$K_{l3}^{\pm}$ Form Factor Measurement at NA48/2

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on behalf of the NA48/2 Collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Rencontres de Moriond
QCD and High Energy Interactions
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Outline

- Physics Motivation
- NA48/2 Experiment
- $K_{l3}^{\pm}$ Form Factor Analysis
  - Event selection
  - Background sources
  - Radiative corrections
  - DATA-MC comparison
  - Systematic checks
  - Preliminary result
- $K_{l3}^{\pm}$ Form Factors at NA62
- Summary
\( K \to \pi l\nu \) \((K_{l3})\) decays provide the most accurate and theoretically cleanest way to access \(|V_{us}|\). The master formula for \(K_{l3}\) decay rates:

\[
\Gamma(K_{l3}(\gamma)) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 I_K^l(\lambda_0)(1 + \delta_{SU(2)}^l + \delta_{EM}^l)^2
\]

**Experimental Inputs:**

- \(\Gamma(K_{l3}(\gamma))\) Branching ratios and kaon lifetimes.
- \(I_K^l(\lambda_0)\) Phase space integral depends on the form factors.

**Theory Inputs:**

- \(S_{EW}\) Universal short distance EW corrections \((1.0232 \pm 0.0003)\).
- \(f_+(0)\) Form factor at zero momentum transfer.
- \(\delta_{SU(2)}^l\) Form factor correction for isospin breaking (charged mode only).
- \(\delta_{EM}^l\) Long distance EM effects.
$K_{l3}$ decays are described by two form factors $f_{\pm}(t)$, and the matrix element can be written as:

$$M = \frac{G_F}{2} V_{us} (f_{+}(t)(P_K + P_{\pi})^\mu \bar{u}_l \gamma_\mu (1 + \gamma_5) u_\nu + f_{-}(t)m_l \bar{u}_l (1 + \gamma_5) u_\nu)$$

$t = q^2$ is the square of the four-momentum transfer to the lepton neutrino system. $f_{-}(t)$ can only be measured in $K_{\mu3}$ decays because of $m_e << m_K$.

$f_{+}(t)$ is the vector form factor and $f_0(t)$ the scalar form factor with:

$$f_0(t) = f_{+}(t) + \frac{t}{(m_K^2 - m_{\pi}^2)} f_{-}(t)$$

By construction $f_{+}(0) = f_0(0)$.

$f_{+}(0)$ cannot be measured directly, therefore the form factors are normalised to $f_{+}(0)$:

$$\bar{f}_{+}(t) = \frac{f_{+}(t)}{f_{+}(0)} \quad \quad \bar{f}_0(t) = \frac{f_0(t)}{f_{+}(0)}$$
Parametrizations using **physical quantities** are called **class 1** parametrizations. They depend on free parameters with a physical meaning.

**Pole Parametrization:**
Assumes the exchange of vector and scalar resonances $K^*$ with spin-parity $1^-/0^+$ and mass $m_V/m_S$. $f_+(t)$ can be described by $K^*(892)$, for $f_0(t)$ no obvious dominance is seen.

\[
\bar{f}_{+0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t}
\]

**Dispersive Parametrization:**
Based on a dispersive approach with the free parameters $\Lambda_+$ and $\ln C$. Accurate polynomial approximations for the dispersive integrals $G(t)$ and $H(t)$ are available.

\[
\bar{f}_+(t) = \exp \left[ \frac{t}{m_{\pi}^2} (\Lambda_+ + H(t)) \right] \quad \bar{f}_0(t) = \exp \left[ \frac{t}{\Delta_{K\pi}} (\ln C - G(t)) \right]
\]

Parametrizations without a **physical meaning** are called **class 2** parametrizations. They require more free parameters and are expansions in the momentum transfer.

**Linear and quadratic parametrization:**

The expansion in the momentum transfer \( t = q^2 \) is widely used:

\[
\bar{f}_{+,0}(t) = \left[ 1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right] \quad \text{Linear}
\]

\[
\bar{f}_{+,0}(t) = \left[ 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \left( \frac{t}{m_\pi^2} \right)^2 \right] \quad \text{Quadratic}
\]

- More free parameters to be determined ➔ **Correlations**!
- No sensitivity to determine \( \lambda''_0 \) with current experiments ➔ \( \bar{f}_+ \) quadratic / \( \bar{f}_0 \) linear.

**Z-fit parametrization:**

The parametrization function depending on \( t \) and \( t_+ = (m_K + m_\pi)^2 \) sums an infinite number of terms, transforming the original series, naively an expansion involving \( t/t_+ \lesssim 0.3 \), into a series with much smaller expansion parameters *(PRD74(2006) 096006).*
NA48/2 Experiment

- **NA48/2**: fixed target experiment in the North Area of the CERN SPS. In 2003-2004 the main purpose was the search for direct CP violation in $K^\pm \rightarrow 3\pi$ decays.

- The beamline offered simultaneous $K^+$ and $K^-$ beams, coinciding within 1 mm along the 114 meter long decay volume.

- For the form factor measurement a dedicated three-days run with minimum bias trigger and low intensity was used.

- The beam momentum was $(60 \pm 1.8)$ GeV/c in this special run.
Main detector components:

- **Magnetic Spectrometer**
  \[
  \frac{\sigma_p}{p} = 1.02\% + 0.044\% \frac{p}{\text{GeV/c}}
  \]
  \(\sim 1.4\%\) resolution for charged particles with \(p=20\ \text{GeV/c}\)

- **Hodoscope**
  two planes of scintillators for fast triggering
  \(\sigma_t \sim 150\ \text{ps}\)

- **Liquid Krypton EM Calorimeter**
  \[
  \frac{\sigma_E}{E} = \frac{3.2\%}{\sqrt{E/\text{GeV}}} + \frac{9.0\%}{E/\text{GeV}} + 0.42\%
  \]
  \(\sim 1\%\) resolution for particles with \(E=20\ \text{GeV}\)

- **Muon veto system**
  three planes of scintillators, each shielded by 80 cm iron
  99.9% efficient for muon with \(p_\mu > 10\ \text{GeV/c}\)
  \(\sigma_t \sim 350\ \text{ps}\)

**Min Bias Trigger:** Coincidence of two Hodoscope hits \(\times E_{\text{LKr}} > 10\ \text{GeV}\)
Event selection

- **1 good track.**
  - Muon identification using muon veto and E/P
  - Electron identification using E/P
  - $P_\mu > 10$ GeV / $P_e > 5$ GeV

- **1 good $\pi^0 \rightarrow \gamma\gamma$.**
  - Pion mass cut: $|m_{\gamma\gamma} - m_{\pi^0}^{PDG}| < 10$ MeV

**Event reconstruction**

- LKr clusters and muon track consistent in time
- Missing mass cut using calculation with $K_{l3}^{\pm}$ hypothesis
  $$MM_{K_{l3}}^2 = (P_K - P_l - P_{\pi^0})^2 < 10 \text{ MeV}^2$$
- Kaon energy reconstruction under the assumption of a missing undetected neutrino within the range of:
  $$55 \text{ GeV} < E_{K^{\pm}} < 65 \text{ GeV}$$

2.5 $\times$ 10^6 $K_{\mu3}^{\pm}$ events selected
4.0 $\times$ 10^6 $K_{e3}^{\pm}$ events selected
\[ K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \text{ Background} \]

\[ K^{\pm}_{\mu 3} : \]

- \( K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \) with \( \pi \rightarrow \mu \) can fake \( K^{\pm}_{\mu 3} \).
- Without suppression, \( K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \) bkg at the level of 20%.
- Cut in the invariant \( \pi^{\pm} \pi^{0} \) - mass and the transverse momentum of the pion:
  - Background contamination reduced to 0.5%.
  - about 24% loss of \( K^{\pm}_{\mu 3} \) events.

- Background is well localised in the Dalitz plot.
$K^{\pm} \rightarrow \pi^{\pm} \pi^0$ Background

$K^{\pm}_{e3}$:

- Pion with $E/P > 0.95$ can fake a $K^{\pm}_{e3}$ decay.
- Cut in the transverse momentum of the event:
  \[ p_T^{\text{event}} > 0.02 \text{ GeV}/c \]
  → Background contamination reduced to < 0.1%.
  → about 3% loss of $K^{\pm}_{e3}$ events.
\( \mathbf{K}^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0 \) Background

\( \mathbf{K}^{\pm}_{\mu3} \):

- \( \pi \to \mu \) decay with lost photons from \( \pi^0 \)-decays.
- **Small** but introduces slope in the Dalitz plot.
- **No dedicated cut** to reduce the background is applied.
- A correction is applied to take the background into account.
- Without the correction the result shifts by \( \simeq 0.5 \sigma_{\text{stat}} \).

\( \mathbf{K}^{\pm}_{e3} \):

- \( \mathbf{K}^{\pm} \rightarrow \pi^{\pm} \pi^0 \pi^0 \) background is negligible.
To extract the form factors, a fit to the Dalitz plot density is performed.

\[
\rho(E_l^*, E_\pi^*) = \frac{d^2N(E_l^*, E_\pi^*)}{dE_l^* dE_\pi^*} \propto A f_+(t) + B f_+(t) (f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} + C \left[ (f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} \right]^2
\]

- \(E_l^*\) and \(E_\pi^*\) are the energy of the lepton and the pion in the kaon rest frame.
- \(A, B\) and \(C\) are kinematical terms.
- The fit is performed in cells of \(5 \times 5\) MeV^2
- Cells which are outside or crossing the border of the physical region of the Dalitz plot are not used in the fit.

**Applied corrections:**
- Background subtraction.
- Acceptance.
- Radiative corrections.
**K_{13} Data-MC Comparison**

- Pion energy in the kaon rest frame: $K_{\mu3}^\pm$

- Pion energy in the kaon rest frame: $K_{e3}^\pm$

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**Data/MC Comparison**

**E_\pi^* [GeV]**

- Data
- MC Summe
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 50$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 10^3$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 50$

---

**Data/MC Comparison**

**E_\pi^* [GeV]**

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- MC Summe
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 50$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 10^3$
- MC $K^+ \rightarrow \pi^0 \mu^+ \nu \times 50$
$K_{\pm \mu 3}^*$:

The $K_{l3}$ decay rate including first order radiative corrections can be written as:

$$\Gamma_{K_{l3}} = \Gamma_{K_{l3}}^0 + \Gamma_{K_{l3}}^1 = \Gamma_{K_{l3}}^0 (1 + 2\delta_{EM}^{K_{l3}})$$

- Simulation code provided by KLOE (author C. Gatti) 
  (EPJ C45 (2006) 417)
- Parameters used for the normalisation. 
  (JHEP 11 (2008) 006)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\delta_{EM}^{K_{\pm \mu 3}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu 3}^0$</td>
<td>0.700 ± 0.110</td>
</tr>
<tr>
<td>$K_{\pm \mu 3}$</td>
<td>0.008 ± 0.125</td>
</tr>
</tbody>
</table>

- For $K_{\pm \mu 3}$ small effects on the acceptance.
- Percent effect on the Dalitz plot slope.
**K_{e3}^{\pm}**: The $K_{l3}$ decay rate including first order radiative corrections can be written as:

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<tr>
<td>$K_{e3}^0$</td>
<td>0.495 ± 0.110</td>
</tr>
<tr>
<td>$K_{e3}^{\pm}$</td>
<td>0.050 ± 0.125</td>
</tr>
</tbody>
</table>

- For $K_{e3}^{\pm}$ the effects on the acceptance are bigger.
- ~10% effect on the Dalitz plot slope.
### Systematic checks

<table>
<thead>
<tr>
<th>$K_{\mu 3}^\pm$</th>
<th>$\Delta \chi_+^\prime$</th>
<th>$\Delta \chi_+^\prime \times 10^{-3}$</th>
<th>$\Delta \lambda_0$</th>
<th>$\Delta m_V$</th>
<th>$\Delta m_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaon Energy</td>
<td>±0.1</td>
<td>±0.0</td>
<td>±0.3</td>
<td>±1</td>
<td>±8</td>
</tr>
<tr>
<td>Vertex</td>
<td>±1.0</td>
<td>±0.5</td>
<td>±0.1</td>
<td>±2</td>
<td>±7</td>
</tr>
<tr>
<td>Bin size</td>
<td>±0.8</td>
<td>±0.4</td>
<td>±0.7</td>
<td>±3</td>
<td>±10</td>
</tr>
<tr>
<td>Energy scale</td>
<td>±0.3</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0</td>
<td>±1</td>
</tr>
<tr>
<td>Acceptance</td>
<td>±0.2</td>
<td>±0.1</td>
<td>±0.3</td>
<td>±2</td>
<td>±5</td>
</tr>
<tr>
<td>$K_{2\pi}$ background</td>
<td>±1.7</td>
<td>±0.5</td>
<td>±0.6</td>
<td>±3</td>
<td>±0</td>
</tr>
<tr>
<td>2nd Analysis</td>
<td>±0.1</td>
<td>±0.1</td>
<td>±0.2</td>
<td>±2</td>
<td>±5</td>
</tr>
<tr>
<td>FF input</td>
<td>±0.3</td>
<td>±0.8</td>
<td>±0.1</td>
<td>±7</td>
<td>±3</td>
</tr>
<tr>
<td>Systematic</td>
<td>±2.2</td>
<td>±1.1</td>
<td>±1.0</td>
<td>±9</td>
<td>±16</td>
</tr>
<tr>
<td>Statistical</td>
<td>±3.0</td>
<td>±1.1</td>
<td>±1.4</td>
<td>±8</td>
<td>±31</td>
</tr>
</tbody>
</table>

| $K_{e 3}^\pm$ | $\Delta \chi_+^\prime$ | $\Delta \chi_+^\prime \times 10^{-3}$ | | $\Delta m_V$ |
|----------------|------------------------|-------------------------------|---------------|
| Kaon Energy    | ±0.3                   | ±0.1                          | ±6            |
| Vertex         | ±0.2                   | ±0.1                          | ±0            |
| Bin size       | ±0.0                   | ±0.1                          | ±2            |
| Energy scale   | ±0.1                   | ±0.0                          | ±0            |
| Acceptance     | ±0.2                   | ±0.0                          | ±3            |
| 2nd Ana        | ±0.9                   | ±0.4                          | ±1            |
| FF input       | ±0.4                   | ±0.0                          | ±1            |
| Systematic     | ±1.1                   | ±0.4                          | ±7            |
| Statistical    | ±0.7                   | ±0.3                          | ±3            |

$K_{\mu 3}^\pm$ is dominated by statistics, $K_{e 3}^\pm$ is dominated by the systematics.
Preliminary Results for NA48/2

<table>
<thead>
<tr>
<th>Quadratic ($\times 10^{-3}$)</th>
<th>$\lambda'_\pm$</th>
<th>$\lambda''_\pm$</th>
<th>$\lambda_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\mu3}^\pm$</td>
<td>26.3 ± 3.0(^\text{stat}) ± 2.2(^\text{syst})</td>
<td>1.2 ± 1.1(^\text{stat}) ± 1.1(^\text{syst})</td>
<td>15.7 ± 1.4(^\text{stat}) ± 1.0(^\text{syst})</td>
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<td>$K_{e3}^\pm$</td>
<td>27.2 ± 0.7(^\text{stat}) ± 1.1(^\text{syst})</td>
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<table>
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<tr>
<th>Pole (MeV/c(^2))</th>
<th>$m_V$</th>
<th>$m_S$</th>
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<tr>
<td>$K_{\mu3}^\pm$</td>
<td>873 ± 8(^\text{stat}) ± 9(^\text{syst})</td>
<td>1183 ± 31(^\text{stat}) ± 16(^\text{syst})</td>
</tr>
<tr>
<td>$K_{e3}^\pm$</td>
<td>879 ± 3(^\text{stat}) ± 7(^\text{syst})</td>
<td></td>
</tr>
</tbody>
</table>

68% Confidence level contours

- KTeV $K^0$
- KLOE $K^0$
- Istra+ $K^-$
- NA48 $K^0$
- NA48/2 $K^-$ preliminary

$K^\pm$ \quad $m^V$ \quad $m_S$

$K_{\mu3}$ Form Factor Measurement at NA48/2

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**Combined result:**

<table>
<thead>
<tr>
<th>Quadratic ($\times 10^{-3}$)</th>
<th>$\chi'_+$</th>
<th>$\chi''_+$</th>
<th>$\lambda_0$</th>
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<tr>
<td>$K_{\mu 3}^{\pm} K_e^{\pm}$ combined</td>
<td>26.98 ± 1.11</td>
<td>0.81 ± 0.46</td>
<td>16.23 ± 0.95</td>
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<tr>
<td>Pole (MeV/c$^2$)</td>
<td>$m_V$</td>
<td>$m_S$</td>
<td></td>
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<tr>
<td>$K_{\mu 3}^{\pm} K_e^{\pm}$ combined</td>
<td>877 ± 6</td>
<td></td>
<td>1176 ± 31</td>
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- statistical and systematical uncertainties combined.

**Experimental situation:**

- $K_{l 3}^0$ results from KLOE, KTeV and NA48, $K_{l 3}^-$ from ISTRA+.
- **NA48/2** is the first measurement which uses $K_{\mu 3}^{\pm}$ and $K_{e 3}^{\pm}$.
- **NA48/2 preliminary result** with high precision - very competitive with the other results. Offers the combined result with the smallest error.
- The results for $K_{e 3}^{\pm}$ and $K_{\mu 3}^{\pm}$ from NA48/2 are in good agreement.
Outlook: $K_{l3}^{\pm}$ Form Factors at NA62

In the year 2007 NA62 collected data for a dedicated measurement of $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu2})$ and a test of the future $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ experiment.

- 4 months of data taking in 2007 with $K^+$ and $K^-$ beams (mostly $K^+$) with a beam momentum of $P_K = (74 \pm 1.6)\text{GeV/c}$.
- Transverse momentum kick of the magnetic spectrometer was doubled → Improvement in the track momentum resolution.
- Collected about 150000 $K_{e2}$ events.
- The result of the full data sample presented at European Physical Society HEP 2011:

$$R_K = (2.488 \pm 0.007_{\text{stat}} \pm 0.007_{\text{syst}}) \times 10^{-5}.$$  

**Form Factors from NA62 2007 data**

- Huge statistics in $K_{\mu3}^{\pm}$ and $K_{e3}^{\pm}$ of $\mathcal{O}(10^7)$ events.
- Special $K_L$ run (15 h) for systematic studies.
  → $K_{\mu3}^0$ and $K_{e3}^0$ statistics of $\mathcal{O}(10^6)$ events.

NA48 analyses of $K_{l3}^0$ and $K_{l3}^{\pm}$ can be repeated with different/larger data sets.
Summary and outlook

- **NA48/2** provides new **preliminary results** on the $K_{l3}^{\pm}$ form factors in the quadratic and Pole parametrization.

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- For the **first time** a result is presented which studied $K^+$ and $K^-$ decays.

- **NA48/2 preliminary result** with high precision:
  - Very competitive with the other results in $K_{\mu3}^{\pm}$ and smallest error in $K_{e3}^{\pm}$.
  - Offers the combined result with the smallest error.

- **NA62** is ready to give its contribution with high statistics in $K_{l3}^{0}$ and $K_{l3}^{\pm}$. 