HEAVY QUARK STRUCTURE FUNCTIONS IN THE ACOT SCHEME

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We compute the structure functions F_2 and F_L in the ACOT scheme for heavy quark production. We use the complete ACOT results to NLO, and make use of the $\overline{MS}$ massless results at NNLO and N^{3}LO to estimate the higher order mass-dependent corrections. The dominant heavy quark mass effects can be taken into account using massless Wilson coefficients together with an appropriate rescaling prescription. Combining the exact NLO ACOT scheme with these expressions should provide a good approximation to the full calculation in the ACOT scheme at NNLO and N^{3}LO.These proceedings are based on Ref. [1], and further details can be found therein.

1 Introduction

The production of heavy quarks in high energy processes has become an increasingly important subject of study both theoretically and experimentally. The theory of heavy quark production in perturbative Quantum Chromodynamics (pQCD) is more challenging than that of light parton (jet) production because of the new physics issues brought about by the additional heavy quark mass scale. The correct theory must properly take into account the changing role of the heavy quark over the full kinematic range of the relevant process from the threshold region (where the quark behaves like a typical "heavy particle") to the asymptotic region (where the same quark behaves effectively like a parton, similar to the well known light quarks $\{u, d, s\}$).

With the ever-increasing precision of experimental data and the progression of theoretical calculations and parton distribution function (PDF) evolution to next-to-next-to-leading order (NNLO) of QCD there is a clear need to formulate and also implement the heavy quark schemes at this order and beyond. The most important case is arguably the heavy quark treatment in inclusive deep-inelastic scattering (DIS) since the very precise HERA data for DIS structure functions and cross sections form the backbone of any modern global analysis of PDFs. Here, the heavy quarks contribute up to 30% or 40% to the structure functions at small momentum fractions x. Extending the heavy quark schemes to higher orders is therefore necessary for extracting precise PDFs and hence for precise predictions of observables at the LHC. However, we would like to also stress the theoretical importance of having a general pQCD framework including heavy quarks which is valid to all orders in perturbation theory over a wide range of hard energy scales and which is also applicable to other observables than inclusive DIS in a straightforward manner.

An example, where higher order corrections are particularly important is the structure function F_L in DIS. The leading order ($\mathcal{O}(\alpha_S^0)$) contribution to this structure function vanishes for massless quarks due to helicity conservation (Callan-Gross relation). This has several consequences: 1) F_L is useful for constraining the gluon PDF via the dominant subprocess $\gamma^* g \to q\bar{q}$.

The heavy quark mass effects of order $\mathcal{O}(\frac{m^2}{Q^2})$ are relatively more pronounced.[a] 3) Since the first non-vanishing contribution to F_L is next-to-leading order (up to mass effects), the NNLO and N^{3}LO corrections are more important than for F_2. The purpose of this study is to calculate the leading twist neutral current DIS structure functions F_2 and F_L in the ACOT factorization scheme up to order $\mathcal{O}(\alpha_S^3)$ (N^{3}LO) and to estimate the error due to approximating the heavy quark mass terms $\mathcal{O}(\alpha_S^2 \times \frac{m^2}{Q^2})$ and $\mathcal{O}(\alpha_S^3 \times \frac{m^2}{Q^2})$ in the higher order corrections.

2 ACOT Scheme

The ACOT renormalization scheme [4,3] provides a mechanism to incorporate the heavy quark mass into the theoretical calculation of heavy quark production both kinematically and dynamically. In 1998 Collins [5] extended the factorization theorem to address the case of heavy quarks; this work provided the theoretical foundation that allows us to reliably compute heavy quark processes throughout the full kinematic realm. The key ingredient provided by the ACOT scheme is the subtraction term (SUB) which removes the "double counting" arising from the regions of phase space where the LO and NLO contributions overlap. Specifically, at NLO order, we can express the total result as a sum of

$$\sigma_{TOT} = \sigma_{LO} + \{\sigma_{NLO} - \sigma_{SUB}\} \tag{1}$$

where the subtraction term for the gluon-initiated processes is

$$\sigma_{SUB} = f_g \otimes \tilde{P}_{g\to Q} \otimes \sigma_{QV\to Q}. \tag{2}$$

σ_{SUB} represents a gluon emitted from a proton (f_g) which undergoes a collinear splitting to a heavy quark ($\tilde{P}_{g\to Q}$) convoluted with the LO quark-boson scattering $\sigma_{QV\to Q}$. Here, $\tilde{P}_{g\to Q}(x,\mu) = \frac{\alpha_s}{2\pi}\ln(\mu^2/m^2)\, P_{g\to Q}(x)$ where $P_{g\to Q}(x)$ is the usual $\overline{MS}$ splitting kernel, m is the quark mass and μ is the renormalization scale which we typically choose to be $\mu = Q$. An important feature of the ACOT scheme is that it reduces to the appropriate limit both as $m \to 0$ and $m \to \infty$ as we illustrate below. Specifically, in the limit where the quark Q is relatively heavy compared to the characteristic energy scale ($\mu \lesssim m$), we find $\sigma_{LO} \sim \sigma_{SUB}$ such that $\sigma_{TOT} \sim \sigma_{NLO}$. In this limit, the ACOT result naturally reduces to the Fixed-Flavor-Number-Scheme (FFNS) result. In the FFNS, the heavy quark is treated as being extrinsic to the hadron, and there is no corresponding heavy quark PDF ($f_Q \sim 0$); thus $\sigma_{LO} \sim 0$. We also have $\sigma_{SUB} \sim 0$ because this is proportional to $\ln(\mu^2/m^2)$. Thus, when the quark Q is heavy relative to the characteristic energy scale μ, the ACOT result reduces to $\sigma_{TOT} \sim \sigma_{NLO}$. Conversely, in the limit where the quark Q is relatively light compared to the characteristic energy scale ($\mu \gtrsim m$), we find that σ_{LO} yields the dominant part of the result, and the "formal" NLO $\mathcal{O}(\alpha_S)$ contribution $\{\sigma_{NLO} - \sigma_{SUB}\}$ is an $\mathcal{O}(\alpha_S)$ correction. In this limit, the ACOT result will reduce to the $\overline{MS}$ Zero-Mass Variable-Flavor-Number-Scheme (ZM-VFNS) limit exactly without any finite renormalizations. The quark mass m no longer plays any dynamical role and purely serves as a regulator. The σ_{NLO} term diverges due to the internal exchange of the quark Q, and this singularity is canceled by σ_{SUB}.

In the limit $Q^2 \gg m^2$ the mass simply plays the role of a regulator. In contrast, for $Q^2 \sim m^2$ the value of the mass is of consequence for the physics. The mass can enter dynamically in the hard-scattering matrix element, and kinematically in the phase space of the process. As is demonstrated in Ref. [1] for the processes of interest the primary role of the mass is kinematic and not dynamic. It was this idea which was behind the original slow-rescaling prescription of

[a] Similar considerations also hold for target mass corrections (TMC) and higher twist terms. We focus here mainly on the kinematic region $x < 0.1$ where TMC are small [2]. An inclusion of higher twist terms is beyond the scope of this study.

Ref. [6] which considered DIS charm production (e.g., $\gamma c \to c$) introducing the shift $x \to \chi = x[1 + (m_c/Q)^2]$. This prescription accounted for the charm quark mass by effectively reducing the phase space for the final state by an amount proportional to $(m_c/Q)^2$.

This idea was extended in the χ-scheme by realizing that (in most cases) in addition to the observed final-state charm quark, there is also an anti-charm quark in the beam fragments since all the charm quarks are ultimately produced by gluon splitting ($g \to c\bar{c}$) into a charm pair. For this case the scaling variable becomes $\chi = x[1 + (2m_c/Q)^2]$. This rescaling is implemented in the ACOT$_\chi$ scheme [7,8]. As mentioned above, the dominant mass effects are those coming from the phase space, i.e. kinematic mass, these can be taken into account via a generalized slow-rescaling $\chi(n)$-prescription. Assuming that a similar relation remains true at higher orders one can construct the following approximation to the full ACOT result up to N^{3}LO ($\mathcal{O}(\alpha_S^3)$):

$$\text{ACOT}[\mathcal{O}(\alpha_S^{0+1+2+3})] \simeq \text{ACOT}[\mathcal{O}(\alpha_S^{0+1})] + \text{ZMVFNS}_\chi[\mathcal{O}(\alpha_S^{2+3})]. \tag{3}$$

Here, the massless Wilson coefficients at $\mathcal{O}(\alpha\, \alpha_S^2)$ and $\mathcal{O}(\alpha\, \alpha_S^3)$ are substituted for the Wilson coefficients in the ACOT scheme as the corresponding massive coefficients have not yet been computed.

3 Results

In Figures 1a) and 1b) we display the fractional contributions for the final-state quarks (j) to the structure functions F_2 and F_L, respectively, for selected x values as a function of Q; here we have used $n = 2$ scaling. Reading from the bottom, we have the cumulative contributions from the $\{u, d, s, c, b\}$. We observe that for large x and low Q the heavy flavor contributions are minimal, but these can grow quickly as we move to smaller x and larger Q.

In Figure 2a) we display the results for F_2 vs. Q computed at various orders. For large x (c.f. $x = 0.1$) we find the perturbative calculation is particularly stable; we see that the LO result is within 20% of the others at small Q, and within 5% at large Q. The NLO is within 2% at small Q, and indistinguishable from the NNLO and N^{3}LO for Q values above ~ 10 GeV. The NNLO and N^{3}LO results are essentially identical throughout the kinematic range. For smaller x values (10^{-3}, 10^{-5}) the contribution of the higher order terms increases. Here, the NNLO and N^{3}LO coincide for Q values above ~ 5 GeV, but the NLO result can differ by $\sim 5\%$.

In Figure 2b) we display the results for F_L vs. Q computed at various orders. In contrast to F_2, we find the NLO corrections are large for F_L; this is because the LO F_L contribution (which violates the Callan-Gross relation) is suppressed by (m^2/Q^2) compared to the dominant gluon contributions which enter at NLO. Consequently, we observe (as expected) that the LO result for F_L receives large contributions from the higher order terms. Essentially, the NLO is the first non-trivial order for F_L, and the subsequent contributions then converge. For example, at large x (c.f. $x = 0.1$) for $Q \sim 10$ GeV we find the NLO result yields ~ 60 to 80% of the total, the NNLO is a $\sim 20\%$ correction, and the N^{3}LO is a $\sim 10\%$ correction. For lower x values (10^{-3}, 10^{-5}) the convergence of the perturbative series improves, and the NLO results is within $\sim 10\%$ of the N^{3}LO result. Curiously, for $x = 10^{-5}$ the NNLO and N^{3}LO roughly compensate each other so that the NLO and the N^{3}LO match quite closely for $Q \geq 2$ GeV.

4 Conclusions

The results of this study form the basis for using the ACOT scheme in NNLO global analyses and for future comparisons with precision data for DIS structure functions.

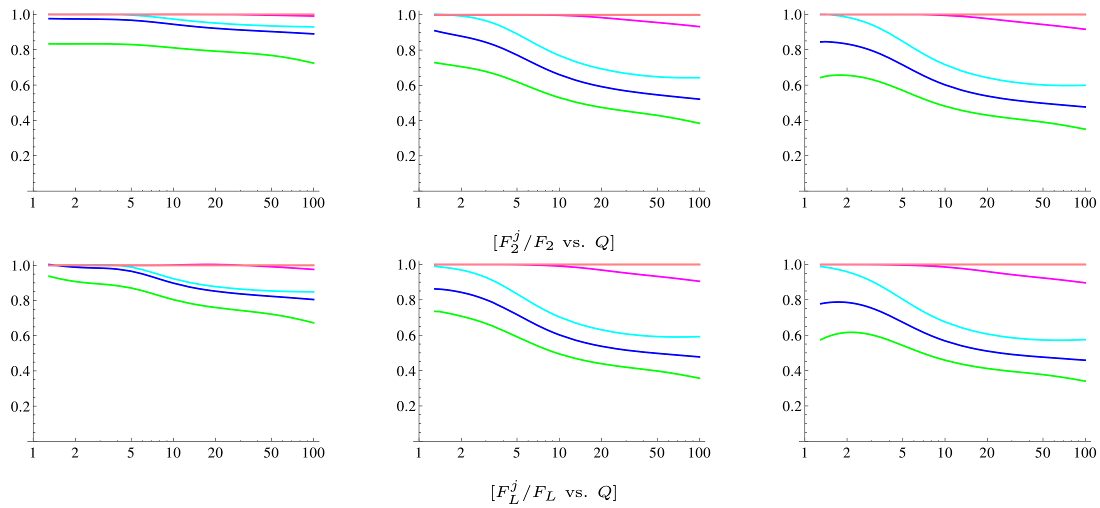

Figure 1: Fractional contribution for each quark flavor to $F^j_{2,L}/F_{2,L}$ vs. Q at N^{3}LO for fixed $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right). Results are displayed for $n = 2$ scaling. Reading from the bottom, we have the cumulative contributions from the $\{u, d, s, c, b\}$ (green, blue, cyan, magenta, pink).

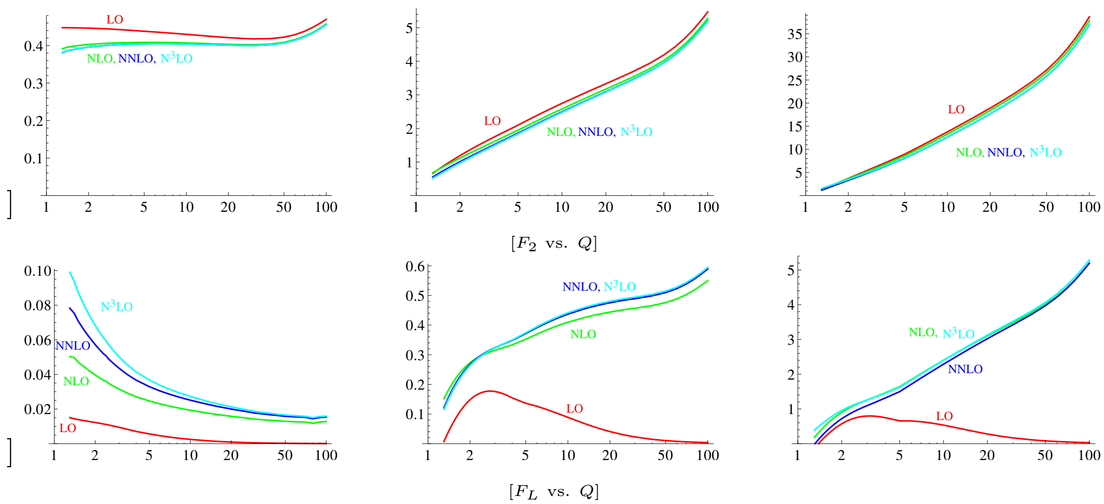

Figure 2: $F_{2,L}$ vs. Q at {LO, NLO, NNLO, N^{3}LO} (red, green, blue, cyan) for fixed $x = \{10^{-1}, 10^{-3}, 10^{-5}\}$ (left to right) for $n = 2$ scaling.

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