

Nuclear Parton Distribution Functions & Heavy Quark Structure Functions

Rencontres de Moriond 2012

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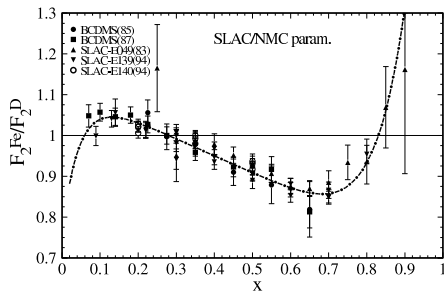
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nuclear PDFs

- **Parton Distribution Functions (PDFs)** - integral part of all calculations involving colliding hadrons
- Non-perturbative objects - cannot be calculated by pQCD (lattice attempts on calculation)
- But once extracted from data they can be used in all hadronic calculations → they are universal courtesy of **Factorization**
$$\sigma = \int f_{a/A}(x, Q) f_{b/B}(x, Q) \hat{\sigma}_{ab}$$
- Extract PDFs via a global fit to data
- Some data are on nuclear targets - νA
- Is there a need to correct for that in global fit?

Nuclear Corrections



- The ratio F_2^{Fe}/F_2^D , measured by EMC, illustrates differences between free and bound structure functions (D - approximate unbound state)
- Four distinct regions
 - 1. Shadowing - $R < 1$, $x \leq 0.1$
 - 2. Antishadowing - $R > 1$, $0.1 \leq x \leq 0.25$
 - 3. EMC region - $R < 1$, $0.25 \leq x \leq 0.8$
 - 4. Fermi motion - $R < 1$, $x \geq 0.8$
- Measure nuclear ratios to study differences between free and bound case

- Nuclear corrections needed in free PDF analysis:
 - νA data allow for flavor decomposition; dimuon data constrains strange quark PDF
 - Precise knowledge of $f_{s/p}(x, Q)$ important for correct predictions of W, Z production ([arXiv:1203.1290](#))
 - σ - small \rightarrow need nuclear target
- nPDFs also - benefit on their own:
 - p-A and A-A collisions (p-Pb, Pb-Pb at LHC ; d-Au, Au-Au at RHIC), important for QGP studies
 - LBL experiments - neutrino oscillation
 - Interesting physics, structure of nucleus - explanations of regions 1-4
- Work on nPDFs done by several groups

nPDF Fits on $I^{\pm}A$ & DY data

Global analyses of nPDFs by four groups:

- **HKN'07** [[PRC76\(2007\)065207](#)]
LO, NLO, error PDFs, $\chi^2/dof = 1.2$
- **EPS'09** [[JHEP0904\(2009\)065](#)]
LO, NLO, error PDFs, $\chi^2/dof = 0.8$
Use also inclusive π^0 data at midrap.
from $d + Au$ and $p + p$ coll. at RHIC
→ gluon
- **DS'04** [[PRD69\(2004\)074028](#)]
first NLO analysis, no error PDFs,
 $\chi^2/dof = 0.76$
- **nCTEQ** [[PRD80\(2009\)094004](#)]
NLO, same data as HKN'07 (up to cuts),
no error PDFs (so far),
 $\chi^2/dof = 0.95$, official release soon

Table from Hirai et al., arXiv:0909.2329

	R	Nucleus	Experiment	EPS09	HKN07	DS04	
DIS	A/D	D/p	NMC		0		
		4He	SLAC E139	0	0	0	
			NMC95	0 (5)	0	0	
		Li	NMC95	0	0		
		Be	SLAC E139	0	0	0	
			EMC-88, 90		0		
		C	NMC 95	0	0	0	
			SLAC E139	0	0	0	
			FNAL-E665		0		
		N	BCDMS 85		0		
			HERMES 03		0		
		Al	SLAC E49		0		
			SLAC E139	0	0	0	
		Fe	EMC 90		0		
			Ca	NMC 95	0	0	0
				SLAC E139	0	0	0
			FNAL-E665		0		
			SLAC E87		0		
			Fe	SLAC E139	0 (15)	0	0
				SLAC E140		0	
	BCDMS 87				0		
	Cu		EMC 93	0	0		
	Kr		HERMES 03		0		
	Ag	SLAC E139	0	0	0		
	Sn	EMC 88		0			
	Au	SLAC E139	0	0	0		
		SLAC E140		0			
	Pb	FNAL-E665		0			
	A/C	Be	NMC 96	0	0	0	
		Al	NMC 96	0	0	0	
Ca		NMC 95		0			
		NMC 96	0	0	0		
Fe		NMC 96	0	0	0		
Sn		NMC 96	0 (10)	0	0		
Pb		NMC 96	0	0	0		
A/Li	C	NMC 95	0	0			
	Ca	NMC 95	0	0			
DY	A/D	C		0	0	0	
		Ca	FNAL-E772	0 (15)	0	0	
		Fe		0 (15)	0	0	
		W		0 (10)	0	0	
	A/Be	Fe	FNAL E866	0	0		
W			0	0			
π pro	dA/pp	Au	RHIC-PHENIX	0 (20)			

- Data choices (previous slide)
- Parametrization

- Convolution Relation - DS'04:

$$f_i^{P/A}(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i^P(x_N/y, Q_0^2)$$

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] \\ + n_v (y/A)^{\alpha_v} (1 - y/A)^{\beta_v} + n_s (y/A)^{\alpha_s} (1 - y/A)^{\beta_s}$$

- Multiplicative Factor - EPS'09, HKN'07:

$$f_i^{P/A}(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i^P(x_N, Q_0^2)$$

$$R_i^{HKN}(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}} \quad (i = u_v, d_v, \bar{q}, g)$$

$$R_i^{EPS}(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

- A-Dependant Functional Form - nCTEQ:

- Uses the same global fit framework & functional form as in the free CTEQ6M analysis

$$\begin{aligned}x f_k^{p/A}(x, Q_0) &= c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4 x})^{c_5}, \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}, \\ \bar{d}(x, Q_0) / \bar{u}(x, Q_0) &= c_0 x^{c_1} (1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}\end{aligned}$$

- Modification - A-dependent parameters :

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}}), \quad k = 1, \dots, 5$$

- For $A = 1$, $c_k = c_{k,0}$
- Construct the nPDF for an (A, Z) nucleus by:
$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- Results in Schienbein, Yu, Kovarik, Keppel, Morfin, Olness, Owens, PRD80(2009)094004

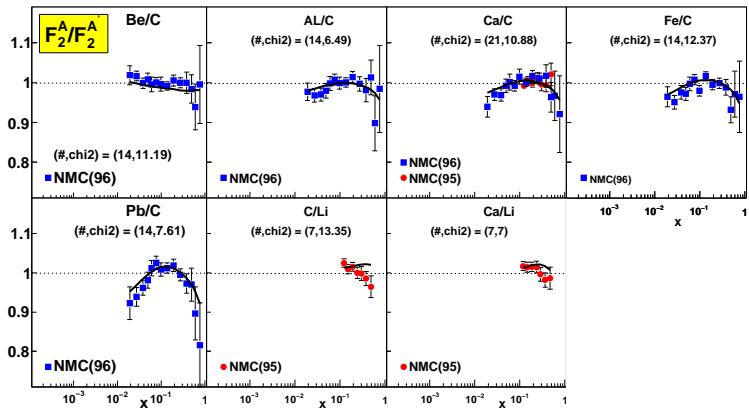
Input

- Use same data as HKN'07 (up to cuts)
- DIS F_2^A/F_2^D data sets: 862 points (before cuts)
- DIS $F_2^A/F_2^{A'}$ data sets: 297 points (before cuts)
- DY data sets $\sigma_{\text{DY}}^{pA}/\sigma_{\text{DY}}^{pA'}$: 92 points (before cuts)

Output

- 708 (1233) data points after (before) cuts
- 32 free parameters; 675 d.o.f.
- Overall $\chi^2/\text{d.o.f.} = 0.95$
- individually:
 - for F_2^A/F_2^D : $\chi^2/\text{pt} = 0.92$
 - for $F_2^A/F_2^{A'}$: $\chi^2/\text{pt} = 0.69$
 - for DY: $\chi^2/\text{pt} = 1.08$

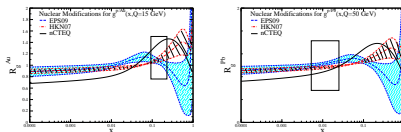
nCTEQ - Global Fit



Summary - decut3

- Factorization works well for DY and $I^{\pm}A$
- decut3 - an excellent fit
- nCTEQ decut3 nPDFs + other sets available at:

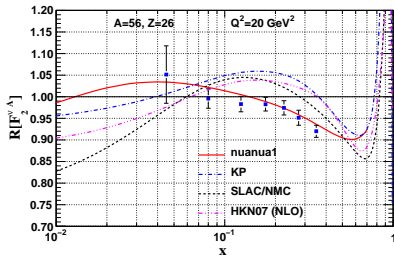
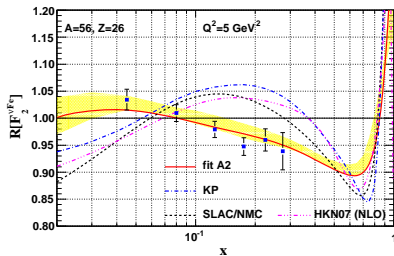
<http://ncteq.hepforge.org/>



- Need to better constrain the gluon nPDF ($\rightarrow \gamma + Q$ production [JHEP01\(2011\)152](#))
- Still need νA data for flavor separation in global analyses of free PDFs
- Use same framework as for decut3 ($I^{\pm}A$ & DY) to fit neutrino data

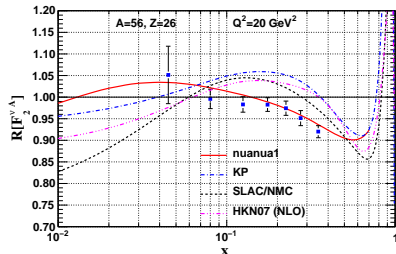
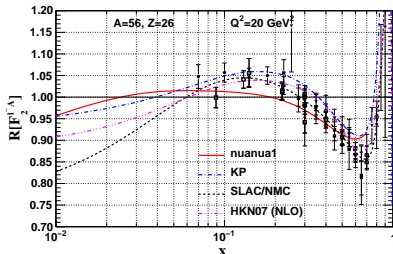
ID	$d\sigma^{\nu A}/dx dy$: Observable	Experiment	# data
33	Pb	CHORUS ν	607 (412)
34	Pb	CHORUS $\bar{\nu}$	607 (412)
35	Fe	NuTeV ν	1423 (1170)
36	Fe	NuTeV $\bar{\nu}$	1195 (966)
37	Fe	CCFR ν di-muon	44 (44)
38	Fe	NuTeV ν di-muon	44 (44)
39	Fe	CCFR $\bar{\nu}$ di-muon	44 (44)
40	Fe	NuTeV $\bar{\nu}$ di-muon	42 (42)
	Total:		4006 (3134)

Fits to only νA data



- Left: Fit to only νFe data ([arXiv:0710.4897](https://arxiv.org/abs/0710.4897))
- Right: New fit to all νA data in A-dependent nPDF framework ([arXiv:0907.2357](https://arxiv.org/abs/0907.2357))
- These fits describe $R[F_2^{\nu A}]$ very well

Are there inconsistencies between νA and $l^\pm A$?



- The neutrino fit does not describe the DY and $l^\pm A$ data
- Can a global fit combining the two data sets help?
- DY, $l^\pm A$ (708 data points) with νA (3134 data points)
- Use different weights to make up for data imbalance

Fits to $l^\pm A$, DY and νA

- Introduce a weight parameter, w
- $\chi^2 = \sum_{l^\pm A \text{ data}} \chi_i^2 + \sum_{\nu A \text{ data}} w \chi_i^2$
- Use a good range of weights: $w = 1/7$, $w = 1/4$, $w = 1/2$ and $w = 1$

Weight	Fit name	l data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nannual	-	-	3134	4192 (1.33)	4192 (1.33)

Fits to $I^\pm A$, DY and νA

- $w = 1/2$ and $w = 1$ - possible compromise?
- Need more rigorous test

Tolerance criterion

Probability distribution for the χ^2 function

$$P_N(\chi^2) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

Determine ξ_{50}^2 and ξ_{90}^2 (ξ_{90}^2) (i.e. $p = 50$, $p = 90$ ($p = 99$)):

$$\int_0^{\xi_p^2} d\chi^2 P_N(\chi^2) = p/100$$

Condition for compatibility of two fits:

The 2nd fit (χ_n^2) should be within the 90% C.L. (99% C.L.) region of the first fit ($\chi_{n,0}^2$)

$$\chi_n^2 < \chi_{n,0}^2 \xi_{90}^2 / \xi_{50}^2 \quad \Leftrightarrow \quad C_{90} \equiv \frac{\Delta\chi^2}{\frac{\chi_{n,0}^2}{\xi_{50}^2} (\xi_{90}^2 - \xi_{50}^2)} < 1$$

see CTEQ'01, PRD65(2001)014012; MSTW'09, EPJC(2009)63,189-285

Tolerance criterion - Individual Data Set

C_{90}

- x-axis - number corresponding to a data set; dashed line corresponds to C_{99}
- Encircled data sets : 1. $I\text{Fe}$ DIS data , 2. νFe & 3. $\bar{\nu}\text{Fe}$ DIS cross-section data (NuTeV)
- With w increasing νA data fit better while $I^{\pm}A$ fit worse
- **There is no compromise fit between $I^{\pm}A$ and νA even at the 99% C.L.**

Summary - νA and $l^\pm A$ Incompatibility

- No single compromise fit between charged lepton and neutrino data [[PRL106\(2011\)122301](#)]
- Different nuclear correction factors preferred
- Implications for free and bound PDFs extraction
- Further experimental measurements and theoretical study needed to explain this behavior

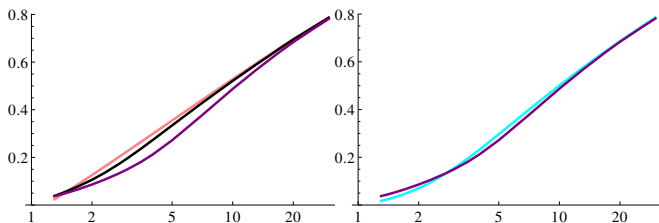
Heavy Quark Structure Functions

Heavy Quark Structure Functions

- Heavy Quark production increasingly important
- Very precise HERA DIS data \rightarrow PDF global analyses
- Increasing precision - experiment & theory \rightarrow NNLO or higher
 - Heavy quark structure functions - 30% - 40% contribution
- Additional heavy mass scale; changing role over full kinematic range:
 - threshold region $m_Q \sim Q$
 - asymptotic region $m_Q \ll Q$
- [arXiv:1203.0282](https://arxiv.org/abs/1203.0282)

- ZM-VFNS - a patchwork of schemes $S^{(n_i, n_r)}$
 $S^{(3,3)} \rightarrow S^{(4,4)} \rightarrow S^{(5,5)} \rightarrow S^{(6,6)}$, with all quarks massless
- ZM-VFNS $_\chi$ is ZM-VFNS with $x \rightarrow \chi$, where $\chi = x(1 + nm^2/Q^2)$
- FFNS - only gluons and light q's in initial state ; no resummation of logs
- ACOT $\sigma_{TOT} = \sigma_{LO} + \{\sigma_{NLO} - \sigma_{SUB}\}$ -
 $m/\mu \rightarrow 0$ ACOT \rightarrow ZM - VFNS & $\mu \sim m$, ACOT \rightarrow FFNS

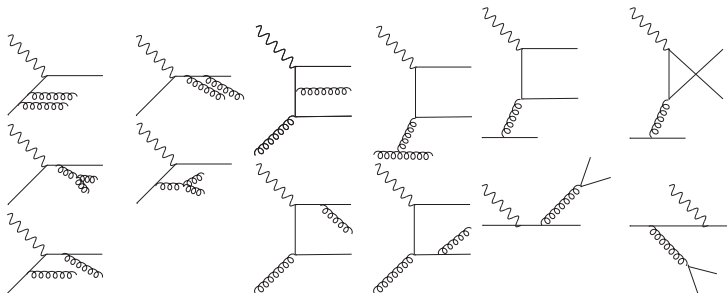
Dynamic & Kinematic mass



$F_2^c(x, Q)$ vs. Q for the NLO ACOT calculation for $x = 10^{-3}$
Left : varying kinematic mass; Right: varying dynamic mass

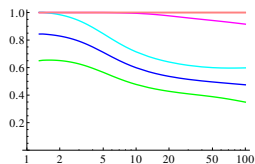
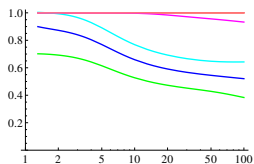
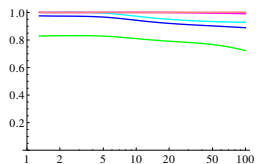
- Dynamic mass not important, while kinematic mass important one
- Include kinematic mass via rescaling $\rightarrow \chi = x(1 + nm^2/Q^2)$
- $\text{ACOT}[\mathcal{O}(\alpha_s^{0+1})] + \text{ZM-VFNS}_\chi[\mathcal{O}(\alpha_s^{2+3})] \simeq \text{ACOT}[\mathcal{O}(\alpha_s^{0+1+2+3})]$

Decomposition

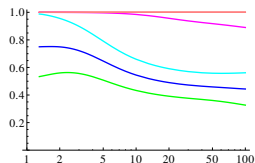
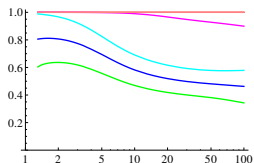
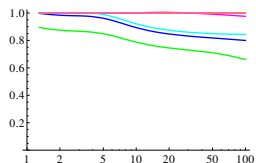


- $F^c = \sum_{i=0}^3 F^{i4} + \sum_{j=1}^3 F^{4j} + F^{44}$
- $F^b = \sum_{i=0}^4 F^{i5} + \sum_{j=1}^4 F^{5j} + F^{55}$
- Apply $x \rightarrow \chi$ for the heavy quarks

Decomposition



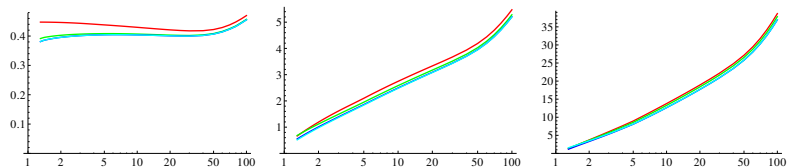
F_2^j/F_2 vs. Q



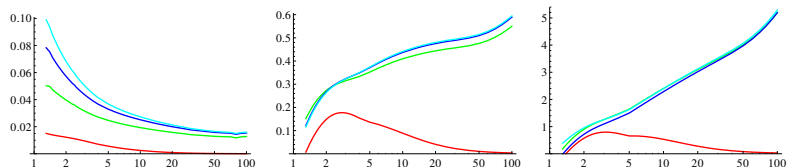
F_L^j/F_L vs. Q

- $\{u, d, s, c, b\} = \{\text{green, blue, cyan, magenta, pink}\}$
- $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right)

Comparison of LO, NLO, NNLO, N³LO



F_2 vs. Q .



F_L vs. Q .

- $\{\text{LO, NLO, NNLO, N}^3\text{LO}\} = \{\text{red, green, blue, cyan}\}$
- $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right)

Conclusions F_L, F_2

- Extended ACOT to N³LO by combining the exact ACOT scheme at NLO with a $\chi(n)$ -rescaling
- Obtained precise predictions for F_L, F_2
- Results for F_2 very stable across full kinematic range
- For F_L higher order terms larger, but still generally small in region probed by HERA