$e^+e^- \to J/\Psi + \eta_c$ in Bethe-Salpeter framework

LI Shi-Yuan
Department of Physics, Shandong University, Jinan 250100, P.R. China

It is shown that the off-shell states as well as the relative movement of the $c\bar{c}$ in bound states are important for this exclusive production process.

1 Introduction

Heavy quarkonium physics has been the traditional arena of Quantum Chromodynamics (QCD) since J/Ψ was discovered, in both perturbative (PT) and non-perturbative (NPT) aspects. The heavy quark mass provides a hard scale for PT QCD calculations. The mass term of fermions in the QCD Lagrangian is irrelevant to the colour symmetry, so that the confinement and NPT QCD mechanism for heavy and light quarks could be similar. These facts make the heavy quark a good tool in investigating the unsolved NPT QCD. A further simplification is expected from the large mass of heavy quark, that it may be non-relativistic (NR) in the bound states. As a matter of fact, not only the relative momentum between heavy quarks in the bound states are considered small, but also these quarks are treated as almost on shell. The off shell states and the creation/annihilation of the heavy quarks are not taken into account in the static bound state. This is justified for the *rest* quarkonia, as investigating their decay widths. In these processes, the largest energy scale is the quarkonium mass M. However, in the hard production processes of heavy quarkonium at high energies, the hard scale are much larger than M. If there is no well justified factorization formula[a] for a certain process, the NR description of the bound states could not be straightforward applicable. In this case, more general framework, which is relativistic and robust to introduce enough number of parameters to describe the bound system, especially the off shell states of the heavy quarks, is needed. The Bethe-Salpeter (BS) wave function framework is one of the good choices. If various relativistic effects are found to be small or could be factorized from the static bound state, the BS framework will naturally leads to the NR descriptions.

In this paper, the cross section of the exclusive production process $e^+e^- \to J/\Psi + \eta_c$ at B factories [1,2,3] is studied in the BS framework. The various relativistic effects, especially the off-shell heavy quark states and large relative momentum are taken into account.

The double charm pair production process in B factory energies is of special significance. These four (anti)charms can be respectively grouped into two colour-singlet pairs, hence the colour-octet mechanism never plays important rôle because of the relatively much smaller colour-octet matrix elements. One can concentrate on the effects of relative movement and heavy

---

[a] factorization here means that all the effects of the off shell states and creation/annihilation of the heavy quarks, as well as the large relative momentum between them, are well separated into the hard part which is calculable via PT QCD, so that the bound state can be described by NR effective theories, such as [4].

quark off shell states without considering the indefinte colour-octet matrix element paremeters. It has been found that the decay width, as well as the energy distribution of the J/Ψ in $\Upsilon \rightarrow J/\Psi + X$ process [5,6] can be easily understood by considering such kind of process [7,8]. It has also been suggested [9] double charm pair production is helpful to analyze the Tevatron polarization 'paradox', which is refreshed by Tevatron RUNII [10] recently. When the discrepancy between the data and the lowest order NRQCD calculation for the exclusive process $e^+e^- \rightarrow J/\Psi + \eta_c$ was presented, Many groups recognized the importance of the relativistic corrections, besides the higher order PT QCD corrections. It has been generally adopted that exclusive process could be more sensitive to the inner movements and more difficult to factorize.

The ways of incorporating the relativistic corrections can be grouped into two: One is in the NRQCD framework, the other is to employ various relativistic wave functions of J/Ψ and η_c. Works in the NRQCD frameworks [11,12] show that the relativistic correction is important, at the same time the $O(\alpha_s^3)$ PQCD corrections are found also very large. The large higher order corrections [13,14] indicate the requirement of all order summations. For the to-date review of works in this framework, one refers to [15]. Works in the framework of light-cone wave functions [16,17,18,19,20], employ the same hard partonic process as the lowest order NRQCD approach. When the scale parameter in the wave function is large, the momentum fraction difference $|x_1 - x_2|$ is of $O(1)$, and this approach can explain the data well. In both the above frameworks, the charm quarks in the bound states are treated as almost on shell, i.e., factorization is assumed.

On the other hand, the bound states can be described by BS wave functions [21,22], with the heavy quark limit (HQL) employed and the partonic process the same as that of the lowest order NRQCD approach. But the results [21,22] are consistent with data, i.e., much larger than that of the NRQCD. The HQL generally is considered as to make NR description of the quarks in the bound states valid. This fact implies that different approaches for 'NR limit' (HQL vs. NRQCD) could lead to completely different results. There could be effects of the full BS approach not incorporated in the NRQCD framework (but kept in HQL) which enhance the cross section. Such experiences have been seen in history. For the electron in the hydrogen atom, the most important 'relativistic effect' comes from the generators of the little group of the Lorentz group, i.e., spin. Such a quantum number can only be naturally deduced from the relativistic wave function/equation of the electron, i.e., the Dirac spinor/equation. The Sommerfeld relativistic corrections to the Bohr theory can never incorporate this.

Of course by the mention of this history we do not imply NRQCD can not properly incorporate the spin effect of the bound state, but want to point out that it could be better to start from the completely relativistic framework, i.e., the full BS wave function *without HQL*, to investigate the relativistic effects in bound state, namely the off-shell states and the large relative movement of $c\bar{c}$. The off-shell states are never covered in all the above approaches, while the large relativistic corrections in NRQCD and wide $|x_1 - x_2|$ spectrum in light cone approaches indicate that the large relative movement is important. Once taking such a step, the Feynman diagram in Figure 1 ($O(\alpha_s^0)$) naturally comes up. These three charm quark lines can never be all on shell at the same time that the four-momenta are all conserved for the two quark-hadron vertices (requiring at least one of the charm propagators off-shell at the order of $\sqrt{s}$). Hence all the above mentioned approaches, which neglect the consideration of off-shell states of $c\bar{c}$, set this diagram to be zero by hand. On the contrary, taking into account the the off-shell state and the large relative movement of $c\bar{c}$ in a self-consistent BS wave function framework, one has to make clear the contribution of this diagram. Its contribution is not necessarily the leading, but depending on how large the effects of the off-shell states and relative movements. So we need the concrete calculations. Details of Section 2,3 are referred to the long write up.

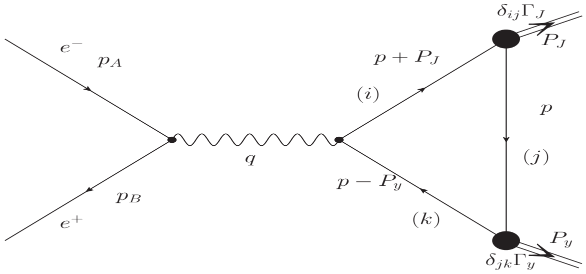

Figure 1: One of two Feynman diagrams.

2 The BS WF framework for the calculation

The BS wave function is

$$\chi(P,q)=\int\frac{d^4x}{(2\pi)^4}e^{-iqx}\frac{1}{\sqrt{3}}\delta_{ij}<0\mid T\psi^i(\frac{x}{2})\bar{\psi}^j(-\frac{x}{2})\mid B>=:S_F(p_1)\Gamma(P,q)S_F(-p_2) \qquad (1)$$

where $p_1=\frac{P}{2}+q,\ p_2=\frac{P}{2}-q,\ P=p_1+p_2,\ q=\frac{1}{2}(p_1-p_2)$. Eq. 1 defines the BS vertex for the coupling of the bound state particle with the composite particles. $S_F(p)$ denotes the quark propagator. For incorporation of the information of the bound states from their decay processes, we employ the Covariant Instantaneous Ansatz (CIA) framework as [23,24,25] in calculations.

3 The calculation procedure and the result

The invariant amplitude is proportional to $\int\frac{d^4p}{(2\pi)^4}\frac{D(\hat{q}_J)D(\hat{q}_y)\phi(\hat{q}_J)\phi(\hat{q}_y)}{((p-P_y)^2-m^2)(p^2-m^2)((p+P_J)^2-m^2)}$, lepton current l_μ, and $h^\mu=4m\varepsilon_{\mu\alpha\beta\sigma}P_y^\alpha\epsilon_\lambda^\beta P_J^\sigma$. The EM (U(1)) gauge invariance is explicit. When replacing the four-vector l_μ by the total four-momentum of the virtual photon, which equals to $(P_J+P_y)_\mu$, the amplitude is zero. The reason is $(P_J+P_y)_\mu h^\mu=0$, because of the totally antisymmetric tensor $\varepsilon_{\mu\alpha\beta\sigma}$. We first integrate over the poles and then do the numerical integral for the remaining 3-dimensional momentum. This provides a calculation procedure other than the Feynman parameters when the vertices are very complex. The cross section as function of β is shown in the Table. Here β is parameter for the distribution of $c\bar{c}$ relative momentum.

4 summary and discussion

The results show that the off shell states of $c\bar{c}$ in bound states can contribute significantly in this production process, when the relative movement is large enough (but because of the normalization factor, the results are static even β very large); no other framework can take into account this effect. Higher order calculations are straightforward, but the treating of the propagator and vertices need to be investigated.

Acknowledgments

The author thanks Shashank and Ming for Collaboration. This work is partially supported by NSFC, NSF of Shandong Province, and funds of Shandong University based on MoE.

References

1. K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **89** (2002) 142001.
2. K. Abe *et al.* [Belle Collaboration], Phys. Rev. D **70** (2004) 071102.

β	I^2	N^2	I^2/N^2	$\sigma(fb)$
$1/2$	3.39×10^{-14}	1.29×10^{6}	2.63×10^{-20}	1.4×10^{-6}
$1/\sqrt{2}$	7.90×10^{-9}	1.47×10^{7}	5.37×10^{-16}	2.9×10^{-2}
1	1.36×10^{-6}	1.52×10^{8}	8.95×10^{-15}	4.9×10^{-1}
1.15	8.00×10^{-6}	2.83×10^{8}	2.83×10^{-14}	1.5
1.3	9.80×10^{-6}	8.06×10^{7}	1.22×10^{-13}	6.6
$\sqrt{2}$	9.80×10^{-5}	1.08×10^{9}	9.09×10^{-14}	5.0
1.5	1.00×10^{-4}	3.36×10^{9}	2.97×10^{-14}	1.6
1.6	3.59×10^{-4}	8.85×10^{9}	4.05×10^{-14}	2.2
1.7	1.08×10^{-3}	1.89×10^{10}	5.73×10^{-14}	3.1
1.8	2.95×10^{-3}	3.40×10^{10}	8.69×10^{-14}	4.7
1.9	7.32×10^{-3}	4.96×10^{10}	1.48×10^{-13}	8.1
2	1.05×10^{-2}	4.72×10^{10}	2.23×10^{-13}	12.2
2.1	3.41×10^{-2}	2.50×10^{10}	1.37×10^{-12}	74.6
2.25	8.59×10^{-2}	5.73×10^{11}	1.50×10^{-13}	8.2
2.5	1.81×10^{-1}	6.59×10^{12}	2.75×10^{-14}	1.5
3	1.41	2.08×10^{14}	6.81×10^{-15}	0.4

3. B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **72** (2005) 031101.
4. G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125.
5. R. Fulton *et al.* [CLEO Collaboration], Phys. Lett. B **224** (1989) 445.
6. R. A. Briere *et al.* [CLEO Collaboration], Phys. Rev. D **70** (2004) 072001.
7. S. Y. Li, Q. B. Xie and Q. Wang, Phys. Lett. B **482** (2000) 65.
8. W. Han and S. Y. Li, Phys. Rev. D **74** (2006) 117502.
9. Q. B. Xie and S. Y. Li, *Prepared for 3rd Joint Meeting of Chinese Physicists Worldwide: Role of Physics in the New Millenium: Research, Education and Society, Hong Kong, China, 31 Jul - 4 Aug 2000*
10. A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **99** (2007) 132001.
11. G. T. Bodwin, J. Lee and C. Yu, Phys.Rev. D77 (2008) 094018.
12. Z. G. He, Y. Fan and K. T. Chao, Phys. Rev. D **75** (2007) 074011.
13. B. Gong and J. X. Wang, Phys.Rev. D77 (2008) 054028.
14. Y. J. Zhang, Y. j. Gao and K. T. Chao, Phys. Rev. Lett. **96** (2006) 092001.
15. H. R. Dong, F. Feng and Y. Jia, arXiv:1204.4128 [hep-ph].
16. J. P. Ma and Z. G. Si, Phys. Rev. D **70** (2004) 074007.
17. A. E. Bondar and V. L. Chernyak, Phys. Lett. B **612** (2005) 215.
18. V. V. Braguta, A. K. Likhoded and A. V. Luchinsky, Phys. Rev. D **72** (2005) 074019.
19. V. V. Braguta, A. K. Likhoded and A. V. Luchinsky, Phys. Lett. B **646** (2007) 80.
20. G. T. Bodwin, D. Kang and J. Lee, Phys. Rev. D **74** (2006) 114028.
21. E. Mengesha and S. Bhatnagar, Int.J.Mod.Phys. E20 (2011) 2521-2533.
22. X. H. Guo, H. W. Ke, X. Q. Li and X. H. Wu, arXiv:0804.0949 [hep-ph].
23. S. Bhatnagar and S. Y. Li, arXiv:hep-ph/0612084.
24. S. Bhatnagar and S. Y. Li, J. Phys. G **32** (2006) 949.
25. S. Bhatnagar, S. Y. Li and J. Mahecha, Int.J.Mod.Phys. E20 (2011) 1437-1454.