Forward-backward asymmetry of B decays in SM and new physics models
Cai-Dian LÜ (吕才典)
IHEP, Beijing
based on work with Run-Hui Li, Wei Wang, Yu-Ming Wang
PRD79,094024 (2009), PRD83,034034 (2011)
Outline

• Introduction
• \( B \rightarrow K_1(K^*_0, K_2, K_3, K_4) \) \( l^+ l^- \) decays in SM
  Angular distributions
    Branching ratios, polarizations, FB asymmetry
• New physics contributions
• Summary
Introduction

- Unlike $b \rightarrow s \gamma$ or $B \rightarrow K^* \gamma$, which have only limited physical observables
- $b \rightarrow s l^+l^-$, and especially $B \rightarrow K^* l^+l^-$, with a number of observables accessible (exp. also easier), provides a wealth of information of weak interactions, ranging from the forward-backward asymmetries, isospin asymmetries, and polarization fractions
Flavor changing Electroweak penguin operators

\[ O_7 = \frac{e m_b}{8 \pi^2} \bar{s} \sigma^{\mu \nu} (1 + \gamma_5) b F_{\mu \nu} + \frac{e m_s}{8 \pi^2} \bar{s} \sigma^{\mu \nu} (1 - \gamma_5) b F_{\mu \nu} \]

\[ O_9 = \frac{\alpha_{em}}{2 \pi} (\bar{l} \gamma_\mu l)(\bar{s} \gamma^\mu (1 - \gamma_5) b), \]

\[ O_{10} = \frac{\alpha_{em}}{2 \pi} (\bar{l} \gamma_\mu \gamma_5 l)(\bar{s} \gamma^\mu (1 - \gamma_5) b) \]

No tree level flavor changing neutral current in SM
With QCD corrections from the four quark operators

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i O_i \]

\[ O_1 = \bar{u} \gamma^\mu L_u \cdot \bar{s} \gamma_{\mu} L_b \]
\[ O_2 = \bar{s} \gamma^\mu L_u \cdot \bar{u} \gamma_{\mu} L_b \]
\[ O_3 = \bar{s} \gamma^\mu L_b \cdot \sum_q \bar{q} \gamma_{\mu} L_q \]
\[ O_4 = \bar{s} \gamma^\mu L_b \cdot \sum_q \bar{q} \gamma_{\mu} L_q \]
\[ O_5 = \bar{s} \gamma^\mu L_b \cdot \sum_q \bar{q} \gamma_{\mu} R_q \]
\[ O_6 = \bar{s} \gamma^\mu L_b \cdot \sum_q \bar{q} \gamma_{\mu} R_q \]
Properties of resonances $K^*_j$

In addition to the lowest $K^*$,

<table>
<thead>
<tr>
<th>$K^*_j$</th>
<th>$J^P$</th>
<th>$n^{2S+1}L_J$</th>
<th>$m$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$\mathcal{B}(K^*_j \to K\pi)$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^*(1410)$</td>
<td>$1^-$</td>
<td>$2^3S_1?$</td>
<td>1414 ± 15</td>
<td>232 ± 21</td>
<td>6.6 ± 1.3</td>
</tr>
<tr>
<td>$K_0^*(1430)$</td>
<td>$0^+$</td>
<td>$1^3P_0, 2^3P_0?$</td>
<td>1425 ± 50</td>
<td>270 ± 80</td>
<td>93 ± 10</td>
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<tr>
<td>$K_2^*(1430)$</td>
<td>$2^+$</td>
<td>$3^P_2$</td>
<td>1432.4 ± 1.3</td>
<td>109 ± 5</td>
<td>49.9 ± 1.2</td>
</tr>
<tr>
<td>$K^*(1680)$</td>
<td>$1^-$</td>
<td>$1^3D_1$</td>
<td>1717 ± 27</td>
<td>322 ± 110</td>
<td>38.7 ± 2.5</td>
</tr>
<tr>
<td>$K_3^*(1780)$</td>
<td>$3^-$</td>
<td>$1^3D_3$</td>
<td>1776 ± 7</td>
<td>159 ± 21</td>
<td>18.8 ± 1.0</td>
</tr>
<tr>
<td>$K_4^*(2045)$</td>
<td>$4^+$</td>
<td>$1^3F_4$</td>
<td>2045 ± 9</td>
<td>198 ± 30</td>
<td>9.9 ± 1.2</td>
</tr>
</tbody>
</table>
About $K_2^*(1430)$ and $f_2'(1525)$

\[ \Gamma = 100 \text{MeV}, \quad 73 \text{ MeV} \]

\[ \rightarrow K \pi, \quad \rightarrow K K \]

<table>
<thead>
<tr>
<th>$l$</th>
<th>$s$</th>
<th>$J$</th>
<th>$^{2s+1}L_J$</th>
<th>$J^{PC}$</th>
<th>Meson</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$^1S_0$</td>
<td>0--</td>
<td>Pseudoscalar ($P$)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>$^3S_1$</td>
<td>1--</td>
<td>Vector ($V$)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$^1P_1$</td>
<td>1++</td>
<td>Axial-vector ($A(^1P_1)$)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>$^3P_1$</td>
<td>1++</td>
<td>$K_0^*(1430)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>$^3P_2$</td>
<td>2++</td>
<td>Tensor ($T$)</td>
</tr>
</tbody>
</table>

$K_1(1270), K_1(1400)$
\[ B \rightarrow K_2^* l^+ l^- (B_s \rightarrow f'_2 l^+ l^-) \]

- 5 polarization states: \( Jz = -2, -1, 0, 1, 2 \)
- 3 contribute to \( \Bar{B}^0 \rightarrow K_2^* l^+ l^- \), \( Jz = -1, 0, 1 \), because of angular momentum conservation

Similar to \( K^* \) mesons. \( \Bar{B}^0 \rightarrow K_2^* l^+ l^- \) formulism can be got by some substitution in \( \Bar{B}^0 \rightarrow K^* l^+ l^- \) formulism in pQCD approach.
Form factors needed for exclusive decays

• For a scalar meson $K^*_0$,
• Only three form factors involved

\[
\langle K_0^*(P_2) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}(P_B) \rangle = -i \left[ P_\mu - \frac{m_B^2 - m_{K_0^*}^2}{q^2} q_\mu \right] F_1(q^2) + \frac{m_B^2 - m_{K_0^*}^2}{q^2} q_\mu F_0(q^2)
\]

\[
\langle K_0^*(P_2) | \bar{s} \sigma_{\mu\nu} q^{\nu} \gamma_5 b | \bar{B}(P_B) \rangle = [(m_B^2 - m_{K_0^*}^2) q_\mu - q^2 P_\mu] \frac{F_T(q^2)}{m_B + m_{K_0^*}}
\]
For (axial)vector or tensor, there are more complications
• The $B \rightarrow K^{*} J$ form factors are mostly resorts to the Lattice QCD simulations, which is quite limited at this stage.

• In the heavy quark limit and the large energy limit, interactions of the heavy and light systems can be expanded in small ratios $\Lambda/E$ and $\Lambda/m_B$

• At the leading power, the large energy symmetry is obtained and such symmetry to a large extent simplifies the heavy-to-light transition

• This soft-collinear effective theory constrains the independent Lorentz structures

• Reduces the seven independent hadronic form factors for each $B \rightarrow K^{*} J (J \geq 1)$ type to two universal functions
\[ B \rightarrow K^*J \ (J \geq 1) \]

\[
A_{0j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv A_{0j}^{K^*,\text{eff}} \approx \left( 1 - \frac{m_{K^*}^2}{m_B E} \right) \xi_{\perp j}^{K^*}(q^2) + \frac{m_{K^*}}{m_B} \xi_{\parallel j}^{K^*}(q^2),
\]

\[
A_{1j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv A_{1j}^{K^*,\text{eff}} \approx \frac{2E}{m_B + m_{K^*}} \xi_{\perp j}^{K^*}(q^2),
\]

\[
A_{2j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv A_{2j}^{K^*,\text{eff}} \approx \left( 1 + \frac{m_{K^*}}{m_B} \right) \left[ \xi_{\perp j}^{K^*}(q^2) - \frac{m_{K^*}}{E} \xi_{\parallel j}^{K^*}(q^2) \right],
\]

\[
V_{K^*j}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv V_{K^*,\text{eff}}^{j} \approx \left( 1 + \frac{m_{K^*}}{m_B} \right) \xi_{\perp j}^{K^*}(q^2),
\]

\[
T_{1j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv T_{1j}^{K^*,\text{eff}} \approx \xi_{\perp j}^{K^*}(q^2),
\]

\[
T_{2j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv T_{2j}^{K^*,\text{eff}} \approx \left( 1 - \frac{q^2}{m_B^2 - m_{K^*}^2} \right) \xi_{\perp j}^{K^*}(q^2),
\]

\[
T_{3j}^{K^*}(q^2) \left( \frac{|\vec{p}_{K^*}|}{m_{K^*}} \right)^{J-1} \equiv T_{3j}^{K^*,\text{eff}} \approx \xi_{\perp j}^{K^*}(q^2) - \left( 1 - \frac{m_{K^*}^2}{m_B^2} \right) \frac{m_{K^*}}{E} \xi_{\parallel j}^{K^*}(q^2).
\]
For the case of B to scalar meson transition, in the large energy limit, the soft-collinear effective theory applies

- Three form factors reduce to only one

\[
\frac{m_B}{m_B + m_{K^*_0}} F_T(q^2) = F_1(q^2) = \frac{m_B}{2E} F_0(q^2) = \xi_{K^*_0}(q^2)
\]
Form factors calculated in pQCD to leading order of $1/m_b$.

<table>
<thead>
<tr>
<th>$F$</th>
<th>$F(0)$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{BK_2}^+$</td>
<td>$0.21^{+0.04+0.05}_{-0.04-0.03}$</td>
<td>$1.73^{+0.02+0.05}_{-0.02-0.03}$</td>
<td>$0.66^{+0.04+0.07}_{-0.05-0.01}$</td>
</tr>
<tr>
<td>$A_{0}^{BK_2}$</td>
<td>$0.18^{+0.04+0.04}_{-0.03-0.03}$</td>
<td>$1.70^{+0.00+0.05}_{-0.02-0.07}$</td>
<td>$0.64^{+0.00+0.04}_{-0.06-0.10}$</td>
</tr>
<tr>
<td>$A_{1}^{BK_2}$</td>
<td>$0.13^{+0.03+0.03}_{-0.02-0.02}$</td>
<td>$0.78^{+0.01+0.05}_{-0.01-0.04}$</td>
<td>$-0.11^{+0.02+0.04}_{-0.03-0.02}$</td>
</tr>
<tr>
<td>$A_{2}^{BK_2}$</td>
<td>$0.08^{+0.02+0.02}_{-0.02-0.01}$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$T_{1}^{BK_2}$</td>
<td>$0.17^{+0.04+0.04}_{-0.03-0.03}$</td>
<td>$1.73^{+0.00+0.05}_{-0.03-0.07}$</td>
<td>$0.69^{+0.00+0.05}_{-0.08-0.11}$</td>
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<td>$T_{2}^{BK_2}$</td>
<td>$0.17^{+0.03+0.04}_{-0.03-0.03}$</td>
<td>$0.79^{+0.00+0.02}_{-0.04-0.09}$</td>
<td>$-0.06^{+0.00+0.00}_{-0.10-0.16}$</td>
</tr>
<tr>
<td>$T_{3}^{BK_2}$</td>
<td>$0.14^{+0.03+0.03}_{-0.03-0.03}$</td>
<td>$1.61^{+0.01+0.04}_{-0.03-0.04}$</td>
<td>$0.52^{+0.05+0.15}_{-0.01-0.01}$</td>
</tr>
<tr>
<td>$V_{B_s f'_2}$</td>
<td>$0.20^{+0.04+0.05}_{-0.03-0.03}$</td>
<td>$1.75^{+0.00+0.03}_{-0.02-0.05}$</td>
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<td>$A_{0}^{B_s f'_2}$</td>
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<td>$A_{1}^{B_s f'_2}$</td>
<td>$0.12^{+0.02+0.02}_{-0.02-0.02}$</td>
<td>$0.80^{+0.02+0.07}_{-0.00-0.03}$</td>
<td>$-0.11^{+0.05+0.09}_{-0.00-0.00}$</td>
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<td>$A_{2}^{B_s f'_2}$</td>
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<td>$0.16^{+0.03+0.04}_{-0.03-0.02}$</td>
<td>$0.82^{+0.04+0.04}_{-0.09-0.08}$</td>
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<td>$T_{3}^{B_s f'_2}$</td>
<td>$0.13^{+0.03+0.03}_{-0.02-0.02}$</td>
<td>$1.64^{+0.02+0.06}_{-0.00-0.06}$</td>
<td>$0.57^{+0.04+0.05}_{-0.01-0.09}$</td>
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</tbody>
</table>
And different models give quite different results

<table>
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<td>$V^{BK_2^*}$</td>
<td>0.38</td>
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<td>0.22</td>
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<tr>
<td>$T_3^{BK_2^*}$</td>
<td>−0.25</td>
<td>0.01^{+0.02}_{-0.01}</td>
<td>0.10 ± 0.02</td>
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<td></td>
</tr>
</tbody>
</table>
In fact, if applying LO SCET, we need only these functions, by Hatanaka and Yang, Phys. Rev. D 79, 114008 (2009); Eur. Phys. J. C 67, 149 (2010)

<table>
<thead>
<tr>
<th>$K^*_j$</th>
<th>$\hat{\xi}_\parallel$</th>
<th>$\hat{\xi}_\perp$</th>
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<tbody>
<tr>
<td>$K^*(1410)$</td>
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<td>-</td>
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<td>$0.23 \pm 0.05$</td>
</tr>
<tr>
<td>$K^*_4(2045)$</td>
<td>$0.13 \pm 0.03$</td>
<td>$0.19 \pm 0.05$</td>
</tr>
</tbody>
</table>
Branching ratios are proportional to form factors, have large uncertainties, but Angular distribution is not.

**Partial decay width**

\[
\frac{d^4\Gamma}{dq^2dq \cos \theta_K dq \cos \theta_I d\phi} = \frac{3}{8} |\mathcal{M}_B|^2
\]

$|\mathcal{M}_B|^2$ is decomposed into 11 terms:

\[
|\mathcal{M}_B|^2 = \left[ I_1^c C^2 + 2I_1^s S^2 + (I_2^c C^2 + 2I_2^s S^2) \cos(2\theta_I) \right.
+ 2I_3^s S^2 \sin^2 \theta_I \cos(2\phi) + 2\sqrt{2}I_4^C S \sin(2\theta_I) \cos \phi \\
+ 2\sqrt{2}I_5^C S \sin(\theta_I) \cos \phi + 2I_6^s S^2 \cos \theta_I \\
+ 2\sqrt{2}I_7^C S \sin(\theta_I) \sin \phi + 2\sqrt{2}I_8^C S \sin(2\theta_I) \sin \phi \\
+ 2I_9^s S^2 \sin^2 \theta_I \sin(2\phi) \right]
\]
Angular distribution

\[ I_7 = \sqrt{2} \beta_l [\text{Im}(A_{L0} A_{L\parallel}^*) - \text{Im}(A_{R0} A_{R\parallel}^*)] \]

\[ I_8 = \frac{1}{\sqrt{2}} \beta_l^2 [\text{Im}(A_{L0} A_{L\perp}^*) + \text{Im}(A_{R0} A_{R\perp}^*)] \]

\[ I_9 = \beta_l^2 [\text{Im}(A_{L\parallel} A_{L\perp}^*) + \text{Im}(A_{R\parallel} A_{R\perp}^*)] \]

- \[ A_{Ri} = A_{Li}|_{C_{10} \rightarrow -C_{10}} \]
- Up to one-loop matrix element and resonances taken out, only \[ C_{9}^{\text{eff}} \] contributes an imaginary part.

Without higher order QCD corrections

\[ I_7 = 0, \quad I_8 \quad \text{and} \quad I_9 \quad \text{is tiny} \]

They could be chosen as the window to observe those effects that can change the behavior of the Wilson coefficients, such as NP effects.
B → K* μ⁺ μ⁻: polarization

arXiv: 1112.3515
\( B \rightarrow K^* \mu^+ \mu^- : A_{FB} \)

![Graph showing the angular distribution of the branching fraction](image)

**arXiv: 1112.3515**
BRs, fL

With the recent pQCD results for $\bar{B}^0 \to K^*_2$ form factors:

Branching ratios:

$$
\mathcal{B}(B \to K^*_2\mu^+\mu^-) = (2.5^{+1.6}_{-1.1}) \times 10^{-7},
$$

$$
\mathcal{B}(B \to K^*_2\tau^+\tau^-) = (9.6^{+6.2}_{-4.5}) \times 10^{-10}.
$$

Longitudinal Polarization fractions:

$$
f_L \equiv \frac{\Gamma_0}{\Gamma} = \frac{\int dq^2 \frac{d\Gamma_0}{dq^2}}{\int dq^2 \frac{d\Gamma}{dq^2}}
$$

$$
f_L(B \to K^*_2\mu^+\mu^-) = (66.6 \pm 0.4)\%,
$$

$$
f_L(B \to K^*_2\tau^+\tau^-) = (57.2 \pm 0.7)\%.
$$
D. Forward-backward asymmetry

The differential forward-backward asymmetry of $\bar{B} \rightarrow \bar{K}_2^* l^+ l^-$ is defined by

$$\frac{dA_{FB}}{dq^2} = \left( \int_0^1 - \int_{-1}^0 \right) d\cos\theta_l \frac{d^2 \Gamma}{dq^2 d \cos\theta_l} = \frac{3}{4} I_6$$

- The forward backward asymmetry varies from positive to negative as $q^2$ grows up.
- The 0-cross point is sensitive to new physics.
Forward and Backward Asymmetry

\[
\frac{dA_{FB}}{dq^2} = \left[ \int_0^1 - \int_{-1}^0 \right] d\cos \theta_l \frac{d^2\Gamma}{dq^2 d\cos \theta_l}
\]

The zero-crossing point \( s_0 \) of FBAs is determined by the equation

\[
C_9 A_1(s_0)V(s_0) + C_7 L \frac{m_b(m_B + m_{K^*})}{s_0} A_1(s_0) T_1(s_0) + C_7 L \frac{m_b(m_B - m_{K^*})}{s_0} T_2(s_0)V(s_0) = 0
\]

\[
s_0 = (3.1 \pm 0.1) \text{ GeV}^2,
\]

Small uncertainty
Using the form factor relations derived from heavy quark symmetry and large energy limit, the 0-crosing point of forward-backward asymmetry simplify to

$$\text{Re} [C_9] + 2 \frac{m_b m_B}{s_0} C_{7L} \mathcal{R}_j^{K*}(s_0) = 0.$$ 

with

$$\mathcal{R}_j^{K*}(q^2) = \frac{m_B + m_{K^*}}{m_B} \frac{T_1^{K^*}(q^2)}{V^{K^*}(q^2)} = 1$$

And in model calculations:

$$\mathcal{R}_{\text{PQCD}}^{K_2} \sim 1.03, \quad \mathcal{R}_{\text{LCSR}}^{K_2} \sim 1.11,$$
Similarly for $B_s \to f_2' \mu^+ \mu^-$:

$$\mathcal{B}(B_s \to f_2' \mu^+ \mu^-) = (1.8^{+1.1}_{-0.7}) \times 10^{-7},$$

$$f_L(B_s \to f_2' \mu^+ \mu^-) = (63.2 \pm 0.7)\%,$$

$$s_0(B_s \to f_2' \mu^+ \mu^-) = (3.53 \pm 0.03) \text{ GeV}^2,$$

$$\mathcal{B}(B_s \to f_2' \tau^+ \tau^-) = (5.8^{+3.7}_{-2.1}) \times 10^{-10},$$

$$f_L(B_s \to f_2' \tau^+ \tau^-) = (53.9 \pm 0.4)\%.$$
Forward backward asymmetry in $B \rightarrow K_1 l^+l^-$ decays

The 0-cross point is at $q^2 = 3.55$ GeV$^2$
NP scenario: Vector-like quark model (VQM)

Expanding SM including a SU(2)$_L$ singlet down type quark, Yukawa sector of SM is modified to

\[ \mathcal{L}_Y = \bar{Q}_Y Y D^c_R + h_d \bar{Q}_Y H D^c_R + m_D \bar{D}_L D_R + h.c. \]

This modification brings FCNC for the mass eigenstates at tree level.

The interaction for $b\rightarrow sZ$ in VQM is

\[ \mathcal{L}_{b\rightarrow s} = \frac{g c_L s \lambda_{sb} s}{\cos \theta_W} \bar{s} \gamma^\mu P_L b Z_\mu + h.c., \]

with which the effective Hamiltonian for $b \rightarrow s l^+ l^-$ is given as

\[ \mathcal{H}_{b \rightarrow s l^+ l^-}^Z = \frac{2 G_F}{\sqrt{2}} \lambda_{sb} c_L s (\bar{s} b) V - A [c_L (\bar{\ell} \ell) V - A + c_R (\bar{\ell} \ell) V + A] \]
NP scenario: Vector-like quark model (VQM)

The VQM effects can be absorbed into the Wilson coefficients $C_9$ and $C_{10}$

$$C_9^{\text{VLQ}} = C_9^{\text{SM}} - \frac{4\pi}{\alpha_{\text{em}}} \frac{\lambda_{sb} c_s^{\ell} (c_L^\ell + c_R^\ell)}{V_{ts} V_{tb}},$$

$$C_{10}^{\text{VLQ}} = C_{10}^{\text{SM}} + \frac{4\pi}{\alpha_{\text{em}}} \frac{\lambda_{sb} c_s^{\ell} (c_L^\ell - c_R^\ell)}{V_{ts} V_{tb}}.$$

Lepton section in VQM is the same as in SM.
NP scenario: Family non-universal Z’ model

Expand SM by simply including an additional $U(1)'$ symmetry. The current is

$$J^{\mu}_{Z'} = g' \sum_i \bar{\psi}_i \gamma^\mu [\epsilon_i^{\psi_L} P_L + \epsilon_i^{\psi_R} P_R] \psi_i,$$

which couples to a family non-universal Z’ boson.

After rotating to the mass eigen basis, FCNC appears at tree level in both LH and RH section.

Interaction for b-s-Z’ is given as

$$\mathcal{L}_{\text{FCNC}}^{Z'} = -g' (B_{sb}^L \bar{s}_L \gamma_\mu b_L + B_{sb}^R \bar{s}_R \gamma_\mu b_R) Z'^\mu + \text{h.c.}$$

The effective Hamiltonian for $b \rightarrow s l^+ l^-$ is given as

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{8 G_F}{\sqrt{2}} (\rho_{sb}^L \bar{s}_L \gamma_\mu b_L + \rho_{sb}^R \bar{s}_R \gamma_\mu b_R) (\rho_{\ell \ell}^L \bar{\ell}_L \gamma_\mu \ell_L + \rho_{\ell \ell}^R \bar{\ell}_R \gamma_\mu \ell_R)$$
NP scenario: Family non-universal $Z'$ model

Different from VQM, the couplings in both the quark and lepton section are free parameters.

Too many free parameters. So we set $\rho_{sb}^R = 0$ in our analysis to reduce freedoms.

$Z'$ also only affects $C_9$ and $C_{10}$ phenomenally:

$$C_9^{Z'} = C_9 - \frac{4\pi}{\alpha_{em}} \frac{\rho_{sb}^L (\rho_{ll}^L + \rho_{ll}^R)}{V_{tb} V_{ts}^*}, \quad C_{10}^{Z'} = C_{10} + \frac{4\pi}{\alpha_{em}} \frac{\rho_{sb}^L (\rho_{ll}^L - \rho_{ll}^R)}{V_{tb} V_{ts}^*}$$
Constrain the model parameters by exp.

**Data used for fitting**

\[
\begin{align*}
&b \rightarrow cl\bar{\nu} & b \rightarrow s l^+ l^- & \overline{B}^0 \rightarrow K^* l^+ l^- \\
&(10.58 \pm 0.15) \times 10^{-2} & (3.66^{+0.76}_{-0.77}) \times 10^{-6} & (1.09^{+0.12}_{-0.11}) \times 10^{-6}
\end{align*}
\]

<table>
<thead>
<tr>
<th>(q^2 (\text{GeV}^2))</th>
<th>(\mathcal{B} (10^{-7}))</th>
<th>(F_L)</th>
<th>(-A_{FB})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 2]</td>
<td>1.46 ± 0.41</td>
<td>0.29 ± 0.21</td>
<td>0.47 ± 0.32</td>
</tr>
<tr>
<td>[2, 4.3]</td>
<td>0.86 ± 0.32</td>
<td>0.71 ± 0.25</td>
<td>0.11 ± 0.37</td>
</tr>
<tr>
<td>[4.3, 8.68]</td>
<td>1.37 ± 0.61</td>
<td>0.64 ± 0.25</td>
<td>0.46 ± 0.26</td>
</tr>
<tr>
<td>[10.09, 12.86]</td>
<td>2.24 ± 0.48</td>
<td>0.17 ± 0.17</td>
<td>0.43 ± 0.20</td>
</tr>
<tr>
<td>[14.18, 16]</td>
<td>1.05 ± 0.30</td>
<td>-0.15 ± 0.28</td>
<td>0.70 ± 0.24</td>
</tr>
<tr>
<td>&gt; 16</td>
<td>2.04 ± 0.31</td>
<td>0.12 ± 0.15</td>
<td>0.66 ± 0.16</td>
</tr>
<tr>
<td>[1, 6]</td>
<td>1.49 ± 0.47</td>
<td>0.67 ± 0.24</td>
<td>0.26 ± 0.31</td>
</tr>
</tbody>
</table>


**Definition of \(\chi^2\)**

\[
\chi_i^2 = \frac{(B_i^{\text{the}} - B_i^{\text{exp}})^2}{(B_i^{\text{err}})^2}
\]
Constrain the VQM parameters

\[
\begin{align*}
\text{Re} \lambda_{sb} & = (0.07 \pm 0.04) \times 10^{-3} \\
\text{Im} \lambda_{sb} & = (0.09 \pm 0.23) \times 10^{-3}
\end{align*}
\] \implies \left\{
\begin{align*}
|\lambda_{sb}| & < 0.3 \times 10^{-3} \\
\text{Phase less constrained}
\end{align*}\right.

Constrains on the Wilson coefficients

with \( \chi^2/d.o.f. = 2.4 \)

\[
\begin{align*}
|\Delta C_9| & = |C_9 - C_9^{SM}| < 0.2 \\
|\Delta C_{10}| & = |C_{10} - C_{10}^{SM}| < 2.8
\end{align*}
\]
Constrain the $Z'$ model parameters

Assume $\Delta C_9, \Delta C_{10}$ as real

$\Delta C_9 = 0.88 \pm 0.75, \quad \Delta C_{10} = 0.01 \pm 0.69$

Both $\Delta C_9$ and $\Delta C_{10}$ are complex numbers.

$\Delta C_9 = (-0.81 \pm 1.22) + (3.05 \pm 0.92)i$

$\Delta C_{10} = (1.00 \pm 1.28) + (-3.16 \pm 0.94)i$

$\text{Im}[C_{10}]$ has little effect on $\chi^2$

$\chi^2 / \text{d.o.f.} = 2.3$

Combining the above results $|\Delta C_9| < 3, \quad |\Delta C_{10}| < 3$
New Physics effects in observables

In the NP effects, we choose \( \Delta C_9 = 3e^{i\pi/4,i\pi/4} \) and \( \Delta C_{10} = 3e^{i\pi/4,i3\pi/4} \) as the reference points.

\[
\frac{d\text{Br}}{dq^2}
\]

Br \((10^{-7})\) may be enhanced, however, large uncertainties

In this parameter space, \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) is consistent with the recent measurement. \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 5.1 \times 10^{-8} \)
New Physics effects in observable $B \rightarrow K_2^{*} l^+l^-$

Zero-crossing point of AFB may be changed significantly in new physics model.
New Physics effects in observable $\frac{dA_{FB}}{dq^2}$

Of $B \rightarrow K^{*0} l^+ l^-$

0 Asymmetry is expected in SM, changed significantly in new physics model
SUSY contributions to parameter of $B \rightarrow K^*\tau + \tau^-$

Asymmetry is expected in SM, changed significantly in new physics model.
B$\to K^*_2 l^+l^-$ Polarization fraction $f_L$

some changes of Polarization fraction $f_L$ in new physics model.
Summary

- $B \rightarrow K_1(K_0^*, K_2^*, K_3^*, K_4^*) \, l^+l^-$ are investigated in SM.
  - Branching ratios are estimated to be $10^{-6}$, $10^{-7}$
  - expected to be observed in future Exp.

- FBA, polarization fractions, etc, are investigated, with small uncertainties.

- NP scenarios (VQM, SUSY, Z' model) are investigated.
  - Parameter space constrained with data of $\bar{B}^0 \rightarrow K^*l^+l^-$ and $b \rightarrow s l^+l^-$. 

Zero-crossing point of FBA can be changed dramatically, which are sensitive to NP effects
Thank you
Estimate BRs from exp.

- Experimentally, we have
  \[ \mathcal{B}(\bar{B}^0 \rightarrow K^*_2 \gamma) = (12.4 \pm 2.4) \times 10^{-6}, \]
  \[ \mathcal{B}(\bar{B}^0 \rightarrow K^* \gamma) = (43.3 \pm 1.5) \times 10^{-6}. \]
  \[ \mathcal{B}(\bar{B}^0 \rightarrow K^* l^+ l^-) = (1.09 \pm 0.12) \times 10^{-6} \]

- Assume \( R = \mathcal{B}(K^*_2) / \mathcal{B}(K^*) \) is the same for radiative and semi-leptonic decays, we have
  \[ \mathcal{B}_{\text{exp}}(B^0 \rightarrow K_{2}^{*0} l^+ l^-) = (3.1 \pm 0.7) \times 10^{-7} \]

Compare with KC Yang, PRD79:114008,2009

\[ \mathcal{B}(B^0 \rightarrow K_{2}^{*0}(1430) \mu^+ \mu^-) = (3.5^{+1.1+0.7}_{-1.0-0.6}) \times 10^{-7} \]