Vector Meson Production from Gauge/Gravity Duality

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work with Miguel S. Costa and Nick Evans

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Outline

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- However, at lower energies, once it is of order \( \Lambda_{QCD} \) the coupling is very strong and we cannot use pQCD.
- Our goal is to study the strong interaction at strong coupling.
- More specifically, a recent conjecture by Maldacena relating string theory on \( AdS_5 \times S_5 \) to \( \mathcal{N} = 4SYM \) allows us to study QCD at strong coupling.
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The Pomeron

The Pomeron is the leading order exchange in all total cross sections, and in $2 \rightarrow 2$ amplitudes with the quantum numbers of the vacuum, in the Regge limit $s \gg t$.

It is the sum of an infinite number of states with the quantum numbers of the vacuum.

It leads to an amplitude that as $s \to \infty$ goes as $A(s,t) \sim s^{\alpha(t)}$, $\alpha(t) = \alpha(0) + \alpha'(t)^2$.

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The AdS/CFT Correspondence

Conjectured exact duality between type IIB string theory on $\text{AdS}_5 \times S^5$, and $\mathcal{N}=4$ SYM, on the boundary.

The duality relates states in string theory to operators in the field theory through the relation

\[ e^\int d^4x \phi_i(x) O_i(x) \rvert_{\text{CFT}} = Z_{\text{string}} \left[ \phi_i(x,z) \rvert_{z \sim 0} \right] \]

The metric we will use

\[ ds^2 = e^{2A(z)} \left[ -dx + dx - dx_\perp dx_\perp + dz dz \right] + R^2 d^2 \Omega_5. \]

In the hard-wall model up to a sharp cutoff $z_0 \simeq 1/\Lambda_{\text{QCD}}$

\[ e^{2A(z)} = R^2 / z^2. \]

Correspondence works in the limit $N_C \to \infty$, $\lambda_g \gg 1$, fixed.
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Pomeron in AdS string theory

What is the Pomeron in AdS String theory? (Brower, Polchinski, Strassler, Tan 2006)

It is the Regge trajectory of the graviton.

In flat space, the Pomeron vertex operator

$$V_P = (2\alpha' \partial X + \bar{\partial} X + )^{1+\alpha'}t e^{-ik \cdot X}$$

The Pomeron exchange propagator in AdS is given by

$$K = \frac{2}{s^2 g_0^2 \chi(s, b, z, z')}$$

where

$$\chi(\tau, L) = (\cot(\pi \rho^2) + i)g_0^2 e^{(1-\rho)\tau}L \sinh L \exp(-L^2 \rho \tau) \left(\frac{\rho \tau}{3}ight)^{3/2}$$
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The weak and strong coupling Pomeron exchange kernels have a remarkably similar form.
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At $t = 0$

Weak coupling:

$$K(k_\perp, k'_\perp, s) = \frac{s^{j_0}}{\sqrt{4\pi D \log s}} e^{-(\log k_\perp - \log k'_\perp)^2 / 4D \log s}$$

$$j_0 = 1 + \frac{\log 2}{\pi^2} \lambda, \quad D = \frac{14\zeta(3)}{\pi} \lambda / 4\pi^2$$

Strong coupling:

$$K(z, z', s) = \frac{s^{j_0}}{\sqrt{4\pi D \log s}} e^{-(\log z - \log z')^2 / 4D \log s}$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}, \quad D = \frac{1}{2\sqrt{\lambda}}$$
According to the Froissart bound
\[ \sigma_{\text{tot}} \leq \pi c \log (s/s_0) \]
Hence the Pomeron exchange violates this bound.
Eventually effects beyond one Pomeron exchange become important.
Eikonal approximation in AdS space (Brower, Strassler, Tan; Cornalba, Costa, Penedones)
\[ A(s,t) = 2 \int d^2\ell - i \ell_\perp \cdot q_\perp \int dz d\bar{z} P_{13}(z) P_{24}(\bar{z})(1 - e^{i\chi(s,b,z,\bar{z})}) \]
We can study different scattering processes by supplying \( P_{13} \) and \( P_{24} \).
For example, already applied to DIS [Brower, MD, Sarcević, Tan; Cornalba, Costa, Penedones], and DVCS [Costa, MD].
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The vector mesons consist of a quark-antiquark pair, and have the same quantum numbers as the photon, \( J^{PC} = 1^{--} \). The production of the \( \rho^0, \omega, \phi \) and \( J/\Psi \) was measured at HERA.
We are interested in calculating the differential and exclusive cross sections

\[
\frac{d\sigma}{dt}(x, Q^2, t) = \frac{|W|^2}{16\pi s^2},
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Here \( W \) is the scattering amplitude

\[
W = 2isQQ' \int dl_\perp e^{iq_\perp \cdot l_\perp} \int \frac{dz}{z^3} \frac{d\bar{z}}{\bar{z}^3} \Psi(z) \Phi(\bar{z}) \left[ 1 - e^{i\chi(S,L)} \right].
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This has the previously mentioned form, we just need to supply the wavefunctions \( \Psi(z) \) and \( \Phi(\bar{z}) \) for the photon and the proton.
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In this analysis we use
\[
\Psi_n(z) = -\left( \sqrt{C} \frac{\pi^2}{6} z^2 K_n(Qz) \right) \left( \frac{\sqrt{2}}{\xi J_1(\xi)} \right) z^2 J_n(mz), \quad \Phi(\bar{z}) = \bar{z}^3 \delta(\bar{z} - z_*)
\]
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Conformal Pomeron

We start with the conformal pomeron, with

\[
1 - e^{i\chi} \approx -i\chi = -i(\cot(\frac{\pi \rho}{2}) + ig_0^2 e^{(1-\rho)\tau}) \frac{L}{\sinh L} \frac{\exp\left(\frac{-L^2}{\rho \tau}\right)}{(\rho \tau)^{3/2}}
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Depends on 3 parameters:
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z_*
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\]

- \( C \) is the aforementioned normalization, and \( g_0^2 \) is related to the coupling of the external states to the pomeron.
Hard wall pomeron

- Obtained by placing a sharp cut-off on the radial AdS coordinate at $z = z_0$. 

\[ \chi_{hw}(\tau,0,z,\bar{z}) = \chi(\tau,0,z,\bar{z}) + F(\tau,z,\bar{z}) \chi(0)_{hw}(\tau,0,z,\bar{z}) \]

When $t \neq 0$, we will use an approximation $\chi_{hw}(\tau,t,z,\bar{z}) = C(\tau,z,\bar{z}) D(\tau,t) \chi(0)_{hw}(\tau,t,z,\bar{z})$. 

\[ \chi(\tau,0,z,\bar{z}) = \frac{1}{\ln(\bar{z}/z)^2} \rho \tau \left( \rho \tau \right)^{1/2} \]

\[
\int_0^{z_0} \frac{1}{\tan(\pi \rho^2 / 2)} + i \rho \tau z_{\bar{z}} e^{1 - \rho \tau} e^{-\left(\ln(\bar{z}/z)^2\right)} \rho \tau \left( \rho \tau \right)^{1/2}
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- First notice that at $t = 0$ $\chi$ for conformal pomeron exchange can be integrated in impact parameter

$$\chi(\tau, t = 0, z, \bar{z}) = i\pi g_0^2 \left( \cot \left( \frac{\pi \rho}{2} \right) + i \right) (z \bar{z}) e^{(1-\rho)\tau} \frac{e^{-\frac{(\ln(\bar{z}/z))^2}{\rho\tau}}}{(\rho\tau)^{1/2}}$$
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$$

- Similarly, the $t = 0$ result for the hard-wall model can also be written explicitly

$$
\chi_{hw}(\tau, t = 0, z, \bar{z}) = \chi(\tau, 0, z, \bar{z}) + \mathcal{F}(\tau, z, \bar{z}) \chi(\tau, 0, z, z_0^2/\bar{z}).
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- When $t \neq 0$, we will use an approximation

$$
\chi_{hw}(\tau, l, z, \bar{z}) = C(\tau, z, \bar{z}) D(\tau, l) \chi_{hw}^{(0)}(\tau, l, z, \bar{z})
$$
The function

\[ F(\tau, z, \bar{z}) = 1 - 4\sqrt{\pi \tau} e^{\eta^2} \text{erfc}(\eta), \quad \eta = \frac{-\log(z \bar{z}/\bar{z}_0^2) + 4\tau}{\sqrt{4\tau}} \]

is set by the boundary conditions at the wall and represents the relative importance of the two terms.
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- Varies between $-1$ and $1$, approaching $-1$ at either large $z$, which roughly corresponds to small $Q^2$, or at large $\tau$ corresponding to small $x$.

- It is therefore in these regions that confinement is important!
The Data

Let us now discuss the data we will use later on in the talk.
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- All the data is at small $x$ ($x < 0.01$).
- In this region pomeron exchange is the dominant process.
- We will look at both the differential and total exclusive cross sections.
Note that the same formalism has been applied before to DIS with good results ($\chi^2 = 1.04$ for the best model) [Brower, MD, Sarčević, Tan, 2010, Cornalba, Costa, Penedones, 2010, Brower Moriond 2011], and DVCS ($\chi^2 = 1.00$ and $\chi^2 = 0.51$ for the best models of the cross section and differential cross section respectively) [Costa, MD, 2012, MD Moriond 2012].
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<table>
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</tbody>
</table>

$C_{onf}$

For $H_{ardwall}$

$\chi^2 / 2 \sqrt{\lambda}$
Differential cross section for the $\rho$ meson:

$$\frac{d\sigma}{dt}$$

$W = 75$ GeV

$Q^2 = \{2.7, 3.3, 5, 6.6, 7.8, 11.5, 11.9, 17.4, 19.7, 33, 41\} \text{ GeV}^2$
Differential cross section for the $\phi$ meson (hardwall model):

- $W = 45$ GeV
  - $Q^2 = 5$ GeV$^2$
- $W = 75$ GeV
  - $Q^2 = 2.4, 3.3, 3.6, 5, 5.2, 6.6, 6.9, 9.2, 12.6, 15.8, 19.7$ GeV$^2$
- $W = 102$ GeV
  - $Q^2 = 5$ GeV$^2$
- $W = 116$ GeV
  - $Q^2 = 5$ GeV$^2$
Differential cross section for the $J/\Psi$ meson (hardwall model):

- $W = 42$ GeV, $Q^2 = 6.8$ GeV$^2$
- $W = 57$ GeV, $Q^2 = 8.9$ GeV$^2$
- $W = 90$ GeV, $Q^2 = 3.1$ GeV$^2$, $Q^2 = 16$ GeV$^2$
- $W = 144$ GeV, $Q^2 = 0.05$ GeV$^2$
Cross sections for the conformal model:

\[ Q^2 = 0.4 \text{ GeV}^2 \]
\[ Q^2 = 3.1 \text{ GeV}^2 \]
\[ Q^2 = 5.9 \text{ GeV}^2 \]
\[ Q^2 = 8.4 \text{ GeV}^2 \]
\[ Q^2 = 16 \text{ GeV}^2 \]
\[ Q^2 = 54 \text{ GeV}^2 \]

\[ Q^2 = 3.3 \text{ GeV}^2 \]
\[ Q^2 = 7 \text{ GeV}^2 \]
\[ Q^2 = 13 \text{ GeV}^2 \]

\[ Q^2 = 2.71 \text{ GeV}^2 \]
\[ Q^2 = 3.3 \text{ GeV}^2 \]
\[ Q^2 = 4.52 \text{ GeV}^2 \]
\[ Q^2 = 6.35 \text{ GeV}^2 \]
\[ Q^2 = 7.6 \text{ GeV}^2 \]
\[ Q^2 = 12 \text{ GeV}^2 \]
\[ Q^2 = 15.8 \text{ GeV}^2 \]
\[ Q^2 = 32.15 \text{ GeV}^2 \]

\[ Q^2 = 3.3 \text{ GeV}^2 \]
\[ Q^2 = 4.15 \text{ GeV}^2 \]
\[ Q^2 = 5.2 \text{ GeV}^2 \]
\[ Q^2 = 6.55 \text{ GeV}^2 \]
\[ Q^2 = 7.35 \text{ GeV}^2 \]
\[ Q^2 = 9.2 \text{ GeV}^2 \]
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\[ Q^2 = 25 \text{ GeV}^2 \]
\[ Q^2 = 35 \text{ GeV}^2 \]
\[ Q^2 = 46 \text{ GeV}^2 \]
Cross sections for the hardwall model:
We thus conclude today’s talk.
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- We have seen that we now have 3 processes (DIS, DVCS and vector meson production) where the AdS (BPST) pomeron exchange gives excellent agreement with experiment in the strong coupling region.

- Hence string theory on AdS is giving us interesting insights into non-perturbative scattering.

- The value of the pomeron intercept is in the region $1/2 - 1/4$ which is in the crossover region between strong and weak coupling, and a lot of the equations have a form which is very similar both at weak and at strong coupling (but $\chi$ is different).

- It might therefore be possible to extend some of the insights we gain even into the weak coupling regime.

- The hard wall model, although a simple modification of AdS, seems to capture effects of confinement well. Interesting to repeat some of the calculations using a different confinement model to identify precisely what features are model independent.
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- Eventually it would be good to have a single set of parameters that fits several different processes.
- We can also try to use a different AdS model of confinement (for example the soft wall model) and combine our methods with work by others (for example on the vector meson wavefunctions).
Thank you!