Are current LHC data really a problem for constrained SUSY?

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Constrained SUSY...

Low-energy SUSY models with grand-unification relations among gauge couplings and (soft) mass parameters

- **CMSSM** (Constrained MSSM): 4+1 parameters
- **NUHM** (Non-Universal Higgs Model): 6+1
- **CNMSSM** (Constrained Next-to-MSSM) 5+1
- **CNMSSM-NUHM**: 7+1
- etc

**Virtues:**
- Well-motivated
- Predictive (few parameters)
- Realistic

**In contrast:**
- general MSSM – over 120 params
- MSSM + simplifying assumptions
- pMSSM: MSSM with 19 params (p19MSSM)
- p9MSSM, p12MSSM, pnMSSM, ...

At $M_{\text{GUT}} \simeq 2 \times 10^{18}$ GeV:

- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars $m_{\tilde{g}_i}^2 = m_{\tilde{t}_i}^2 = m_{H_u}^2 = m_{H_d}^2 = m_0^2$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB
  \[
  \mu^2 = \frac{m_{H_u}^2 - m_{H_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{m_{1/2}^2}{2}
  \]
- five independent parameters: $m_{1/2}, m_0, A_0, \tan \beta, \text{sgn}(\mu)$
- well developed machinery to compute masses and couplings

Figure from hep-ph/9709356
Constrained SUSY – still alive?

The constrained MSSM (CMSSM) paradigm is “hardly tenable”

At Open Symposium of the European Strategy Preparatory Group, Krakow, Poland, 10-12 Sept. 2012

Really?
Outline

• The CMSSM in light of the data
• Global fits
• Impact of $m_h \sim 126$ GeV
• Probe CMSSM with DM searches
• Implications of $\text{BR}(B_s \to \mu \mu)$
• Summary

Based on:
• Two ultimate tests of constrained SUSY, 1302.5956
• The Constrained NMSSM with a 125 GeV Higgs boson -- A global analysis, 1211.1693
• Constrained MSSM favoring new territories: The impact of new LHC limits and a 125 GeV Higgs boson, 1206.0264
  With updates
SUSY: most important constraints:

- **Dark matter density**
  - Positive measurement, inconsistent with SM

- **Direct search limits**
  - Lower limit...

- **The Higgs mass**
  - CMS: \( m_h \sim 125.8 \text{ GeV (in ZZ)}; \ m_h = 124.9 \text{ GeV (in } \gamma \gamma) \)
  - ATLAS: \( m_h = 124.3 \text{ GeV (in ZZ)}; \ m_h = 126.8 \text{ GeV (in } \gamma \gamma) \)

- **B\(_s\) \to \mu \mu**
  - \( \tilde{B}(B^0_s \to \mu^+ \mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9} \)

- **Other flavor (b to s gamma, etc)**
- **M\(_W\), EW,...**
- **(g-2)\(_\mu\)on**
How to compare theory with experiment

- Rigid step-function application of limits/allowed ranges (e.g. DM relic abundance, etc)
  - Mahmoudi et al, Hewett et al, ...
- Frequentist (chi^2-based)
  - MasterCode, Fittino, ...
- Bayesian
  - BayesFITS, Allanach, SuperBayes, Balazs, ...

Frequentist: “probability is the number of times the event occurs over the total number of trials, in the limit of an infinite series of equiprobable repetitions”

Bayesian: “probability is a measure of the degree of belief about a proposition”

Both F and B are based on the likelihood function.
The Likelihood function

Central object: Likelihood function

- **Positive measurements:**
  Take a single observable $\xi(m)$ that has been measured
  - $c$ – central value, $\sigma$ – standard exptal error
  - define
    \[ \chi^2 = \frac{(\xi(m) - c)^2}{\sigma^2} \]
  - assuming Gaussian distribution ($d \to (c, \sigma)$):
    \[ \mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{\chi^2}{2} \right] \]
  - when include theoretical error estimate $\tau$ (assumed Gaussian):
    \[ \sigma \to s = \sqrt{\sigma^2 + \tau^2} \]
  - TH error "smears out" the EXPTAL range
  - for several uncorrelated observables (assumed Gaussian):
    \[ \mathcal{L} = \exp \left[ -\sum_i \frac{\chi_i^2}{2} \right] \]

- **Limits:**
  - Smear out bounds.
  - Add theory error.

- **LHC direct limits:**
  - Need careful treatment. Typically use Poisson.
CMSSM: numerical scans

- Perform random scan over 4 CMSSM +4 SM (nuisance) parameters simultaneously
- Use Nested Sampling algorithm to evaluate posterior
- Use 4,000 live points

- Very wide ranges:
  - $100 \text{ GeV} \leq m_0 \leq 20 \text{ TeV}$
  - $100 \text{ GeV} \leq m_{1/2} \leq 10 \text{ TeV}$
  - $-20 \text{ TeV} \leq A_0 \leq 20 \text{ TeV}$
  - $3 \leq \tan \beta \leq 62$

<table>
<thead>
<tr>
<th>Nuisance</th>
<th>Description</th>
<th>Central value ± std. dev.</th>
<th>Prior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_t$</td>
<td>Top quark pole mass</td>
<td>$173.5 \pm 1.0 \text{ GeV}$</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$m_b(m_b)_{\text{SM}}$</td>
<td>Bottom quark mass</td>
<td>$4.18 \pm 0.03 \text{ GeV}$</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\alpha_s(M_Z)^{\overline{MS}}$</td>
<td>Strong coupling</td>
<td>$0.1184 \pm 0.0007$</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$1/\alpha_{\text{em}}(M_Z)^{\overline{MS}}$</td>
<td>Inverse of em coupling</td>
<td>$127.916 \pm 0.015$</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>

Use Bayesian approach (posterior)
Hide and seek with SUSY

The experimental measurements that we apply to constrain the CMSSM’s parameters. Masses are in GeV.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Mean or Range</th>
<th>Error: (Exp., Th.)</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combination of:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMS razor 4.4/fb, $\sqrt{s} = 7$ TeV</td>
<td>See text</td>
<td>See text</td>
<td>Poisson</td>
</tr>
<tr>
<td>CMS $\alpha_T$ 11.7/fb, $\sqrt{s} = 8$ TeV</td>
<td>See text</td>
<td>See text</td>
<td>Poisson</td>
</tr>
<tr>
<td>$m_h$ by CMS $\Omega_x h^2$</td>
<td>125.8 GeV 0.1120</td>
<td>0.6 GeV, 3 GeV 0.0056, 10%</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\delta (g - 2)_\mu^{\text{SUSY}} \times 10^{10}$</td>
<td>28.7</td>
<td>8.0, 1.0</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\text{BR} \left( \overline{B} \rightarrow X_s \gamma \right) \times 10^4$</td>
<td>3.43</td>
<td>0.22, 0.21</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\text{BR} \left( B_u \rightarrow \tau \nu \right) \times 10^4$</td>
<td>1.66</td>
<td>0.33, 0.38</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\Delta M_{B_s}$</td>
<td>17.719 ps$^{-1}$</td>
<td>0.043 ps$^{-1}$, 2.400 ps$^{-1}$</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$</td>
<td>0.23116</td>
<td>0.00012, 0.00015</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.385</td>
<td>0.015, 0.015</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\text{BR} \left( B_s \rightarrow \mu^+ \mu^- \right)_{\text{current}} \times 10^9$</td>
<td>3.2</td>
<td>+1.5 – 1.2, 10% (0.32)</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$\text{BR} \left( B_s \rightarrow \mu^+ \mu^- \right)_{\text{proj}} \times 10^9$</td>
<td>3.5 (3.2$^*$)</td>
<td>0.18 (0.16$^<em>$), 5% [0.18 (0.16$^</em>$)]</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>

SM value: $\approx 3.5 \times 10^{-9}$

10 dof
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  - Lower limit...

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- **\( B_s \rightarrow \mu^+ \mu^- \)**
  - \( \mathcal{B}(B^0_s \rightarrow \mu^+ \mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9} \)

- **Other flavor (b to s gamma, etc)**
- **M_W, EW,...**

- **(g-2)_{\muon}**
Dark matter density

- Unified SUSY: neutralino relic density is typically 1-2 orders of magnitude too large
- Remaining mechanisms of reducing it to correct range:
  - neutralino-stau coannihilation
  - pseudoscalar Higgs A resonance $\Omega h^2 \propto m_A^4 / \tan^2 \beta$
  - focus point/hyperbolic branch region
  - (~1 TeV higgsino LSP at larger MSUSY)
  - Plus (very rare) LSP-stop coannihilation

CMSSM: no more generic DM-favored regions
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- **Other flavor ($b$ to $s$ gamma, etc)**
- **$M_W$, EW,...**
- **$(g-2)_{\text{muon}}$**
Reproducing CMS limits on SUSY

We approximate CMS limits by deriving likelihood maps

First, validate our method:

![Graph showing likelihood maps for CMSSM, μ > 0, tanβ = 3, A₀ = 0]

Next, derive combined CMS limit based on datasets:

α_T 11.7/ fb, √s = 8 TeV
Razor 4.4/ fb, √s = 7 TeV

Applies to both signs of μ
And to similar models: NUHM, CNMSSM,...

Below will use combined CMS limit via likelihood function
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  $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9}$

- **Other flavor (b to s gamma, etc)**
- **M_W, EW,**...

- **(g-2)$_{\muon}$**
In SUSY $m_h$ is a calculated quantity.

1-loop: positive, up to $\sim 45$ GeV

$$\Delta m_h^2 = \frac{3m_t^4}{4\pi^2 v^2} \left[ \ln \left( \frac{M_{SUSY}^2}{m_t^2} \right) + \frac{X_t^2}{M_{SUSY}^2} \left( 1 - \frac{X_t^2}{12M_{SUSY}^2} \right) \right]$$

2-loop: negative, $\sim 3$ GeV

two most complete calculations differ by a 2-5 GeV

(DR-bar (Slavich,...) used in SoftSusy, Spheno, Suspect, and on-shell (Hollik,...) in FeynHiggs)

Substantial theory error!

Two ways to obtain $m_h \sim 126$ GeV:

1. increase $M_{SUSY}$ -> heavy superpartners! or
2. take large $|X_t| \sim |A_t|$ -> stop_1 at $\sim 1$ TeV

Applies to SUSY generally, not just constrained models.
Light Higgs in the CMSSM

Like fn

\[ \mathcal{L}_{\text{mass}} \sim e^{-\frac{(m_h - 125.3 \text{ GeV})^2}{\sigma^2 + \tau^2}} \]

\[ \sigma = 0.6 \text{ GeV}, \tau = 2 \text{ GeV} \]

BayesFITS (2013)

Light Higgs mass \( m_h \)

- \( m_h < 120 \text{ GeV} \)
- \( 120 - 124 \text{ GeV} \)
- \( 124 - 127 \text{ GeV} \)
- \( > 127 \text{ GeV} \)

\( \delta(g-2)_\mu \)

BR(B_s\rightarrow\mu^+\mu^-) = (3.2 \pm 1.5) \times 10^{-9}

\sim 126 \text{ GeV Higgs at/near lowest chi2 (S.C./AF) and at X_SUSY>> 1TeV}
Bye bye SUSY?

...bye bye happiness... Hello emptiness. I feel like I could die.

Best fit to ~126 GeV Higgs -> M_SUSY~ or >> 1 TeV

Consistent with limits from flavor violation, direct SUSY searches

Dark matter relic density: selects some regions
- CMSSM: DM regions (almost) disconnected
- other models: they overlap
Can multi-TeV ranges of parameters be experimentally tested?
CMSSM and 1-tonne DM detectors

\[
\mu > 0
\]

1-\text{tonne} DM detectors to cover most of CMSSM predictions

Generic prediction of multi-TeV SUSY: 
\(~1\text{TeV} LSP\) (higgsino)

\[\text{...over ALL multi-TeV ranges of mass parameters}\]
LHC: Impact on DM Searches

- extended ranges of SUSY parameters
- no other SUSY regions exist

Before LHC

LHC (1/fb)

Posterior pdf
CMSSM, $\mu > 0$
Log Priors
$BR(B_s \rightarrow \mu^+ \mu^-) = (3.2 \pm 1.5) \times 10^{-9}$ (current)

XENON(2011) limit not applied

13/03/2013

Roszkowski, Moriond 9-16/3/13
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- **Other flavor (b to s gamma, etc)**
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BR($B_s \rightarrow \mu^\pm \mu^\mp$)

\[ BR(B^0_s \rightarrow \mu^+\mu^-) = (3.2^{+1.5}_{-1.2}) \times 10^{-9} \]

1.1 \times 10^{-9} < BR(B^0_s \rightarrow \mu^+\mu^-) < 6.4 \times 10^{-9} at 95\% CL

Note this gives weaker upper bound than before.

LHC combination (June 2012): \textcolor{red}{BR(B^0_s \rightarrow \mu^+\mu^-) < 4.2 \times 10^{-9} at 95\% CL}

We approximate the signal with a Gaussian

\[ BR(B_s \rightarrow \mu^+\mu^-) \propto (3.54 \pm 0.30) \times 10^{-9} \]

Note the Gaussian Like allows larger BR than 4.2 bound before.

- sensitive probe of new physics

\[ BR(\overline{B}_s \rightarrow \mu^+\mu^-) \propto \tan^6 \beta / m_A^4 \]
Effect of precise $\text{BR}(\overline{B}_s \rightarrow \mu^+ \mu^-)$

If $\text{BR}(\overline{B}_s \rightarrow \mu^+ \mu^-) \simeq \text{SM value}$ with 5-10% precision (both TH and EXPT)

$\Rightarrow$ A funnel region gone
Effect of precise $\text{BR}(\overline{B}_s \rightarrow \mu^+ \mu^-)$

If $\text{BR}(\overline{B}_s \rightarrow \mu^+ \mu^-) \simeq \text{SM value}$

with 5-10% precision

$\Rightarrow$ A funnel region gone

Ways to rule out the CMSSM:

- No DM signal in 1-tonne detectors
- DM signal at $\sim 500$ to 750 GeV

SC: for $\mu < 0$ $\sigma_p^{\text{SI}}$ lower (cancellations)

NUHM, CNMSSM: similar ranges of $\sigma_p$ but DM-favored regions overlap
... a question on many people’s mind...

But what about fine-tuning/naturalness?!

- I prefer to follow what the data implies, rather than theoretical prejudice

- Naturalness: fundamental Higgs -> SUSY
- $126\text{ GeV} \rightarrow M_{\text{SUSY}} \sim 1\text{TeV}$ or $> 1\text{TeV}$

- Fine-tuning is needed at any scale above the EW scale!
  - $1\text{ TeV}$ is not a magic number

- If SUSY is discovered, the FT issue will have to be understood
- If SUSY is not discovered, the issue will become irrelevant

- There are ideas around of how to live comfortably with high fine-tuning
To take home:

- **CMSSM**: consistent with all experimental constraints, except g-2, R(\gamma \gamma).
  
  (Other simple constrained SUSY models: similar story.)

- **Higgs of 126 GeV --> SUSY at multi-TeV scale.**
  
  Plus a window of light stop_1 (~1TeV) – best fit region (stau coann.)

- **1-tonne DM detectors to probe most CMSSM parameters.**
  
  Far beyond direct LHC reach.

  Other simple constrained SUSY models: similar story.

- **1TeV (higgsino) DM** – generic prediction of constrained SUSY models (and also MSSM)

- **precise determination of BR(B_s to \mu \mu) can be very helpful in CMSSM (but not beyond)**