Higgs production at NNLOPS

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Why going NNLO?

- sometimes NLO not enough:
  - large NLO/LO “K-factor”
    [perturbative expansion “not (yet) stable”]
  - very high precision needed
    ⇒ NNLO

- NNLO is the frontier:
  first $2 \to 2$ NNLO computations in 2012-13!

- paramount example: Higgs production
  [corrections so large that NNNLO relevant - see next talk]
Why going NNLO?

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  - large NLO/LO “K-factor”
    [perturbative expansion “not (yet) stable”]
  - very high precision needed
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  [corrections so large that NNNLO relevant - see next talk]

aim: build an event generator that is NNLO accurate (NNLOPS)

- the approach presented here works for “2 → 1” processes at the LHC.
- In 1309.0017 we used it for Higgs production.
1. $H+j @ NLO, H+jj @ LO \Rightarrow$ use $H+j @ NLOPS (\text{POWHEG})$

$$d\sigma_{\text{POWHEG}} = d\Phi_n \bar{B}_{\text{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k^\text{min}_T) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

[+$p_T$-vetoing subsequent emissions, to avoid double-counting]
1. \( H+j @ NLO, H+jj @ LO \) \( \Rightarrow \) use \( H+j @ NLOPS (POWHEG) \)

\[
d\sigma_{\text{POWHEG}} = d\Phi_n \bar{B}_{\text{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k_{T\text{min}}) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}
\]

\[\downarrow \quad \text{[+ } p_T\text{-vetoing subsequent emissions, to avoid double-counting]}\]

\[
\bar{B}_{\text{NLO}}(\Phi_n) d\Phi_n = \alpha_s^3(\mu_R) \left[ B + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right] d\Phi_n
\]

\( H+j \) is a 2-scales problem (\( \rightarrow \) choice of \( \mu \) not unique)
1. $H+j @ \text{NLO}$, $H+jj @ \text{LO} \implies \text{use } H+j @ \text{NLOPS (POWHEG)}$

$$d\sigma_{\text{POWHEG}} = d\Phi_n \, \tilde{B}_{\text{NLO}}(\Phi_n) \left\{ \Delta(\Phi_n; k_{T}^{\text{min}}) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \right\}$$

\[ + \text{p}_T\text{-vetoing subsequent emissions, to avoid double-counting} \]

$$\tilde{B}_{\text{NLO}}(\Phi_n) \, d\Phi_n = \alpha_s^3(\mu_R) \left[ B + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right] d\Phi_n$$

\[ H+j \text{ is a 2-scales problem (\rightarrow choice of } \mu \text{ not unique) } \]

*.want to reach NNLO accuracy for e.g. $y_H$, i.e. when fully inclusive over QCD radiation*

- need to allow the 1st jet to become unresolved
- the above approach needs to be modified: as it stands, $\tilde{B}(\Phi_n)$ is not finite when $q_T \to 0!$
2. integrate over phase space regions where $H$ is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all $q_T$ spectrum)

**MiNLO: Multiscale Improved NLO** [Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to **a-priori** choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy)
- how: correct weights of different NLO terms with CKKW-inspired approach:

\[
\bar{B}_{\text{MiNLO}} = \alpha^2_S(m_h) \bar{B} + \alpha(S) V(\mu_R) + \alpha(NLO) S \int d\Phi r_R
\]

\[
\bar{B} = \alpha^3_S(\mu_R) \left[ B(1 - 2\Delta_g(q_T,m_h)) + \alpha(NLO) S \int d\Phi r_R \right]
\]

$\bar{\mu}_R = (m_h q_T)^{1/3}$

$\alpha(S) = \frac{1}{2\pi} \left[ A f \log m_h q_T^2 + B f \right]$

$\Delta_g(q_T,m_h) = -\int m_h q_T^2 dq_T^2 \alpha(S) (q_T^2)\frac{1}{\pi}$

$\mu_F = q_T$ (Sudakov FF included on $H + j$ Born kinematics)

$\bar{H} J$-MiNLO yields finite results also when 1st jet is unresolved ($q_T \to 0$)

\[\text{¯B}_{\text{MiNLO}} \text{ideal to extend validity of } H + j \text{POWHEG}\]
2. **integrate** over phase space regions where $H$ is produced with **arbitrarily soft/collinear jet** (i.e. finite results when integrating over all $q_T$ spectrum)

**MiNLO: Multiscale Improved NLO** [Hamilton,Nason,Zanderighi, 1206.3572]

- original goal: method to **a-priori** choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy)
- how: correct weights of different NLO terms with CKKW-inspired approach:
  - for all PS points, build the “more-likely” shower history that would have produced it (can be done by clustering kinematics with $k_T$-algo)
  - correct original NLO including $\alpha_S$ couplings evaluated at nodal scales and Sudakov FFs
  - make sure that **NLO accuracy is not spoiled**!
2. integrate over phase space regions where $H$ is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all $q_T$ spectrum)

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$$\bar{B}_{\text{NLO}} = \alpha_3^3(\mu_R) \left[ B + \alpha_{\text{S}}^{(\text{NLO})} V(\mu_R) + \alpha_{\text{S}}^{(\text{NLO})} \int d\Phi R \right]$$

![Diagram of MiNLO](image-url)
2. integrate over phase space regions where \( H \) is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all \( q_T \) spectrum)

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\tilde{B}_{NLO} = \alpha_S^3(\mu_R) \left[ B + \alpha_S^{(NLO)} V(\mu_R) + \alpha_S^{(NLO)} \int d\Phi_r R \right]
\]

\[
\tilde{B}_{MiNLO} = \alpha_s^2(m_h) \alpha_s(q_T) \Delta_g^2(q_T, m_h) \left[ B \left( 1 - 2\Delta_g^{(1)}(q_T, m_h) \right) + \alpha_s^{(NLO)} V(\bar{\mu}_R) + \alpha_s^{(NLO)} \int d\Phi_r R \right]
\]

\* \( \bar{\mu}_R = (m_h^2 q_T)^{1/3} \)

\* \( \log \Delta_f(q_T, m_h) = -\int_{q_T^2}^{m_h^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \left[ A_f \log \frac{m_h^2}{q^2} + B_f \right] \)

\* \( \Delta_f^{(1)}(q_T, m_h) = -\frac{\alpha_S^{(NLO)}}{2\pi} \left[ \frac{1}{2} A_{1,f} \log^2 \frac{m_h^2}{q_T^2} + B_{1,f} \log \frac{m_h^2}{q_T^2} \right] \)

\* \( \mu_F = q_T \)
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- original goal: method to *a-priori* choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy)
- how: correct weights of different NLO terms with CKKW-inspired approach:

$$\bar{B}_{NLO} = \alpha_S^3(\mu_R) \left[ B + \alpha_S^{(NLO)} V(\mu_R) + \alpha_S^{(NLO)} \int d\Phi_R R \right]$$

$$\bar{B}_{MiNLO} = \alpha_S^2(m_h) \alpha_S(q_T) \Delta^2 g(q_T, m_h) \left[ B \left( 1 - 2\Delta_g^{(1)}(q_T, m_h) \right) + \alpha_S^{(NLO)} V(\bar{\mu}_R) + \alpha_S^{(NLO)} \int d\Phi_R R \right]$$

- $\bar{B}_{MiNLO}$ yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{MiNLO}$ ideal to extend validity of $H+j$ POWHEG
"Improved" MiNLO & NLOPS merging

- accuracy of $\text{HJ-MiNLO}$ for inclusive observables carefully investigated
  
  [Hamilton, Nason, Oleari, Zanderighi, 1212.4504]

- $\text{HJ-MiNLO}$ describes inclusive observables at order $\alpha_S$ (relative to inclusive $H$ @ LO)

- to reach genuine NLO when inclusive, “spurious” terms must be of relative order $\alpha_S^2$, i.e.

  $$O_{\text{HJ-MiNLO}} = O_{\text{H@NLO}} + O(\alpha_S^{b+2})$$

  if $O$ is inclusive ($H@LO \sim \alpha_S^b$).

- “Original MiNLO” contains ambiguous $O(\alpha_S^{b+3/2})$ terms.

- Possible to improve $\text{HJ-MiNLO}$ such that $H@NLO$ is recovered ($NLO(0)$), without spoiling NLO accuracy of $H+j(NLO(1))$.

- Proof based on careful comparisons of MiNLO with general resummation formula

- Need to include $B$ in MiNLO-Sudakovs

- Need to evaluate $\alpha_S(NLO)$ in HJ-MiNLO at scale $q_T$, and $\mu_F = q_T$

- Effectively as if we merged NLO(0) and NLO(1) samples, without merging different samples (no merging scale used: there is just one sample).

- Other NLOPS-merging approaches:
  - [Hoeche, Krauss, et al., 1207.5030]
  - [Frederix, Frixione, 1209.6215]
  - [Lonnblad, Prestel, 1211.7278 - Platzer, 1211.5467]
  - [Alioli, Bauer, et al., 1211.7049]
  - [Hartgring, Laenen, Skands, 1303.4974]
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  - proof based on careful comparisons of MiNLO with general resummation formula
  - need to include $B_2$ in MiNLO-Sudakovs
  - need to evaluate $\alpha_S^{(\text{NLO})}$ in HJ-MiNLO at scale $q_T$, and $\mu_F = q_T$

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**HJ-MiNLO** differential cross section \((d\sigma/dy)_{HJ-MiNLO}\) is NLO accurate

\[
W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\text{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\text{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)
\]

thus, reweighting each event with this factor, we get NNLO+PS

- obvious for \(y_H\), by construction
- \(\alpha_S^4\) accuracy of **HJ-MiNLO** in 1-jet region not spoiled, because \(W(y) = 1 + \mathcal{O}(\alpha_S^2)\)
- if we had NLO\(^{(0)}\) + \(\mathcal{O}(\alpha_S^{2+3/2})\), 1-jet region spoiled because

\[
[NLO^{(1)}]_{\text{NNLOPS}} = \text{NLO}^{(1)} + \mathcal{O}(\alpha_S^{4.5})
\]

---

\* Variants for \(W\) are possible:

\[
W(y,p_T) = h(p_T) \int d\sigma_{\text{NNLO}} A \delta(y - y(\Phi)) \int d\sigma_{\text{MiNLO}} A \delta(y - y(\Phi)) + (1 - h(p_T)) d\sigma_A = d\sigma_{h(p_T)}, d\sigma_B = d\sigma(1 - h(p_T)), h = \left(\frac{\beta m}{\beta m_H}\right)^2 (\frac{\beta m}{\beta m_H})^2 + p_T^2\]

\* \(\beta\) cannot be too small, otherwise resummation spoiled

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HJ–MiNLO* differential cross section \( \frac{d\sigma}{dy}_{\text{HJ–MiNLO}} \) is NLO accurate

\[
W(y) = \left( \frac{d\sigma}{dy} \right)_{\text{NNLO}} \div \left( \frac{d\sigma}{dy} \right)_{\text{HJ–MiNLO}} = \frac{c_2 \alpha_S^2 + c_3 \alpha_S^3 + c_4 \alpha_S^4}{c_2 \alpha_S^2 + c_3 \alpha_S^3 + d_4 \alpha_S^4} \approx 1 + \frac{c_4 - d_4}{c_2} \alpha_S^2 + \mathcal{O}(\alpha_S^3)
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[\text{NLO}\(^{(1)}\)]_{\text{NNLOPS}} = \text{NLO}\(^{(1)}\) + \mathcal{O}(\alpha_S^{4.5})
\]

* Variants for \( W \) are possible:

\[
W(y, p_T) = h(p_T) \int d\sigma_A^{\text{NNLO}} \delta(y - y(\Phi)) + (1 - h(p_T)) \int d\sigma_A^{\text{MiNLO}} \delta(y - y(\Phi))
\]

\[
d\sigma_A = d\sigma \ h(p_T), \quad d\sigma_B = d\sigma \ (1 - h(p_T)), \quad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}
\]

* \( h(p_T) \) controls where the NNLO/NLO K-factor is spread
* \( \beta \) cannot be too small, otherwise resummation spoiled
In 1309.0017 we used

\[
W(y, p_T) = h(p_T) \left( \frac{\int d\sigma^{\text{NNLO}} \delta(y - y(\Phi)) - \int d\sigma^{\text{MiNLO}}_B \delta(y - y(\Phi))}{\int d\sigma^{\text{MiNLO}}_A \delta(y - y(\Phi))} + (1 - h(p_T)) \right)
\]

\[
d\sigma_A = d\sigma h(p_T), \quad d\sigma_B = d\sigma (1 - h(p_T)), \quad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}
\]

- one gets exactly \((d\sigma/dy)_{\text{NNLOPS}} = (d\sigma/dy)_{\text{NNLO}}\) (no \(\alpha_S^5\) terms)
- we used \(h(p_T^{j_1})\)

inputs for following plots:
- results are for 8 TeV LHC
- scale choices: NNLO input with \(\mu = m_H/2\), HJ-MiNLO “core scale” \(m_H\) (other powers are at \(q_T\))
- PDF: everywhere MSTW2008 NNLO
- NNLO always from HNNLO
- 6M events reweighted at the LH level
- plots after \(k_T\)-ordered PYTHIA 6 at the PS level (hadronization and MPI switched off)
- **NNLO** with $\mu = m_H/2$, HJ-MiNLO “core scale” $m_H$
- $(7_{\text{Mi}} \times 3_{\text{NN}})$ pts scale var. in NNLOPS, 7pts in NNLO

[NNLO from HNNLO, Catani,Grazzini]

Notice: band is 10% (at NLO would be $\sim$ 20-30%)

[Until and including $O(\alpha_S^4)$, PS effects don’t affect $y_H$ (first 2 emissions controlled properly at $O(\alpha_S^4)$ by MiNLO+POWHEG)]
\( \beta = \infty \) (W indep. of \( p_T \))

\[ \beta = 1/2 \]

- **HqT**: NNLL+NNLO, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{\text{res}} \equiv m_H/2 \) [HqT, Bozzi et al.]
- \( \beta = 1/2 \) & \( \infty \): uncertainty bands of HqT contain NNLOPS at low-/moderate \( p_T \)
- \( \beta = 1/2 \): HqT tail harder than NNLOPS tail (\( \mu_{\text{HqT}} < "\mu_{\text{MiNLO}}" \))
- \( \beta = 1/2 \): very good agreement with HqT resummation ["\( \sim \) expected", since \( Q_{\text{res}} \equiv m_H/2 \)]
\[ \beta = \infty \text{ (W indep. of } p_T) \]

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**HqT**: NNLL+NNLO, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{\text{res}} \equiv m_H/2 \)

**\( \beta = 1/2 \)**: NNLOPS tail \( \rightarrow \) NLOPS tail [ \( W(y, p_T \gg m_H) \rightarrow 1 \) ]

larger band (affected just marginally by NNLO, so it’s \( \sim \) genuine NLO band)
\( \varepsilon(p_T, \text{veto}) = \frac{\Sigma(p_T, \text{veto})}{\sigma_{\text{tot}}} = \frac{1}{\sigma_{\text{tot}}} \int d\sigma \theta(p_T, \text{veto} - p_T^{j_1}) \)

- **JetVHeto**: NNLL resum, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{\text{res}} \equiv m_H/2 \), (a)-scheme only

- nice agreement, differences never more than 5-6 %

Separation of \( H \rightarrow WW \) from \( t\bar{t} \) bkg: x-sec binned in \( N_{\text{jet}} \)

0-jet bin \( \Leftrightarrow \) jet-veto accurate predictions needed!
Conclusions

**NNLOPS:**
- **MiNLO-improved POWHEG** simulation allows to define a procedure to reach NNLOPS
- shown first results for Higgs production
- the code is public and can be found in the **POWHEG-BOX (V2)** repository

*with this formalism NNLOPS doable for DY and $H+V$*

---

**MiNLO & improved MiNLO:**
- original motivation: assign scales and Sudakov FF in $B+n$ jets NLO computations
- ideal as starting point for **POWHEG**
- have shown results where MiNLO used on top of $pp \rightarrow H+j$. However procedure is more general (and indeed has been already used also on more complex cases)
- $B+j$ “improved” **MiNLO** allows to merge NLO$^{(0)}$ and NLO$^{(1)}$ samples, without the need of a merging scale
- merging for higher multiplicity requires further study
Conclusions

- **NNLOPS:**
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  merging for higher multiplicity requires further study

Thank you for your attention!
CKKW in a nutshell

- ME weight $B(\Phi_n) \Rightarrow \text{“most-likely” shower history (via } k_T\text{-algo): } Q > q_3 > q_2 > q_1 \equiv Q_0$

\[ \alpha_5^5(Q) B(\Phi_3) \rightarrow \alpha_2^2(Q) B(\Phi_3) \frac{\Delta g(Q_0, Q)}{\Delta g(Q_0, q_2)} \frac{\Delta g(Q_0, Q)}{\Delta g(Q_0, q_3)} \frac{\Delta g(Q_0, q_3)}{\Delta g(Q_0, q_1)} \]

\[ \Delta g(Q_0, q_2) \Delta g(Q_0, q_2) \Delta g(Q_0, q_3) \Delta g(Q_0, q_3) \Delta g(Q_0, q_1) \Delta g(Q_0, q_1) \]

\[ \alpha_S(q_1) \alpha_S(q_2) \alpha_S(q_3) \]

where typically

\[ \log \Delta_f(q_T, Q) = - \int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \alpha_S(q^2) \left[ A_{1,f} \log \frac{Q^2}{q^2} + B_{1,f} \right] \]

- Fill phase space below $Q_0$ with vetoed shower
Find “most-likely” shower history (via $k_T$-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$
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From CKKW to MiNLO

- Find “most-likely” shower history (via $k_T$-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$

- Evaluate $\alpha_S$ at nodal scales

$$\alpha_S^n(\mu_R) B(\Phi_n) \Rightarrow \alpha_S(q_1) \alpha_S(q_2) \ldots \alpha_S(q_n) B(\Phi_n)$$

* scale compensation requires $\mu^2_R = (q_1 q_2 \ldots q_n)^{2/n}$ in $V$
From CKKW to MiNLO

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* scale compensation requires $\mu^2_R = (q_1q_2\ldots q_n)^2/n$ in $V$

- Sudakov FFs in internal and external lines of Born “skeleton”

\[ B(\Phi_n) \Rightarrow B(\Phi_n) \times \{\Delta(Q_0,Q)\Delta(Q_0,q_i)\ldots\} \]

* Upon expansion, $O(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

\[ B(\Phi_n) \Rightarrow B(\Phi_n)\left(1 - \Delta^{(1)}(Q_0,Q) - \Delta^{(1)}(Q_0,q_i) + \ldots\right) \]
From CKKW to MiNLO

- Find “most-likely” shower history (via $k_T$-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$

- Evaluate $\alpha_S$ at nodal scales

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\alpha_S^n(\mu_R) B(\Phi_n) \Rightarrow \alpha_S(q_1) \alpha_S(q_2) \cdots \alpha_S(q_n) B(\Phi_n)
\]

* scale compensation requires $\bar{\mu}_R^2 = (q_1 q_2 \cdots q_n)^2/n$ in $V$

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B(\Phi_n) \Rightarrow B(\Phi_n) \times \{ \Delta(Q_0, Q) \Delta(Q_0, q_i) \cdots \}
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B(\Phi_n) \Rightarrow B(\Phi_n) \left( 1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + \ldots \right)
\]

- $X + \text{jets cross-section finite without generation cuts}$
  $\leftarrow \bar{B}$ with MiNLO prescription: ideal starting point for NLOPS (POWHEG) for $X + \text{jets}$
MiNLO details

MiNLO: All $\alpha_S$ in Born term are chosen with CKKW (local) scales $q_1, \ldots, q_n$

$$\alpha^n_S(\mu_R)B \Rightarrow \alpha_S(q_1)\alpha_S(q_2)\ldots\alpha_S(q_n)B$$

- Normal NLO structure ($\mu = \mu_R$):
  $$\sigma(\mu) = \underbrace{\alpha^n_S(\mu)B}_{\text{Born}} + \underbrace{\alpha^{n+1}_S(\mu)(C + nb_0 \log(\mu^2/Q^2)B)}_{\text{Virtual}} + \underbrace{\alpha^{n+1}_S(\mu)R}_{\text{Real}}$$

- Explicit $\mu$ dependence of virtual term as required by RG invariance:
  $$\alpha^n_S(\mu')B = \left[\alpha_S(\mu) - nb_0\alpha^{n+1}_S(\mu)\log(\mu'^2/\mu^2)\right]B + \mathcal{O}(\alpha^{n+2}_S)$$
  $$\Rightarrow \sigma(\mu') - \sigma(\mu) = \mathcal{O}(\alpha^{n+2}_S)$$

- In MiNLO “scale compensation” kept if

$$\left(C + nb_0 \log(\mu^2_R/Q^2)B\right) \Rightarrow \left(C + nb_0 \log(\bar{\mu}^2_R/Q^2)B\right)$$

with $\bar{\mu}^2_R = (q_1q_2\ldots q_n)^2/n$
“Improved” MiNLO & NLOPS merging

- Resummation formula

\[
\frac{d\sigma}{dq_T^2\,dy} = \sigma_0 \frac{d}{dq_T^2}\left\{ [C_{ga} \otimes f_a](x_A, q_T) \times [C_{gb} \otimes f_b](x_B, q_T) \times \exp S(q_T,Q) \right\} + R_f \\
S(q_T,Q) = -2 \int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \left[ A_f \log \frac{Q^2}{q^2} + B_f \right]
\]

- If \(C_{ij}^{(1)}\) included and \(R_f\) is LO\(^{(1)}\), then upon integration we get NLO\(^{(0)}\):

Take derivative, then compare with MiNLO:

\[
\sim \sigma_0 \frac{1}{q_T^2} [\alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L] \exp S(q_T,Q) + R_f \\
L = \log(Q^2/q_T^2)
\]

- highlight terms are needed to reach NLO\(^{(0)}\):

\[
\int^{Q^2} d q_T^2 \frac{L^m \alpha_S^n (q_T) \exp S}{q_T^2} \sim \left( \alpha_S(Q^2) \right)^{n-(m+1)/2}
\]

- if I don’t include \(B_2\) in MiNLO \(\Delta_g\), I miss a term \((1/q_T^2)\alpha_S^2 B_2 \exp S\)

- upon integration, violate NLO\(^{(0)}\) by a term of relative \(O(\alpha_S^{3/2})\)

- “wrong” scale in \(\alpha_S^{(\text{NLO})}\) in MiNLO produces again same error

Alternative proof also available in the paper.
MiNLO details

Few technicalities for original MiNLO:
- \( \mu_F = Q_0 \) (as in CKKW)
- Cluster with CKKW also \( V \) and \( R \) kinematics
  - Actual implementation uses FKS mapping for first cluster of \( \Phi_{n+1} \)
  - Ignore CKKW Sudakov for 1\(^{st}\) clustering of \( \Phi_{n+1} \) (inclusive on extra radiation)
- Some freedom in choice of \( \alpha_s^{(\text{NLO})} \) (entering \( V, R \) and \( \Delta^{(1)} \)):
  * suggested average of LO \( \alpha_s \)
  * not free for “improved” MiNLO
- Used full NLL-improved Sudakovs (\( A_1, B_1, A_2 \))

Improved MiNLO: where are terms coming from when differentiating resum. formula?
- \( 1/q_T^2 \), always from integration in Sudakov
- \( \alpha_s \) from \( C^{(0)} \times B_1, ... \)
- \( \alpha_s^2 \) from \( C^{(0)} \times B_2, ... \)
- ... 
- \( \alpha_s L \) from \( A_1 \) term in exponent
- \( \alpha_s L^2 \) from \( A_2 \) term in exponent
- ...
$p_T^H$ spectrum:

- $\mu_{\text{HJ-MiNLO}} = m_H, m_H, p_T$
- At high $p_T$, $\mu_{\text{HJ-MiNLO}} = p_T$
- If $\beta = 1/2$, NNLOPS $\rightarrow$ HJ-MiNLO at high $p_T$
- NNLO/NLO $\sim 1.5$, because HNNLO with $\mu = m_H/2$, $\mu_{\text{HJ-MiNLO,core}} = m_H$