Complete Next-to-Leading-Order Study on the Yield and Polarization of Υ(1S, 2S, 3S) at the Tevatron and LHC

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5 Summary
WHY HEAVY QUARKONIUM NEEDED?

- Color-singlet and Color-octet mechanism was proposed based on NRQCD
- Heavy quarkonium production is a good place to testify the theoretical framework
- The $p_t$ distribution and polarization for $J/\psi$ and $\Upsilon$ production measured at the Tevatron and LHC are drawing people’s attention
- Especially at the LHC, it would generate large amount of effective data about heavy quarkonium as its upgrading
- It seems that the QCD NLO calculations can adequately describe the experimental data
- But there are still some difficulties
Experimental measurement for $\Upsilon$ at present

- In early works\textsuperscript{[1,2]}, measurement on the polarization of $\Upsilon(1S)$ at the Tevatron was presented, while the corresponding LO NRQCD prediction\textsuperscript{[3]} does not coincide with it.

- The $p_t$ distribution for the yield of $\Upsilon$ have been presented by the four collaborations at the LHC: ATLAS, ALICE, CMS, and LHCb.

- Some of them have done the measurement up to large $p_t$ region, such as ATLAS measurement.

- There are also the polarization measured for $\Upsilon(1S, 2S, 3S)$ by CMS\textsuperscript{[4]}: $\lambda_\theta, \lambda_{\theta \phi}, \lambda_\phi$, and $\tilde{\lambda}$, in the helicity, C-S, and PX frame.

\textsuperscript{1} D. E. Acosta et al. (CDF), PRL 88, 161802(2002).
\textsuperscript{2} V. M. Abazov et al. (D0), PRL 101, 182004(2008).
\textsuperscript{3} E. Braaten and J. Lee, PRD 63, 071501 (2001).
\textsuperscript{4} S. Chatrchyan et al. (CMS), PRL 110, 081802 (2013).
Theoretical study recently (at NLO)

- the polarization for direct $J/\psi$ hadroproduction were studied by Butenschoen et al.\cite{5} and K.-T. Chao et al.\cite{6}  
- the polarization for prompt $J/\psi$ hadroproduction were studied by B. Gong\cite{7}  
- a NLO calculation for the yield of $\Upsilon(1S)$ via complete CO states (include $^3P_J^{[8]}$)\cite{8},

Therefore, no complete inclusive NLO QCD study on the polarization of $\Upsilon(1S, 2S, 3S)$ hadroproduction has been performed, which could be directly used to compare with the experimental measurement.

\cite{5}, Butenschoen et al. PRL 108, 172002(2012).
\cite{7}, B. Gong et al., PRL 110, 042002(2013).
\cite{8}, K. Wang et al., PRD 85, 114003 (2012).
The Fortran codes produced by FDC (Feynman Diagram Calculation) package

- We have made it a complete Fortran package after we have finished the work of charmonium prompt production, which is specially used to do heavy quarkonium calculation at NLO.
- The package could be easily applied for $\Upsilon$ case.
- We also developed a shell-script package to do numerical calculation automatically in supercomputers, which can avoid lots of manual work during the program running process.
Advantages on the calculation for $\Upsilon$ in comparison with $J/\psi$

- Both QCD coupling constant $\alpha_s$ and $\nu^2$ are smaller in bottomonium case than $J/\psi$ case.
- The fix order prediction on $\Upsilon$, are also supposed to be very good for $p_t$ up 60 GeV.

Therefore, we can expect that, for $\Upsilon$ case, it should have a better convergence behavior in a wide $p_t$ region than $J/\psi$.

Besides, it is found that the measurement for $\Upsilon$ at the LHC is easier than that for $J/\psi$, though the situation is opposite at the Tevatron.
Disadvantages on the calculation for $\Upsilon$ in comparing with $J/\psi$

Although our Fortran program used for $J/\psi$ can be easily applied for $\Upsilon$, some other troubles can not be avoided:

- more complicated feed-down contributions for $\Upsilon(1S, 2S)$;
- no directly measured data for $\chi_b$ production, therefore it is hard to decide its contribution to $\Upsilon$ production.
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Calculation, Fit and the LDMEs

<table>
<thead>
<tr>
<th>STATES</th>
<th>LO sub-process</th>
<th>number of Feynman diagrams</th>
<th>NLO sub-process</th>
<th>number of Feynman diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^3S_1^{(1)}$</td>
<td>$g + g \rightarrow (b\bar{b})_n + g$</td>
<td>6</td>
<td>$g + g \rightarrow (b\bar{b})_n + g$(one-loop)</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$g + g \rightarrow (bb)_n + g + g$</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$g + g \rightarrow (bb)_n + b + b$</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$g + g \rightarrow (bb)_n + q + \bar{q}$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$g + q(\bar{q}) \rightarrow (bb)_n + g + q(\bar{q})$</td>
<td>6</td>
</tr>
<tr>
<td>$^1S_0^{(8)}$ (also $^3P_1^{8}$)</td>
<td>$g + g \rightarrow (b\bar{b})_n + g$</td>
<td>(12,16,12)</td>
<td>$g + g \rightarrow (b\bar{b})_n + g$(one-loop)</td>
<td>(369,644,390)</td>
</tr>
<tr>
<td>or $^3S_1^{(8)}$</td>
<td>$g + q(\bar{q}) \rightarrow (bb)_n + q(\bar{q})$</td>
<td>(2,5,2)</td>
<td>$g + q(\bar{q}) \rightarrow (bb)_n + q(\bar{q})$(one-loop)</td>
<td>(61,156,65)</td>
</tr>
<tr>
<td>or $^3P_1^{1}$</td>
<td>$q + \bar{q} \rightarrow (b\bar{b})_n + g$</td>
<td>(2,5,2)</td>
<td>$q + \bar{q} \rightarrow (b\bar{b})_n + g$(one-loop)</td>
<td>(61,156,65)</td>
</tr>
</tbody>
</table>

Table: The sub-processes for Υ prompt production at LO and NLO corrections and the number of the corresponding diagrams. The number in the round brackets in the column 'number of Feynman diagrams', denotes the number of diagrams for the states $^1S_0^{8}$ (also $^3P_1^{8}$), $^3S_1^{8}$ (for both the direct part and the $\chi_b$ state), $^3P_1^1$ ( for $\chi_b$ state) from left to right, respectively. There are almost 5,000 diagrams in total.
Calculation content

Our calculation contains the direct part: $^3S_1^{[1]}$, $^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_j^{[8]}$, for $\Upsilon(1S, 2S, 3S)$ and the feed-down part $^3S_1^{[8]}$, $^3P_j^{[1]}$ for $\chi_b J(1P, 2P)$.

Our study focus on the helicity frame only and covers the experimental measurement on the Tevatron and LHC:

- $\sqrt{s} = 1.96 \, TeV$, $|y| < 0.6$ (CDF Run II, PRL 108, 151802 (2012)),
- $\sqrt{s} = 1.8 \, TeV$, $|y| < 0.4$ (CDF Run I, PRL 88, 161802(2002)),
- $\sqrt{s} = 7 \, TeV$, $|y| < 0.6$ and $0.6 < |y| < 1.2$ (CMS, PRL 110, 081802 (2013)),
- $|y| < 2$ (CMS, PRD 83, 112004 (2011)),
- $|y| < 1.2$ (ATLAS, PRD 87, 052004 (2013)),
- $2.0 < y < 4.5$ (LHCb, EPJC 72, 2025 (2012)).
Fit method

We did a combined fit to get the CO LDMEs through the $p_t$ distribution of the yield and polarization covering all the 7 experimental measurement above mentioned. Three fits are performed for $\Upsilon(3S, 2S, 1S)$ hadroproduction step by step.

- For $\Upsilon(3S)$, no feed-down considered, we got the 3 direct CO LDMEs $\langle \mathcal{O}^{1S_0^{[8]}} \rangle$, $\langle \mathcal{O}^{3S_1^{[8]}} \rangle$, $\langle \mathcal{O}^{3P_0^{[8]}} \rangle$.

- For $\Upsilon(2S)$ (or $\Upsilon(1S)$), no direct measurement for the $\chi_b(2P)$ (or $\chi_b(1P)$) state, we treated $\langle \mathcal{O}\chi_b(2P)^{3S_1^{[8]}} \rangle$ (or $\langle \mathcal{O}\chi_b(1P)^{3S_1^{[8]}} \rangle$) as another unknown parameter to be fitted, thus we got 4 CO LDMEs for $\Upsilon(2S)$ (or $\Upsilon(1S)$).
Fit details

- The direct part constrained to be positive.
- Three values of NRQCD factorization scale $\mu_\Lambda$ are taken, $m_b$, $m_b v$, $\Lambda_{QCD}$.
- The region of $p_t > 8$ GeV was chosed to fit.
The error band

Knowing that the approximately linear dependence of the three short-distance parts, the CO LDMEs directly obtained by our fit are with large uncertainty. Just as we did for $J/\psi$ fit, the covariance-matrix method are adopted, so we get the array $\Lambda$ with independent errors:

$$d\sigma = \sum O_i d\hat{\sigma}_i = \sum OVV^{-1}d\hat{\sigma} = \sum \Lambda V^{-1}d\hat{\sigma}.$$ 

This method are used only for $\Upsilon(3S, 2S)$. For $\Upsilon(1S)$, the feed-down part contributes the most, so the bands caused from the error of CO LDMEs we fitted are so small that it need not to be rotated. In our paper, we present the figures of $\Upsilon(3S, 2S)$ by using the rotated array $\Lambda$, and for $\Upsilon(1S)$ we just use the CO LDMEs obtained directly.
Long-distance-matrix-elements (LDMEs)

Under the choice of $\mu_\Lambda = m_b \nu$, the LDMEs are obtained as:

<table>
<thead>
<tr>
<th>H</th>
<th>$\langle O^H(1S_0^{[8]}) \rangle$</th>
<th>$\langle O^H(3S_1^{[8]}) \rangle$</th>
<th>$\langle O^H(3P_0^{[8]}) \rangle / m^2_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(1S)$</td>
<td>11.15 ± 0.43</td>
<td>$-0.41 \pm 0.24$</td>
<td>$-0.67 \pm 0.00$</td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td>3.55 ± 2.12</td>
<td>0.30 ± 0.78</td>
<td>$-0.56 \pm 0.48$</td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td>$-1.07 \pm 1.07$</td>
<td>2.71 ± 0.13</td>
<td>0.39 ± 0.23</td>
</tr>
<tr>
<td>$\chi_{b0}(2P)$</td>
<td>$-$</td>
<td>2.76 ± 0.67</td>
<td>$-$</td>
</tr>
<tr>
<td>$\chi_{b0}(1P)$</td>
<td>$-$</td>
<td>1.27 ± 0.16</td>
<td>$-$</td>
</tr>
</tbody>
</table>

**Table:** The CO LDMEs for bottomonia production in this work (in unit of $10^{-2} \text{ GeV}^3$).
The $p_t$ distribution of yield for $\Upsilon(3S)$

$$\left( \frac{d\sigma}{dp_t} \times B(\Upsilon \rightarrow \mu\mu) \right)(nb/Gev)$$

Figure: Differential cross section for $\Upsilon(3S)$ hadroproduction.
Our results

The result for \( \Upsilon(3S) \)

The \( p_t \) distribution of polarization for \( \Upsilon(3S) (\lambda_\theta) \)

Polarization parameter \( \lambda \) for \( \Upsilon(3S) \) hadroproduction.
The $p_t$ distribution of yield for $\Upsilon(2S)$

$$\left( \frac{d\sigma}{dp_t} \times B(\Upsilon \to \mu\mu) \right) \text{(nb/Gev)}$$

**Figure:** Differential cross section for $\Upsilon(2S)$ hadroproduction.
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Our results

The result for $\Upsilon(2S)$

The $p_t$ distribution of polarization for $\Upsilon(2S)$ ($\lambda_\theta$)

Polarization parameter $\lambda$ for $\Upsilon(2S)$ hadroproduction.
The $p_t$ distribution of yield for $\Upsilon(1S)$

$$\left(\frac{d\sigma}{dp_t} \times B(\Upsilon \rightarrow \mu\mu)\right)(nb/Gev)$$

**Figure**: Differential cross section for $\Upsilon(1S)$ hadroproduction.
Our results

The $p_t$ distribution of polarization for $\Upsilon(1S)$ ($\lambda_\theta$)

Polarization parameter $\lambda$ for $\Upsilon(1S)$ hadroproduction.
Some comments

- With different choices of the NRQCD factorization scale $\mu_\Lambda$, we find that $\mu_\Lambda$ dependence is very small in $p_t$ distribution of the yield and polarization for $\Upsilon$.

- For $p_t$ distribution of $\Upsilon(1S, 2S, 3S)$ yield, the experimental measurements at the Tevatron and LHC can be explained very well in a wide range of $p_t$.

- For $\Upsilon(1S, 2S)$, the predictions for polarization can explain the CMS data well, meanwhile still have some distance from the CDF data.

- For $\Upsilon(3S)$, the polarization can not be explained.
More study needed

- The relativistic corrections to $J/\psi$ hadroproduction (G.-Z. Xu et al. PRD 86, 094017(2012)) is negative and large in small $p_t$ range, and this infers that the relativistic corrections to $\Upsilon(3S)$ is the largest one among $\Upsilon(1S, 2S, 3S)$ and detailed study may change the result of fit.

- The uncertainty from the unknown fraction of $\chi_{bJ}$ feed-down in the fits for $\Upsilon(1S, 2S)$ could be large. Therefore a further precise measurement on the fraction of $\chi_{bJ}$ feed-down or on direct $\Upsilon$ production will be very helpful to fix the polarization puzzle.

- Other unknown feed-down contribution such as $\chi_{bJ}(3P)$, could make the polarization of $\Upsilon(3S)$ better.
Thanks!