Subtraction method for parton shower to $2 \to 2$ matrix element matching in $k_\perp$-factorisation

M. Deák
Departamento de Física de Partículas, Facultade de Física,
Universidade de Santiago de Compostela, Campus Sur,
15706 Santiago de Compostela, Spain

We present a subtraction method for including next to leading order corrections to a 2 to 2 jet production process in kt-factorisation equivalent to a jet matching procedure in Monte Carlo generators. We study the improvement in soft cut dependance.

1 Motivation

Motivation for this work is to increase the precision of the fixed order calculation of multi-jet in $k_\perp$-factorisation by going beyond the fixed order of the QCD perturbation theory for the matrix element of the hard subprocess. The calculation is done in the context of a Monte Carlo generator, but the ideas have a general application. We push the ideas of[1] to take a step forward to inclusion of higher order corrections to this process.

The goal is to remove the dependence of observable cross sections on the soft cutoff. The soft cutoff is necessary to regularise matrix elements of $2 \to 2$ processes which exhibit collinear singularities [1]. The cutoff is applied in a form of cutoffs on each of the final state transversal momenta $p_{3\perp}, p_{4\perp} > p_{cut\perp}$. In $k_\perp$-factorisation the transversal momenta of the final state are not compensated and the contribution to the observed cross section of one jet depends on $p_{cut\perp}$ since $p_{3\perp}$, although fixed, does not fix $p_{4\perp}$. Consequently $p_{4\perp}$ can run through the integration region near the cutoff $p_{cut\perp}$ and generate big contributions to the observed cross section $\sigma(p_{3\perp})$.

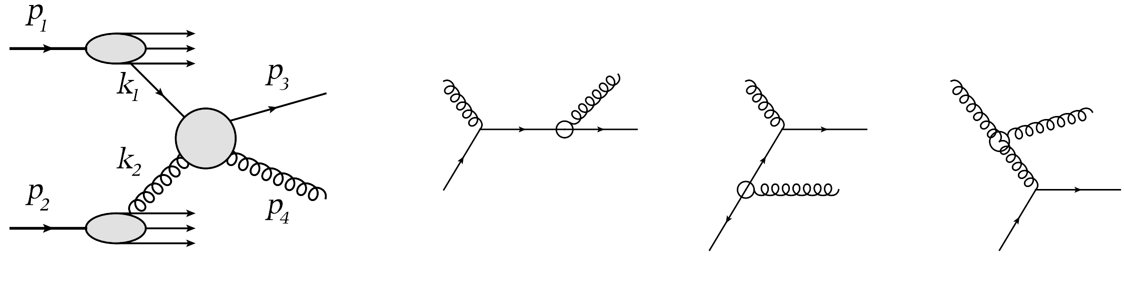

Figure 1: The notation and diagrams for the process.

One can express the above using simple formula. In $k_\perp$-factorisation on the other hand

(notation in the figure 1)

$$\frac{d^2\sigma(\mathbf{p}_3)}{d\mathbf{p}_3^2} = \int_0^1 d\xi_1 \int_0^1 d\xi_2 \int_{p_{cut\perp}} d^2\mathbf{p}_4 \, \frac{d^2\hat{\sigma}(\xi_1,\xi_2,\mathbf{p}_3,\mathbf{p}_4)}{d\mathbf{p}_3^2 d\mathbf{p}_4^2} \times \tilde{H}(\xi_1,\xi_2,\mathbf{p}_3+\mathbf{p}_4,\mu^2(\mathbf{p}_3,\mathbf{p}_4)) \; . \tag{1}$$

In the equations above, H and $\tilde{H}$ are the non-perturbative factors related to the initial state radiation, μ^2 is the hard scale of the subprocess and ξ_1 and ξ_2 are the proton momentum fractions carried by the initial state partons.

A solution of the problem sketched above is regularisation of the hard $2 \to 2$ subprocess by virtual corrections.

A way to regularise the cross section is matrix element-shower jet matching. It has to be applied when including loop corrections or match parton showers with hard subprocesses with multi-jet final states in Monte Carlo generators based on collinear factorisation [2]. The peculiarity of $k_\perp$-factorisation causes that it has to be applied already for $2 \to 2$ processes.

In next chapters we describe regularisation of the cross section by the inclusion of virtual corrections using a Monte Carlo program framework and study of the cut dependence before and after regularisation.

2 Finite terms – Leading order shower versus the exact tree level matrix element

From previous introductory section follows, that calculations we are about to perform are mostly important for cases when we want to look at production of one or more jets using a $2 \to 2$ hard subprocess. For this purpose we are going to concentrate on one simplified example of the partonic subprocess $qg^* \to qg$. Note, that for simplification we made the quark on-shell. The chosen process $qg^* \to qg$ gives a very important contribution to forward jet production – a process important from the point of view of small-x physics [1].

We are going to take the leading order in α_S matrix element of the example process $qg^* \to qg$. Then we calculate the limits corresponding to all the collinear divergencies interpreting them as leading order in α_S parton shower of $2 \to 1$ process-generated $2 \to 2$ matrix elements. This allows us to remove the divergencies present in the full $2 \to 2$ matrix element and replace them with the full parton shower in which the divergencies are regulated by virtual corrections.

3 Virtual corrections

To calculate the virtual corrections we use a Monte Carlo parton shower implementation of the CCFM equation in the Monte Carlo program Cascade [3].

Using a Monte Carlo implementation of the CCFM equation, we can generate extra external legs to a $2 \to 1$ process with above mentioned virtual corrections included. It is important to mention that we are going to calculate $qg^* \to qg$ convoluted with unintegrated parton density functions.

For the diagrams with emitted quark in the initial state we have used Monte Carlo implementation of the one loop CCFM equation and for a quark emitted from the final state leg an implementation of the DGLAP equation [3]. The virtual corrections in the latter cases are calculated in a similar way using corresponding final state or initial state evolution equation.

Using formulas from the previous section we can calculate the difference between the leading α_s approximation of the first emission from the $2 \to 1$ matrix element, figure 1, and the $2 \to 2$ process generated from $2 \to 1$ process by an extra emission in a shower algorithm. By doing the latter we calculate the virtual corrections present in the shower in form of the Sudakov and Non-Sudakov formfactors by

$$d\sigma^{virtual}_{qg^*\to qg} = d\sigma^{shower}_{qg^*\to qg} - d\sigma^{0}_{qg^*\to qg}, \tag{2}$$

where $d\sigma^{shower}_{qg^*\to qg}$ is the differential cross section of the $2 \to 2$ process calculated using the full shower algorithm and $d\sigma^{0}_{qg^*\to qg}$ is the leading α_s part of the emission.

In the next step we add add the difference $d\sigma^{virtual}_{qg^*\to qg}$ to the exact $2 \to 2$ matrix element:

$$d\sigma^{corrected}_{qg^*\to qg} = d\sigma^{exact}_{qg^*\to qg} + d\sigma^{virtual}_{qg^*\to qg}. \tag{3}$$

The differential cross section $d\sigma^{corrected}_{qg^*\to qg}$ includes virtual corrections resummed in Sudakov and Non-Sudakov formfactors present in parton showers.[a]

4 Results of the Monte Carlo implementation

We have studied the dependence of the transversal momenta cross sections of final state quark and gluon on the cut $p_{q\perp}, p_{g\perp} > p_{cut\perp}$. We have chosen values of $p_{cut} = 1$ and 2 GeV and we have plotted ratios of cross sections for these two choices. We have fixed a cut on $|\mathbf{Q}| > p^{\mathbf{Q}}_{cut\perp} = 1\ GeV$ (defined by $\mathbf{Q} = (1-\nu)\mathbf{p}_4 - \nu\mathbf{p}_3$ and $\nu = (p_2 \cdot p_4)/(p_2 \cdot k_1)$ and related to final state collinear singularity).

A general feature of the plots in figures 2 to 3 is that the transverse momentum spectra of the quark and the gluon, calculated using the exact matrix element and the leading order shower, agree very well.

From the plots one can also see that the inclusion of the virtual corrections present in the parton showers reduces the dependence on the regulation cut. The reduction of the dependence on the cut is a consequence of cancelation of this dependence in equation (2), this result shows that the contribution of finite terms is relatively small.

We observe reduction of the dependence on the cutoff around $40-50\%$ in the quark case, figure 3 left, and even bigger reduction of the dependence, $70-80\%$ in the gluon case, figure 3 right.

The steep rise of the gluon transversal momentum spectrum explains the remaining dependence on the cut in the transversal momentum spectrum of the quark and other features of our plots, exactly in the spirit of the equation 1. A small change in cut on $p_{g\perp}$ produces a big change in the integral 1 which causes a shift in the observed cross section. This does not happen in the gluon case since the $p_{q\perp}$ spectrum exhibits a turnover at larger transversal momentum value.

5 Summary and Outlook

We have presented a simple prescription based on subtraction of cross sections for including loop corrections and IR regularisation of a $2 \to 2$ process in $k_\perp$-factorisation. We have also shown a successful application of the prescription to the process $qg^* \to qg$ important for forward jet production. We have studied transversal momentum cross sections for 2 produced jets with different cuts applied.

We observe a decrease in the dependence of the observed transversal momentum cross sections on the transversal momentum cutoff. In the case of the quark transversal momentum the improvement is around $40-50\%$. In the case of the gluon momentum even around $70-80\%$.

[a] Other way how to see equation (3) is by defining $d\sigma^{finite}_{qg^*\to qg} = d\sigma^{exact}_{qg^*\to qg} - d\sigma^{0}_{qg^*\to qg}$ and rewriting $d\sigma^{corrected}_{qg^*\to qg} = d\sigma^{shower}_{qg^*\to qg} + d\sigma^{finite}_{qg^*\to qg}$.

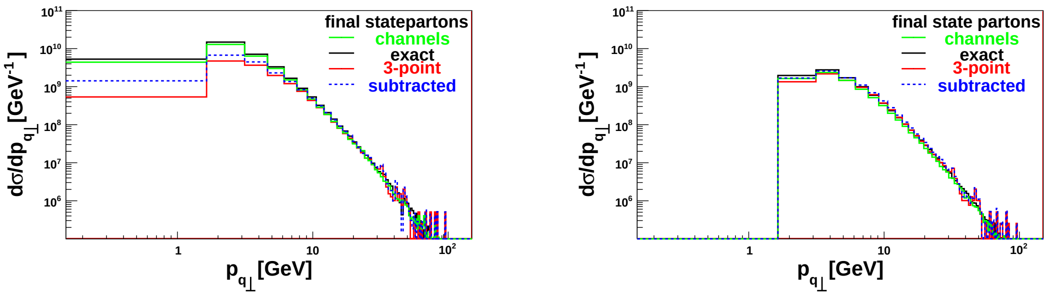

Figure 2: Transversal momentum of the quark $p_{q\perp}$ when cuts on $p_{q,g\perp} > 1\ GeV$. The dashed line labeled subtracted shows the result of equations (2) and (3).

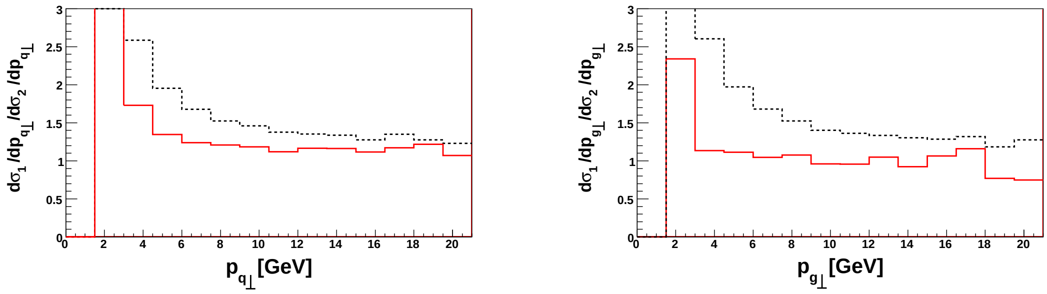

Figure 3: Ratios of the cross section of the quark transversal momentum $p_{q\perp}$ when cuts on $p_{q,g\perp} > 1\ GeV$ and $2\ GeV$. Red being the result of subtraction and black the full $2 \to 2$ matrix element. The dashed line labeled subtracted shows the result of equations (2) and (3).

A side observation of the calculation is that the contribution of the finite terms of the matrix element to the transversal momentum spectra is relatively small.

Next steps should involve application of the method to forward jet phenomenology. For this purpose extension to more complicated matrix elements will be necessary. For practical reasons and to remove the cutoff dependence completely it will be also good to formulate a prescription for jet matching on an event-by-event basis and implement it in a Monte Carlo generator.

6 Acknowledgements

1. M. Deak, F. Hautmann, H. Jung, and K. Kutak. Forward Jet Production at the Large Hadron Collider. *JHEP*, 09:121, 2009.
   M. Deak, F. Hautmann, H. Jung, and K. Kutak. Forward-Central Jet Correlations at the Large Hadron Collider. 2010.
2. S. Catani, F. Krauss, R. Kuhn, and B. R. Webber. QCD Matrix Elements + Parton Showers. *JHEP*, 11:063, 2001.
3. H. Jung, S. Baranov, M. Deak, A. Grebenyuk, F. Hautmann, et al. The CCFM Monte Carlo generator CASCADE version 2.2.03. *Eur.Phys.J.*, C70:1237–1249, 2010.