Hadronic Cross Section Measurements with ISR and the Implications on $g_{\mu} - 2$

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on behalf of the B-Factories

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Outline

1. Introduction
   - Optical theorem
   - Experimental setup at \textit{BaBAR}

2. Recent Results
   - $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$
   - $e^+e^- \rightarrow K^+K^-$

3. Summary
The contributions to $a_\mu$ and its uncertainty

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

$$(g_\mu - 2)/2 =: a_\mu = 0 \quad \text{(Dirac)}$$

$$\vec{\mu} = \vec{g} \frac{e}{2m} \vec{s}$$

$$(g_\mu - 2)/2 =: a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}}$$
The contributions to $a_\mu$ and its uncertainty

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$$(g_\mu - 2)/2 =: a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadronic}$$

![Diagram of muon scattering process]

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

$$(g_\mu - 2)/2 =: a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}}$$

\begin{align*}
\text{QED} & \\
\text{weak}
\end{align*}

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

$$\frac{(g_\mu - 2)}{2} = a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{weak}} + a_{\mu}^{\text{hadronic}}$$

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<table>
<thead>
<tr>
<th>Interaction</th>
<th>Contribution [$\cdot10^{-11}$]</th>
<th>Uncertainty [$\cdot10^{-11}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>116 584 718.951</td>
<td>0.080</td>
</tr>
<tr>
<td>weak</td>
<td>154</td>
<td>2</td>
</tr>
<tr>
<td>hadronic</td>
<td>6930</td>
<td>49</td>
</tr>
<tr>
<td>BNL E821</td>
<td>116 592 089</td>
<td>63</td>
</tr>
</tbody>
</table>

Deviation: 287

$\rightarrow 3.6\sigma$

Just a fluctuation?

$3.6\sigma$ effect, thus reduction of uncertainties necessary!
The optical theorem

\[
a^\text{had}_\mu = \frac{1}{4\pi^3} \int_0^\infty K(s)\sigma^\text{had}(s)ds, \quad K(s) \propto \frac{1}{s}
\]
The optical theorem

\[ a_{\mu}^{\text{had}} = \frac{1}{4\pi^3} \int_0^\infty K(s)\sigma_{\text{had}}(s)ds, \quad K(s) \propto \frac{1}{s} \]

\[ \sigma_{\text{had}} \] (left) from Nuovo Cim., C034S1:31-40, 2011 [5] and relative contributions to \( a_{\mu}^{\text{had}} \) (right).
The **BABAR** Experiment

**Experimental specifications**

Energy: $\sqrt{s} = 10.6\text{ GeV} \quad (E_{e^-} = 9.0\text{ GeV}, E_{e^+} = 3.1\text{ GeV})$,

Luminosity: $\mathcal{L} = 454\text{ fb}^{-1} \quad (\Upsilon(4S))$
**Initial State Radiation (ISR) events at BABAR**

**ISR selection**

- Detected high energy photon: $E_\gamma > 3\text{GeV}$  
  $\rightarrow$ defines $E_{CM}$ & provides strong background rejection

- Event topology: $\gamma_{ISR}$ back-to-back to hadrons  
  $\rightarrow$ high acceptance

- Kinematic fit including $\gamma_{ISR}$  
  $\rightarrow$ very good energy resolution (4 – 15MeV)

- $e^+e^-$-boost into the laboratory reference frame  
  $\rightarrow$ high efficiency at production threshold of hadronic system

- Continuous measurement from threshold to $\sim 4.5\text{GeV}$  
  $\rightarrow$ provides common, consistent systematic uncertainties
Most important channels

Most important channels

Left panel: Relative contributions to $a_{\mu}^{\text{had}}$ (from Nuovo Cim., C034S1:31-40, 2011 [5]).
Most important channels

Left panel: Relative contributions to $\delta a^\text{had}_{\mu}$ (from Nuovo Cim., C034S1:31-40, 2011 [5]).
\[ e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^- \]

Phys. Rev. D\textbf{85}, 112009 (2012), based on 454 fb\(^{-1}\)

supersedes our previous publication,

based on 89 fb\(^{-1}\) of the data:

Internal structure in various $E_{CM}$ energy slices

First column (4 entries/event):
$a_1(1260)$
Recent Results

\[ e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^- \]

Internal structure in various \( E_{CM} \) energy slices

First column (4 entries/event):

\( a_1(1260) \)

Second column (4 entries/event):

- strong \( \rho^0 \) contribution
- e.g. for \( M_{4\pi} > 1.4 \text{ GeV}/c^2 \):
- \( 1/4th \) of entries in \( \rho^0 \) peak
- \( \rho^0 \rho^0 \) is forbidden
- \( \rightarrow \rho^0 \) in each event!
Internal structure in various $E_{CM}$ energy slices

First column (4 entries/event):
$a_1(1260)$

Second column (4 entries/event):
strong $\rho^0$ contribution
e.g. for $M_{4\pi} > 1.4 \text{ GeV/}c^2$:
$1/4$th of entries in $\rho^0$ peak
$\rho^0 \rho^0$ is forbidden
$\rightarrow \rho^0$ in each event!

Third column (1 entry/event):
$2\pi$ lie within $\rho^0$ mass
$\rightarrow$ other $\pi^+\pi^-$'s mass plotted

$f_2(1270), a_1(1260), f_0(980)\ldots$?
$\rightarrow$ Partial Wave Analysis needed
Cross section for $e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

- **Systematic uncertainties**
  - 2.4% in peak region (1.1-2.8 GeV)
  - 11% (0.6-1.1 GeV)
  - 4% (2.8-4.0 GeV)

- $J/\psi$ visible
Recent Results

**Cross section for** $e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

- **Systematic uncertainties**
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- **$J/\psi$ visible**

- **< 1.4 GeV:** agreement with previous BABAR results, SND and CMD-2 data
- **> 1.4 GeV:** highest precision (DM2, 20%)
$e^+e^- \rightarrow K^+K^-$

PRELIMINARY
about to be submitted to PRD, based on $232\ \text{fb}^{-1}$
Cross section $\sigma(e^+e^- \rightarrow K^+K^-)$

Uncertainty of 0.8% near $\phi$ peak!

→ luminosity from $\mu\mu\gamma$ and $K$-ID
A phenomenological fit to the form factor
Comparison to other experiments

Recent Results

$e^+e^- \rightarrow K^+K^-$

PRELIMINARY

Normalization difference:

- $2.4 \sigma_{\text{syst}}$ CMD2
- $7.9 \sigma_{\text{syst}}$ BABAR

Stat. only

$\phi$ mass values compatible within calibration uncertainties

Normalization difference:

- $1.4 \sigma_{\text{syst}}$ SND
- $14.6 \sigma_{\text{syst}}$ BABAR

Stat. only

PRELIMINARY

PRELIMINARY

I. Logashenko Tau08

$\sigma (\text{nb})$
The $\Phi$ parameters

$m_\Phi$ and $\Gamma_\Phi$ obtained from the fit of the form factor

**BABar**

$m_\Phi = 1019.51 \pm 0.02(\pm 0.11) \text{ MeV}$

$\Gamma_\Phi = 4.29 \pm 0.04(\pm 0.07) \text{ MeV}$

**PDG**

$m_\Phi = 1019.455 \pm 0.020 \text{ MeV}$

$\Gamma_\Phi = 4.26 \pm 0.04 \text{ MeV}$

$\rightarrow$ good agreement

From integrated $\Phi$ peak: $\Gamma_{ee}^{ee} \times B(\Phi \rightarrow K^+ K^-) = \frac{\alpha^2 \beta^3(s,m_K)}{324} \frac{m_\Phi^2}{\Gamma_\Phi} \alpha_\Phi^2 C_{FS}$

**BABar:**

$\Gamma_{ee}^{ee} \times B(\Phi \rightarrow K^+ K^-) = 0.6344 \pm 0.0059_{\text{exp}} \pm 0.0028_{\text{fit}} \pm 0.0015_{\text{cal}} \text{ keV (1.1\%)}$

**CMD2:**

$\Gamma_{ee}^{ee} \times B(\Phi \rightarrow K^+ K^-) = 0.605 \pm 0.002 \pm 0.013 \text{ keV (2.1\%)}$
Recent Results

Charged kaon form factor at large $Q^2$

Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

- Power law: $F_K \sim \alpha_S(Q^2)Q^{-n} $ with $n=2$
  → in good agreement with the data ($2.5-5\text{ GeV} \quad n = 2.10 \pm 0.23$)
- HOWEVER: data on $|F_K|^2$ factor $\sim 20$ above prediction!
- No trend in data up to $25\text{ GeV}^2$ for approaching the asymp. QCD prediction
Impact on $g_\mu - 2$

\[ a_{\mu}^{\text{had}} (K^+ K^-) = 216.3 \pm 2.7 \pm 6.8 \]
\[ \downarrow \]
\[ a_{\mu}^{\text{had}} (K^+ K^-) = 229.5 \pm 1.4 \pm 2.2 \]
calculation only based on BABAR 2013 data!

\[ a_{\mu}^{\text{had}} (\pi^+ \pi^- \pi^+ \pi^-) = 133.5 \pm 1.0 \pm 5.2 \]
\[ \downarrow \]
\[ a_{\mu}^{\text{had}} (\pi^+ \pi^- \pi^+ \pi^-) = 136.4 \pm 0.3 \pm 3.6 \]
calculation only based on BABAR 2012 data!

[PR 477, 1 (2009).]

$\rho, \omega, \phi, \ldots$

$0.0 \text{ GeV}, \infty$

$9.5 \text{ GeV}$

$3.1 \text{ GeV}$

$2.0 \text{ GeV}$
Impact on $g_\mu - 2$

$$a_{\mu}^{\text{had}}(K^+K^-) = 216.3 \pm 2.7 \pm 6.8$$
$$\downarrow$$
$$a_{\mu}^{\text{had}}(K^+K^-) = 229.5 \pm 1.4 \pm 2.2$$

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$$a_{\mu}^{\text{had}}(\pi^+\pi^-\pi^+\pi^-) = 133.5 \pm 1.0 \pm 5.2$$
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$$a_{\mu}^{\text{had}}(\pi^+\pi^-\pi^+\pi^-) = 136.4 \pm 0.3 \pm 3.6$$

Calculation only based on BABAR 2012 data!

(all $a_{\mu}$ units in $10^{-11}$)

Dominant contribution to $\delta a_{\mu}^{\text{had}}$:

$$a_{\mu}^{\text{had}}(\pi^+\pi^-\pi^0\pi^0) = 180.1 \pm 0.3 \pm 12.4$$
$$\downarrow$$

BABAR analysis in progress
Summary

- ISR physics has proven to be a very productive field even years after the end of data taking at the B-factories.
- Precision measurements of $\sigma(e^+e^- \rightarrow 4\pi)$ and $\sigma(e^+e^- \rightarrow 2K)$ have greatly improved $a_{\mu}^{SM}$.
- More hadronic final states (e.g. $2\pi2\pi^0$) in preparation.
- $g_{\mu} - 2$ puzzle needs to be solved:
  - Data from new experiments (e.g. BES-III).
  - Light-By-Light scattering needs to be studied.
  - E989 at Fermilab and J-PARC $g-2$/EDM.
Thank you!

Any questions?
Backup slides
Complete Tenth-Order QED Contribution to the Muon $g - 2$. 

Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL. 

Reevaluation of the Hadronic Contributions to the Muon $g - 2$ and to $\alpha(M_Z)$. 
Exclusive hadronic cross sections measured via ISR from BaBar. 

Electroweak effective couplings for future precision experiments. 

A novel precision measurement of muon g-2 and EDM at J-PARC. 
Breit-Wigner fit function

\[ F_K(s) = \frac{(a_\phi \cdot BW_\phi(s) + a_\phi' \cdot BW_\phi'(s) + a_\phi'' \cdot BW_\phi''(s))}{3} \]
\[ + \frac{(a_\rho \cdot BW_\rho(s) + a_\rho' \cdot BW_\rho'(s) + a_\rho'' \cdot BW_\rho''(s) + a_\rho''' \cdot BW_\rho'''(s))}{2} \]
\[ + \frac{(a_\omega \cdot BW_\omega(s) + a_\omega' \cdot BW_\omega'(s) + a_\omega'' \cdot BW_\omega''(s) + a_\omega''' \cdot BW_\omega'''(s))}{6} \]

with

\[ a_\phi + a_\phi' + a_\phi'' = 1 \]
\[ a_\rho + a_\rho' + a_\rho'' + a_\rho''' = 1 \]
\[ a_\omega + a_\omega' + a_\omega'' + a_\omega''' = 1 \]
Basic method

Definition of $g$:

$$\vec{\mu} = g \frac{e}{2m} \vec{s}.$$  

Motion in magnetic field:

$$\vec{\omega}_c = \frac{e\vec{B}}{m\gamma},$$

$$\vec{\omega}_l = \frac{e\vec{B}}{m\gamma} + a \frac{e\vec{B}}{m},$$

$$\Rightarrow \vec{\omega}_a = a \frac{e\vec{B}}{m}.$$  

$(a = (g - 2)/2)$

Spin precession. [5]
Direct Measurement

Realization in detail

From $\pi^+$ production to $\mu^+$ decay. [5]

Electric field necessary for focussing: BMT equation

$$\bar{\omega}_a = \frac{e}{m_\mu} \left( a_\mu \bar{B} - \left[ a_\mu - \frac{1}{\gamma^2 - 1} \right] \bar{v} \times \bar{E} \right).$$

$\bar{\omega}_a$ is independent of $\bar{E}$ for $\gamma = 29.3 \iff E_\mu = \gamma m_\mu = 3.1$ GeV.
Direct Measurement of \((g_\mu - 2)\)
Experiment E821 at Brookhaven National Laboratory

The result [2]:

\[
a_\mu = (116\,592\,089 \pm 54_{\text{stat}} \pm 33_{\text{syst}}) \cdot 10^{-11}
\]
New Direct Measurement of $\left( g_\mu - 2 \right)$
Experiment E989 at Fermilab

The goal:
Reduce uncertainty by factor 4 (!)
New Direct Measurement of \((g_\mu - 2)\)

Experiment E989 at Fermilab
Ultra-cold muon experiment at J-PARC MLF from [6]

The goal:

Uncertainty of $\sim 10 \cdot 10^{-11}$ (!)
Ultra-cold muon experiment at J-PARC MLF from [6]

New Method:
Produce muons from ionization of muonium, store them and track decay.