 \left[ \frac{d\hat{\sigma}^{\text{res}}}{dq_T^2} \right]_{Q_T \to 0} f.o. \]

\[ \int_0^{Q_T^2} dq_T^2 \left[ \frac{d\hat{\sigma}^{\text{fin}}}{dq_T^2} \right]_{Q_T \to 0} f.o. = 0 \]

Resummation holds in impact parameter space: \( q_T \ll M \Leftrightarrow M_b \gg 1, \log M/q_T \gg 1 \Leftrightarrow \log M_b \gg 1 \)

In the Mellin moments (\( f_N \equiv \int_0^1 f(x)x^{N-1}dx \)) space we have the exponentiated form:

\[ \mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_S) \times \exp \left\{ G_N(\alpha_S, L) \right\} \]

where

\[ g_N(\alpha_S, L) = g^{(1)}_N(\alpha_S L) + g^{(2)}_N(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}_N(\alpha_S L) + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M) \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}_N^{(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}_N^{(2)} + \cdots \right] \]

LL (\( \sim \alpha_S^n L^{n+1} \)): \( g^{(1)}_N, (\sigma^{(0)}) \); NLL (\( \sim \alpha_S^n L^n \)): \( g^{(2)}_N, \mathcal{H}_N^{(1)} \); NNLL (\( \sim \alpha_S^n L^{n-1} \)): \( g^{(3)}_N, \mathcal{H}_N^{(2)} \);

NLL and NNLL respectively matched with “finite” part at: \( \alpha_S \) (LO) and \( \alpha_S^2 \) (NLO)

Perturbative unitarity constrain and resummation scale \( Q(\sim M) \):

\[ \ln(M^2 b^2) \rightarrow \tilde{L} \equiv \ln(Q^2 b^2 + 1) \Rightarrow \exp \left\{ G_N(\alpha_S, \tilde{L}) \right\} \bigg|_{b=0} = 1 \Rightarrow \int_0^{\tilde{L}} dq_T^2 \left( \frac{d\hat{\sigma}}{dq_T^2} \right)_{(N)NLL+(N)LO} = \hat{\sigma}^{(\text{tot})}_{(N)NLO}; \]
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Giancarlo Ferrera – Università di Milano
Moriond QCD 2012 – 15/3/12
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\[ \frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res})}}{dq_T^2} + \frac{d\hat{\sigma}^{(\text{fin})}}{dq_T^2}; \]

\[ \int_0^{Q_T^2} dq_T^2 \left[ \frac{d\hat{\sigma}^{(\text{res})}}{dq_T^2} \right]_{\text{f.o.}} Q_T \to 0 \sim 1 + \sum_n \sum_{m=1}^{2n} c_{nm} \alpha_S^n \log^m \frac{M^2}{Q_T^2}, \]

\[ \int_0^{Q_T^2} dq_T^2 \left[ \frac{d\hat{\sigma}^{(\text{fin})}}{dq_T^2} \right]_{\text{f.o.}} Q_T \to 0 = 0. \]

Resummation holds in impact parameter space: \( q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1 \)

\[ \frac{d\hat{\sigma}^{(\text{res})}}{dq_T^2} = \frac{M^2}{\hat{s}} \int_0^{\infty} db \frac{b}{2} J_0(bq_T) W(b, M), \]

In the Mellin moments \((f_N \equiv \int_0^1 f(x)x^{N-1}dx)\) space we have the exponentiated form:

\[ W_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp \left\{ G_N(\alpha_S, L) \right\} \]

where \( L \equiv \log(M^2 b^2) \)

\[ G_N(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M) \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}_N^{(1)} + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}_N^{(2)} + \cdots \right] \]

LL (\( \sim \alpha_S^n L^{n+1} \)): \( g^{(1)}, (\sigma^{(0)}) \); NLL (\( \sim \alpha_S^n L^n \)): \( g_N^{(2)}, \mathcal{H}_N^{(1)} \); NNLL (\( \sim \alpha_S^n L^{n-1} \)): \( g_N^{(3)}, \mathcal{H}_N^{(2)} \);

NLL and NNLL respectively matched with “finite” part at: \( \alpha_S \) (LO) and \( \alpha_S^2 \) (NLO)

Perturbative unitarity constrain and resummation scale \( Q(\sim M) \):

\[ \ln(M^2 b^2) \to \tilde{L} \equiv \ln(Q^2 b^2 + 1) \Rightarrow \exp \left\{ G_N(\alpha_S, \tilde{L}) \right\} \bigg|_{b=0} = 1 \Rightarrow \int_0^{\infty} dq_T^2 \left( \frac{d\hat{\sigma}}{dq_T^2} \right)_{(N)\text{NLL}+(N)\text{LO}} = \hat{\sigma}^{(\text{tot})}_{(N)\text{NLO}}; \]
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DYqT: $q_T$-resummation at NNLL+NLO:

Bozzi, Catani, de Florian, G.F., Grazzini arXiv:1007.2351

- We have applied for Drell-Yan transverse-momentum distribution the resummation formalism developed by [Catani, de Florian, Grazzini(’01)] already applied for the case of Higgs boson production [Bozzi, Catani, de Florian, Grazzini(’03, ’06, ’08)].

- We have performed the resummation up to NNLL+NLO. It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders;
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  - NNLO result (i.e. $O(\alpha_s^2)$) for the total cross section (upon integration over $q_T$).

- NLO+PS generators (MC@NLO/POWHEG) reach LL (and part of the NLL)+NLO accuracy.
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Resummed results: \( q_T \) spectrum of \( Z \) boson at the Tevatron

- Uncertainty bands obtained varying \( \mu_R, \mu_F, Q \) independently:
  \[ \frac{1}{2} \leq \{ \frac{\mu_F}{m_Z}, \frac{\mu_R}{m_Z}, 2Q/m_Z, \frac{\mu_F}{\mu_R}, Q/\mu_R \} \leq 2 \]
  to avoid large logarithmic contributions
  \( \sim \ln\left(\frac{\mu_F^2}{\mu_R^2}\right), \ln\left(\frac{Q^2}{\mu_R^2}\right) \)
  in the evolution of the parton densities and in the resummed form factor.

- Significant reduction of scale dependence from NLL+LO to NNLL+NLO for all \( q_T \).
- Good convergence of resummed results: NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).

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- At large values of $q_T$, the NLO and NNLL+NLO bands overlap.

- At intermediate values of transverse momenta the scale variation bands do not overlap.

- The resummation improves the agreement of the NLO results with the data.

In the small-$q_T$ region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL+NLO band.

D0 data for the $Z$ $q_T$ spectrum: Fractional difference with respect to the reference result: NNLL+NLO, $\mu_R = \mu_F = 2Q = m_Z$. 

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![Graph showing $q_T$ spectrum comparison](image)
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Non-perturbative effects: $q_T$ spectrum of $Z$ boson at the Tevatron

- Up to now result in a complete perturbative framework.
- Non-perturbative effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP}b^2\}$:
  \[ \exp\{G_N(\alpha_S, \tilde{L})\} \rightarrow \exp\{G_N(\alpha_S, \tilde{L})\} S_{NP} \]

  - $g_{NP} = 0.8$ GeV$^2$ [Kulesza et al. ('02)]

- With NP effects the $q_T$ spectrum is harder.

Quantitative impact of such NP effects is comparable with perturbative uncertainties.
Non perturbative effects: $q_T$ spectrum of $Z$ boson at the Tevatron

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NEW: $q_T$-resummation with leptonic variables dependence

- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

We have included the full dependence on the leptonic variables: possible to apply cuts on vector boson and decay products.

To construct the “finite” part we rely on the fully-differential NNLO result from the code DYNNLO [Catani, Cieri, de Florian, Ferrera, Grazzini (’09)].

Calculation implemented in a numerical program which includes spin correlations, $\gamma^*Z$ interference, finite-width effects and compute distributions in form of bin histograms: analogously to the HRes code (see M. Grazzini talk).
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CMS data for the $Z$ $q_T$ spectrum compared with NNLL+NLO result.
Scale variation:
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ATLAS data for the $Z$ $q_T$ spectrum compared with NNLL+NLO result.
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ATLAS data for the $W$ $q_T$ spectrum compared with NNLL+NLO result.

Lepton $p_T$ spectrum from $W^+$ decay.
NNLL+NLO result compared with the NNLO result.
Important spectrum for the measurement of $M_W$ at the LHC.
Conclusions

- **NNLL+NLO DY $q_T$-resummation** [Bozzi, Catani, de Florian, G.F., Grazzini [arXiv:1007.2351]].
  Reduction of scale uncertainties from NLL+LO to NNLL+NLO accuracy. The NNLL+NLO results consistent with the experimental data in a wide region of $q_T$.

- **NEW**: added full kinematical dependence on the vector boson and on the final state leptons.

- Preliminary comparison with LHC data (implementing experimental cuts): good agreement between data and NNLL+NLO results without any model for Non Perturbative effects.

- Perspectives: more accurate comparisons and public version of the codes.
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Back up slides
The $q_T$ resummation formalism

The main distinctive features of the formalism we use: [Catani, de Florian, Grazzini ('01)], [Bozzi, Catani, de Florian, Grazzini ('03, '06, '08)]:

- Resummation performed at partonic cross section level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, study of $\mu_R$ and $\mu_F$ dependence as in fixed-order calculations.

- Possible to make prediction without introducing non perturbative effects: Landau singularity of the QCD coupling regularized using a Minimal Prescription [Laenen, Sterman, Vogelsang ('00)], [Catani et al. ('96)].

- Resummed effects exponentiated in a universal Sudakov form factor $G_N(\alpha_S, L)$; process-dependence factorized in the hard scattering coefficient $H_N(\alpha_S)$.

- Perturbative unitarity constrain and resummation scale $Q$:

  $$\ln \left( \frac{M^2 b^2}{b_0^2} \right) \to \bar{L} \equiv \ln \left( \frac{Q^2 b^2}{b_0^2} + 1 \right) \Rightarrow \exp \{ G_N(\alpha_S, \bar{L}) \} \bigg|_{b=0} = 1 \Rightarrow \int_0^{\infty} dq_T^2 \left( \frac{d\hat{\sigma}}{dq_T^2} \right)_{NLL+LO} = \delta^{(tot)}_{NLO};$$

- avoids unjustified higher-order contributions in the small-$b$ region: no need for unphysical switching from resummed to fixed-order results.
- allows to recover exactly the total cross-section upon integration on $q_T$
- variations of the resummation scale $Q \sim M$ allows to estimate the uncertainty from higher orders uncalculated logarithmic corrections.
Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron

- **Left side:** NLL+LO result compared with fixed LO result. Resummation cure the fixed order divergence at $q_T \to 0$.
- **Right side:** NNLL+NLO result compared with fixed NLO result.
- The $q_T$ spectrum is slightly harder at NNLL+NLO accuracy than at NLL+LO accuracy.
- Integral of the NLL+LO (NNLL+NLO) curve reproduce the total NLO (NNLO) cross section to better 1% (check of the code).
Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron

- Our calculation implements $\gamma^* Z$ interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- Uncertainty bands obtained by performing renormalization and factorization scale variations: $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$, with $Q = m_Z/2$.
  
  In the region $q_T \lesssim 30$ the NNLL+NLO and NLL+LO bands overlap (contrary to the fixed-order case).
- We observe a significant reduction of scale dependence going from NLL+LO to NNLL+NLO accuracy.
- Suppression of NLL+LO result in the large-$q_T$ region ($q_T \gtrsim 60$ GeV) (strong dependence from the resummation scale, see next plot).

NLL+LO: pdf=MSTW08 NLO, 2-loops $\alpha_S$

NNLL+NLO: pdf=MSTW08 NNLO, 3-loops $\alpha_S$
Resummed results: $q_T$ spectrum of $Z$ boson at the Tevatron

- Uncertainty bands obtained by performing resummation scale variations (estimate of higher-order logarithmic contributions): $m_Z/4 \leq Q \leq m_Z$ with $\mu_F = \mu_R = m_Z$.
- The resummation scale dependence at NNLL+NLO (NLL+LO) is about ±5% (±12%) around the peak and ±5% (±16%) in the $q_T \gtrsim 20$ GeV region and it is larger than the renormalization and factorization scale dependence.
- Going from the NLL+LO to the NNLL+NLO calculation the resummation scale dependence is reduced by roughly a factor 2 in the wide region $5$ GeV $\lesssim q_T \lesssim 50$ GeV.
Fixed order results: $q_T$ spectrum of $Z$ boson at the Tevatron $\sqrt{s} = 1.8$ TeV

- CDF data: $66$ GeV $< M^2 < 116$ GeV, 
  $\sigma_{tot} = 248 \pm 11$ pb [CDF Coll. ('00)]
- D0 data: $75$ GeV $< M^2 < 105$ GeV, 
  $\sigma_{tot} = 221 \pm 11$ pb [D0 Coll. ('00)]

- Factorization and renormalization scale variations:
  $\mu_F = \mu_R = m_Z$, 
  $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$, 
  $q_T \sim m_Z$: LO $\pm 25\%$, NLO $\pm 8\%$ 
  $q_T \sim 20$ GeV: LO $\pm 20\%$, NLO $\pm 7\%$

- Good agreement between NLO results and data up to $q_T \sim 20$ GeV.

- In the small $q_T$ region ($q_T \lesssim 20$ GeV) LO and NLO result diverges to $+\infty$ and $-\infty$ (accidental partial agreement at $q_T \sim 5 - 7$ GeV): need for resummation.

LO and NLO scale variations bands overlap only for $q_T > 70$ GeV
Fixed order results: $q_T$ spectrum of $Z$ boson at the Tevatron

- D0 data [D0 Coll. (’08,’10)].
- Scale variations as before: $\mu_F = \mu_R = m_Z$, $1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R\} \leq 2$.
- Experimental errors very small but bins are larger.
- Qualitatively same situation of Tevatron Run I data.
- LO and NLO scale variations bands overlap only for $q_T > 60$ GeV.
- Good agreement between NLO results and data up to $q_T \sim 20$ GeV.

In the small $q_T$ region ($q_T \lesssim 20$ GeV) effects of soft-gluon resummation are essential.

At Tevatron 90% of the $W^\pm$ and $Z^0$ are produced with $q_T \lesssim 20$ GeV.